NFA to DFA conversion and regular expressions

CSCI 3130 Formal Languages and Automata Theory

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DFAs and NFAS are equally powerful

NFA can do everything a DFA can do

How about the other way?

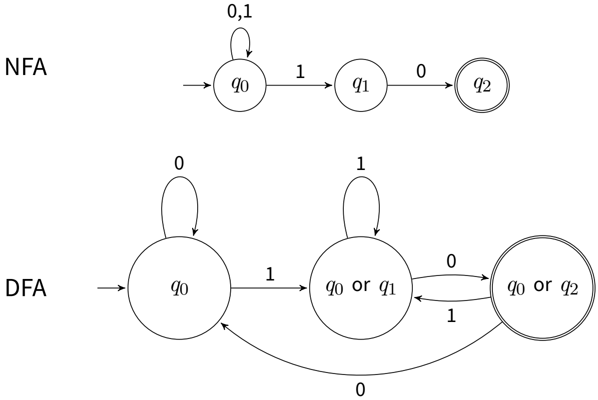
Every NFA can be converted into a DFA for the same language

NFA $\longrightarrow$ DFA in two easy steps

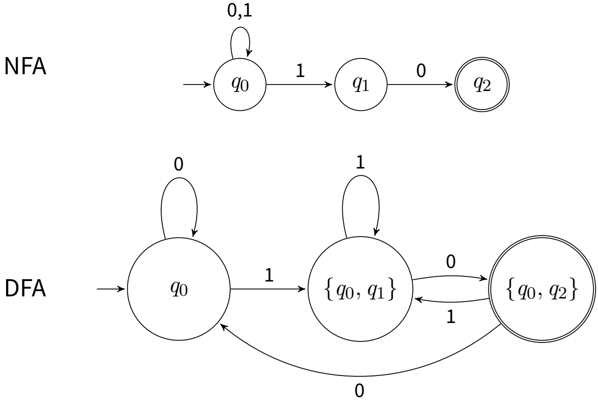
1. Eliminate ε-transitions
2. Convert simplified NFA to DFA

   We will do this first

NFA $\longrightarrow$ DFA: intuition



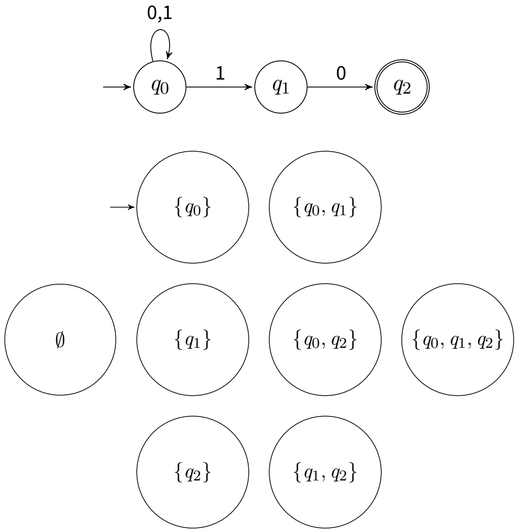
NFA $\longrightarrow$ DFA: intuition



NFA $\longrightarrow$ DFA: states

NFA

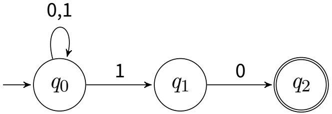
DFA



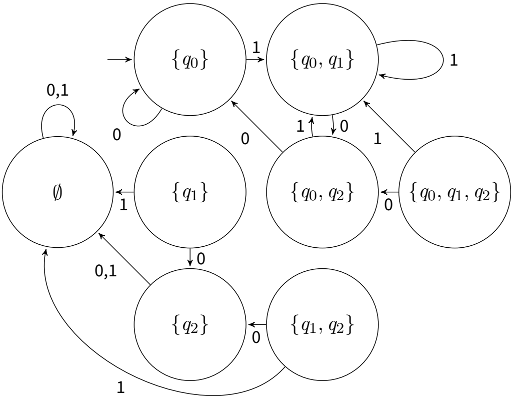
DFA has a state for every subset of NFA states

