Equivalence of DFA and Regular Expressions

CSCI 3130 Formal Languages and Automata Theory

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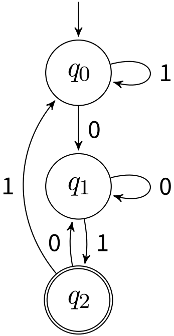
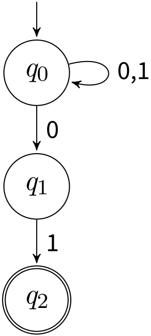
Three ways of doing it

$L = \{x \in \Sigma^* \mid x \text{ ends in } 01\}$ $\Sigma = \{0, 1\}$



DFA

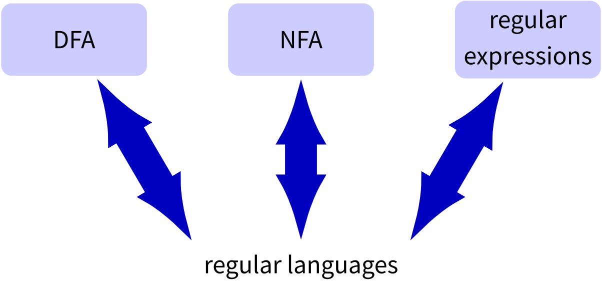


NFA

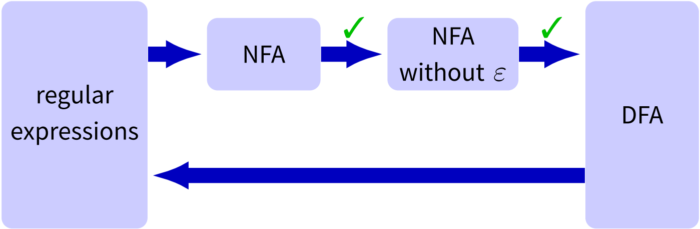
$(0 + 1)^*01$

regular expressions

They are equally powerful

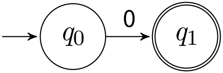


Roadmap

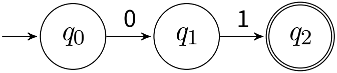


Examples: regular expression $\longrightarrow$ NFA

$R_1 = \mathsf{0}$

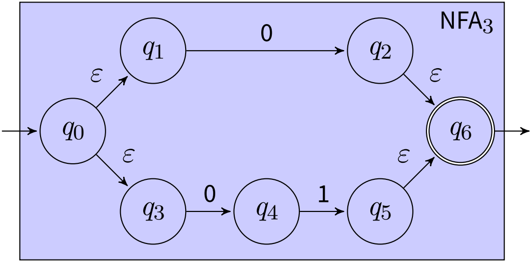


$R_2 = \mathsf{01}$

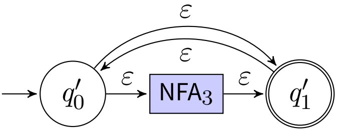


Examples: regular expression → NFA

$R_3 = 0 + 01$



$R_4 = (0 + 01)^*$



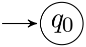
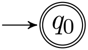
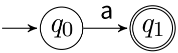
Regular expressions

In general, how do we convert a regular expression to an NFA?

A regular expression over Σ is an expression formed by the following rules

- The symbols $\emptyset$ and ε are regular expressions
- Every a in Σ is a regular expression
- If R asd S are regular expressions, so are $R + S$, RS and R^*