NFA $\longrightarrow$ DFA: transitions

NFA



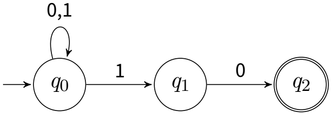
DFA



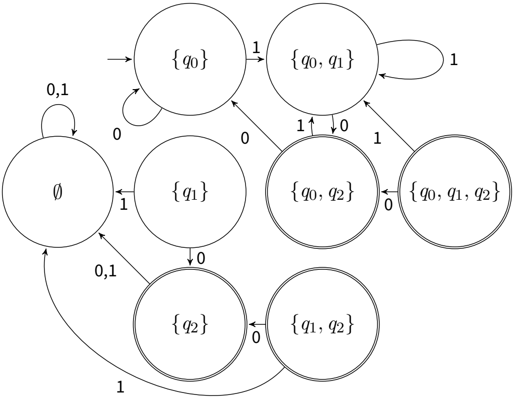
DFA has a state for every subset of NFA states

NFA $\longrightarrow$ DFA: accepting states

NFA



DFA

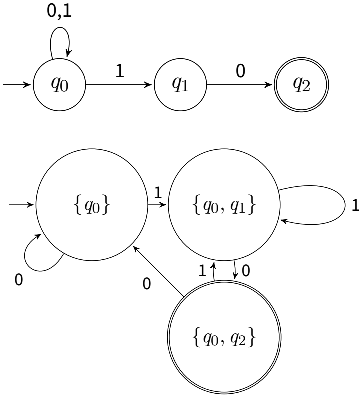


DFA accepts if it contains an NFA accepting state

NFA $\longrightarrow$ DFA: eliminate unreachable states

NFA

DFA



At the end, you may eliminate unreachable states

General conversion

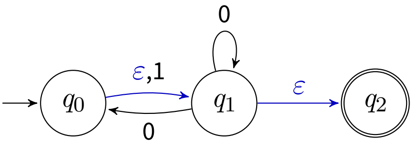
	NFA	DFA
states	$q_0, q_1, \ldots, q_n$	$\emptyset, \{q_0\}, \{q_1\}, \{q_0, q_1\}, \ldots,$ $\{q_0, \ldots, q_n\}$ one for each subset of states
initial state	q_0	$\{q_0\}$
transitions	δ	$\delta'(\{q_{i_1}, \ldots, q_{i_k}\}, a) =$ $\delta(q_{i_1}, a) \cup \cdots \cup \delta(q_{i_k}, a)$
accepting states	$F \subseteq Q$	$F' = \{S \mid S$ contains some state in $F\}$

NFA $\longrightarrow$ DFA in two easy steps

1. Eliminate ε-transitions
2. Convert simplified NFA to DFA ✓

Eliminating ε-transitions

NFA:



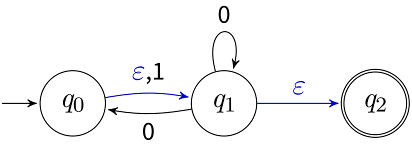
NFA without ε's:

	0	1
q_0	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$
q_1	$\{q_0, q_1, q_2\}$	$\emptyset$
q_2	$\emptyset$	$\emptyset$

Accepting states: q_2, q_1, q_0

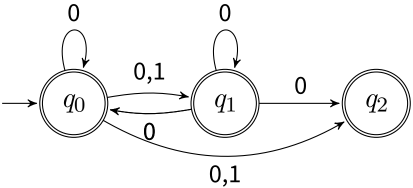
Eliminating ε-transitions

NFA:



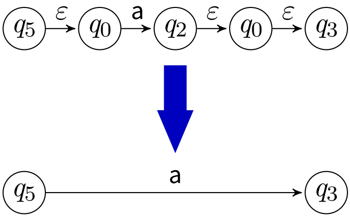
new NFA:

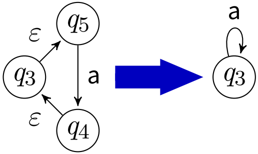
	0	1
q_0	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$
q_1	$\{q_0, q_1, q_2\}$	$\emptyset$
q_2	$\emptyset$	$\emptyset$



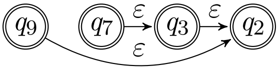
Eliminating ε-transitions

Paths with ε's are replaced with a single transition





States that can reach accepting state by ε are all accepting



Regular expressions

String concatenation

$$s = \mathsf{abb} \qquad t = \mathsf{bab}$$

$$st = \mathsf{abbbab}$$
$$ts = \mathsf{bababb}$$
$$ss = \mathsf{abbabb}$$
$$sst = \mathsf{abbabbbab}$$

$$s = x_1 \ldots x_n, \quad t = y_1 \ldots y_m$$
$$\Downarrow$$
$$st = x_1 \ldots x_n y_1 \ldots y_m$$

Operations on languages

- Concantenation of languages L_1 and L_2

$$L_1 L_2 = \{st : s \in L_1, t \in L_2\}$$

- n-th power of language L

$$L^n = \{s_1 s_2 \dots s_n \mid s_1, s_2, \dots, s_n \in L\}$$

- Union of L_1 and L_2

$$L_1 \cup L_2 = \{s \mid s \in L_1 \text{ or } s \in L_2\}$$

Example

$L_1 = \{0, 01\}$ $\qquad$ $L_2 = \{\varepsilon, 1, 11, 111, \ldots\}$

$$\begin{aligned} L_1 L_2 &= \{0, 01, 011, 0111, \ldots\} \cup \{01, 011, 0111, 01111, \ldots\} \\ &= \{0, 01, 011, 0111, \ldots\} \end{aligned}$$

0 followed by any number of 1s

$L_1^2 = \{00, 001, 010, 0101\}$

$L_2^2 = L_2$

$L_2^n = L_2$ for any $n \geqslant 1$

$$L_1 \cup L_2 = \{0, 01, \varepsilon, 1, 11, 111, \ldots\}$$

Operations on languages

The star of L are contains strings made up of zero or more chunks from L

$$L^* = L^0 \cup L^1 \cup L^2 \cup \ldots$$

Example: $L_1 = \{0, 01\}$ and $L_2 = \{\varepsilon, 1, 11, 111, \ldots\}$
What is L_1^*? L_2^*?

Example

$$L_1 = \{0, 01\}$$

$$L_1^0 = \{\varepsilon\}$$
$$L_1^1 = \{0, 01\}$$
$$L_1^2 = \{00, 001, 010, 0101\}$$
$$L_1^3 = \{000, 0001, 0010, 00101, 0100, 01001, 01010, 010101\}$$

Which of the following are in L_1^*?

00100001 00110001 10010001

Example

$$L_1 = \{0, 01\}$$

$$L_1^0 = \{\varepsilon\}$$
$$L_1^1 = \{0, 01\}$$
$$L_1^2 = \{00, 001, 010, 0101\}$$
$$L_1^3 = \{000, 0001, 0010, 00101, 0100, 01001, 01010, 010101\}$$

Which of the following are in L_1^*?

00100001	00110001	10010001
Yes	No	No

Example

$$L_1 = \{\mathsf{0}, \mathsf{01}\}$$

$$L_1^0 = \{\varepsilon\}$$
$$L_1^1 = \{\mathsf{0}, \mathsf{01}\}$$
$$L_1^2 = \{\mathsf{00}, \mathsf{001}, \mathsf{010}, \mathsf{0101}\}$$
$$L_1^3 = \{\mathsf{000}, \mathsf{0001}, \mathsf{0010}, \mathsf{00101}, \mathsf{0100}, \mathsf{01001}, \mathsf{01010}, \mathsf{010101}\}$$

Which of the following are in L_1^*?

00100001	00110001	10010001
Yes	No	No

L_1^* contains all strings such that any 1 is preceded by a 0

Example

$$\boxed{L_2 = \{\varepsilon, 1, 11, 111, \dots\}}$$

any number of 1s

$$L_2^0 = \{\varepsilon\}$$
$$L_2^1 = L_2$$
$$L_2^2 = L_2$$
$$L_2^n = L_2 \quad (n \geqslant 1)$$

Example

$$L_2 = \{\varepsilon, 1, 11, 111, \dots\}$$

any number of 1s

$$\begin{aligned} L_2^0 &= \{\varepsilon\} \\ L_2^1 &= L_2 \\ L_2^2 &= L_2 \\ L_2^n &= L_2 \quad (n \geqslant 1) \end{aligned}$$

$$\begin{aligned} L_2^* &= L_2^0 \cup L_2^1 \cup L_2^2 \cup \dots \\ &= \{\varepsilon\} \cup L_2 \cup L_2 \cup \dots \\ &= L_2 \end{aligned}$$

$$L_2^* = L_2$$

Combining languages

We can construct languages by starting with simple ones, like $\{0\}$ and $\{1\}$, and combining them

$\{0\}(\{0\} \cup \{1\})^*$ $\Rightarrow$ $0(0+1)^*$
all strings that start with 0

Combining languages

We can construct languages by starting with simple ones, like $\{0\}$ and $\{1\}$, and combining them