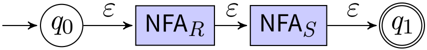
General method

regular expression	$\Longrightarrow$ NFA
$\emptyset$	
ε	
a $\in \Sigma$	

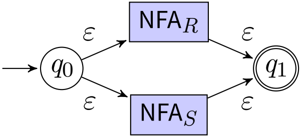
General method

regular expression $\Longrightarrow$ NFA

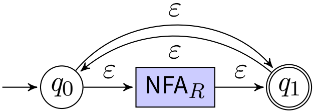
RS



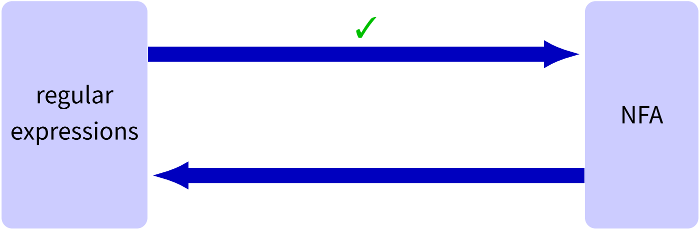
$R + S$



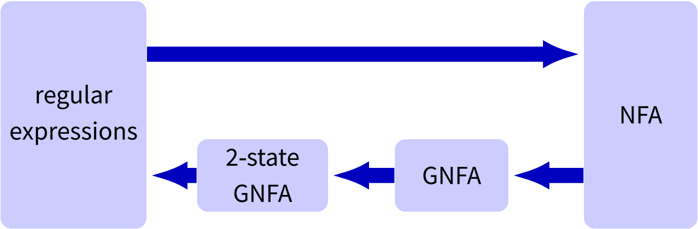
R^*



Roadmap



Roadmap



regular expressions

NFA

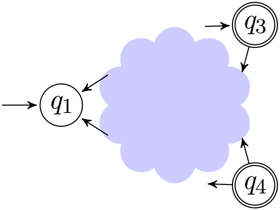
GNFA

2-state GNFA

Simplify the NFA

First we simplify the NFA so that

- It has exactly one accepting state
- No arrows come into the start state
- No arrows go out of the accepting state



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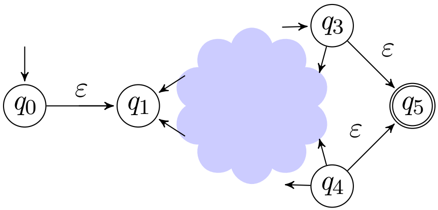
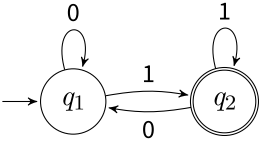
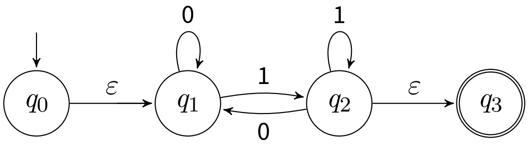


Simplify the NFA



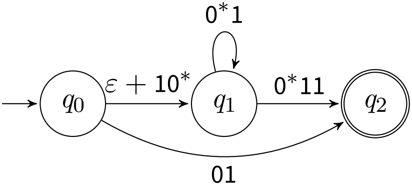
Simplify the NFA



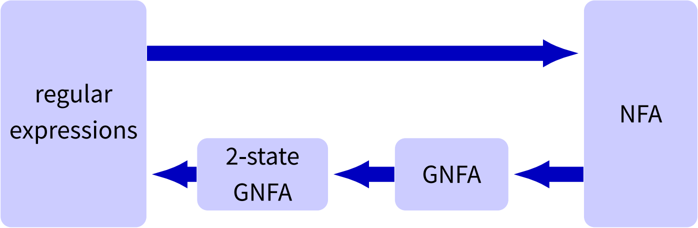
- It has exactly one accepting state ✓
- No arrows come into the start state ✓
- No arrows go out of the accepting state ✓

Generalized NFAs

A generalized NFA is an NFA whose transitions are labeled by regular expressions, like

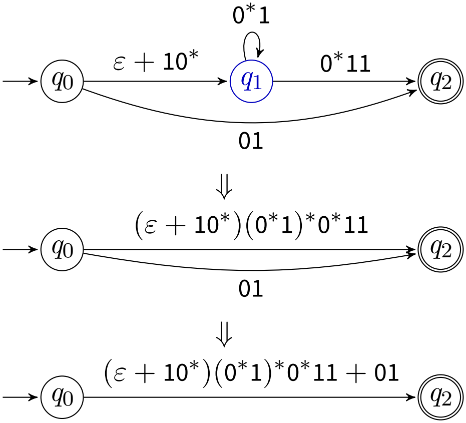


GNFA state elimination



We will eliminate every state but the start and accepting states

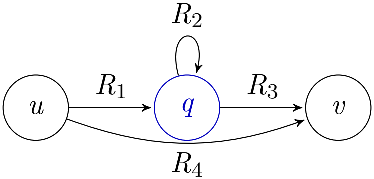
State elimination



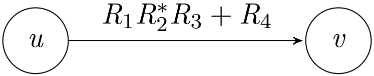
State elimination: general method

To eliminate state q, for every pair of states (u, v)

Replace

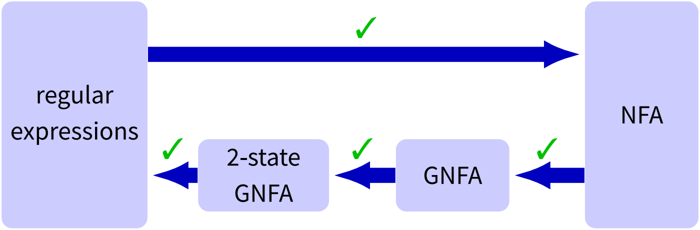


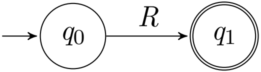
by



Remember to do this even when $u = v$

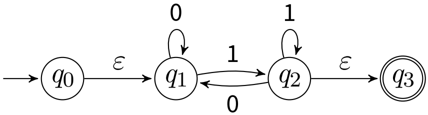
Roadmap





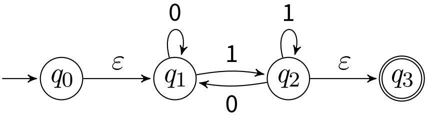
A 2-state GNFA is the same as a regular expression R

Conversion example

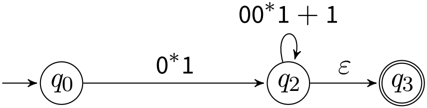


Eliminate q_1:

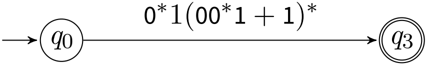
Conversion example



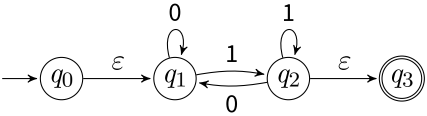
Eliminate q_1:



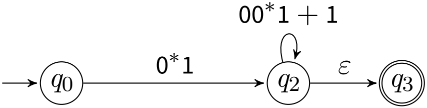
Eliminate q_2:



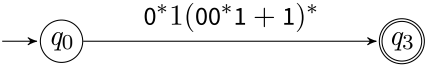
Conversion example



Eliminate q_1:

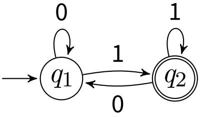


Eliminate q_2:

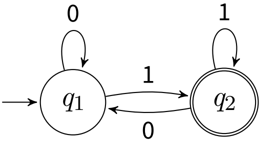


Check:

$0^*1(00^*1 + 1)^* \overset{?}{=}$



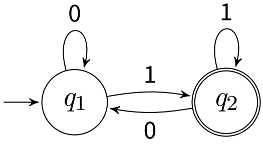
Check your answer!



All strings ending in 1

$(0+1)^*1$

Check your answer!



All strings ending in 1
$(0 + 1)^*1$

$0^*1(00^*1 + 1)^*$

Always ends in 1

$= 0^*1(0^*1)^*$

Does every string ending in 1 have this form?

Yes