$\{0\}(\{0\} \cup \{1\})^* \quad \Rightarrow \quad 0(0+1)^*$

all strings that start with 0

$(\{0\}\{1\}^*) \cup (\{1\}\{0\}^*) \quad \Rightarrow \quad 01^* + 10^*$

0 followed by any number of 1s, or

1 followed by any number of 0s

Combining languages

We can construct languages by starting with simple ones, like $\{0\}$ and $\{1\}$, and combining them

$\{0\}(\{0\} \cup \{1\})^*$ $\Rightarrow$ $0(0+1)^*$
all strings that start with 0

$(\{0\}\{1\}^*) \cup (\{1\}\{0\}^*)$ $\Rightarrow$ $01^* + 10^*$
0 followed by any number of 1s, or
1 followed by any number of 0s

$0(0+1)^*$ and $01^* + 10^*$ are regular expressions
Blueprints for combining simpler languages into complex ones

Syntax of regular expressions

A regular expression over Σ is an expression formed by the following rules

- The symbols $\emptyset$ and ε are regular expressions
- Every a in Σ is a regular expression
- If R asd S are regular expressions, so are $R + S$, RS and R^*

Examples:

$$\emptyset \qquad \varepsilon$$

$$0(0+1)^* \qquad 1^*(\varepsilon+0)$$

$$01^* + 10^* \qquad (0+1)^*01(0+1)^*$$

A language is regular if it is represented by a regular expression

Understanding regular expressions

$$\Sigma = \{0, 1\}$$

$01^* = 0(1)^*$ represents $\{0, 01, 011, 0111, \ldots\}$
0 followed by any number of 1s

01^* is not $(01)^*$

Understanding regular expressions

$0 + 1$ yields $\{0, 1\}$ — strings of length 1

$(0 + 1)^*$ yields $\{\varepsilon, 0, 1, 00, 01, 10, 11, \ldots\}$ — any string

$(0 + 1)^*010$ — any string that ends in 010

$(0 + 1)^*01(0 + 1)^*$ — any string containing 01

Understanding regular expressions

What is the following language?

$$((0+1)(0+1))^* + ((0+1)(0+1)(0+1))^*$$

Understanding regular expressions

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Understanding regular expressions

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Understanding regular expressions

What is the following language?

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$((0+1)(0+1))^*$

$((0+1)(0+1)(0+1))^*$

$(0+1)(0+1)$

strings of length 2

$(0+1)(0+1)(0+1)$

strings of length 3

Understanding regular expressions

What is the following language?

$$((0+1)(0+1))^* + ((0+1)(0+1)(0+1))^*$$

$((0+1)(0+1))^*$

strings of even length

$((0+1)(0+1)(0+1))^*$

strings whose length is a multiple of 3

$(0+1)(0+1)$

strings of length 2

$(0+1)(0+1)(0+1)$

strings of length 3

Understanding regular expressions

What is the following language?

$$((0+1)(0+1))^* + ((0+1)(0+1)(0+1))^*$$

strings whose length is even or a multiple of 3

= strings of length $0, 2, 3, 4, 6, 8, 9, 10, 12, \ldots$

$$((0+1)(0+1))^*$$

strings of even length

$$((0+1)(0+1)(0+1))^*$$

strings whose length is a multiple of 3

$$(0+1)(0+1)$$

strings of length 2

$$(0+1)(0+1)(0+1)$$

strings of length 3

Understanding regular expressions

What is the following language?

$$((0+1)(0+1)+(0+1)(0+1)(0+1))^*$$

Understanding regular expressions

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Understanding regular expressions

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$$(0+1)(0+1)+(0+1)(0+1)(0+1)$$

$$(0+1)(0+1) \qquad\qquad (0+1)(0+1)(0+1)$$

Understanding regular expressions

What is the following language?

$$((0+1)(0+1)+(0+1)(0+1)(0+1))^*$$

$$(0+1)(0+1)+(0+1)(0+1)(0+1)$$

$(0+1)(0+1)$
strings of length 2

$(0+1)(0+1)(0+1)$
strings of length 3

Understanding regular expressions

What is the following language?

$$((0+1)(0+1)+(0+1)(0+1)(0+1))^*$$

$$(0+1)(0+1)+(0+1)(0+1)(0+1)$$

strings of length 2 or 3

$$(0+1)(0+1)$$

strings of length 2

$$(0+1)(0+1)(0+1)$$

strings of length 3

Understanding regular expressions

What is the following language?

$$((0+1)(0+1)+(0+1)(0+1)(0+1))^*$$

strings that can be broken into blocks, where each block has length 2 or 3

$$(0+1)(0+1)+(0+1)(0+1)(0+1)$$

strings of length 2 or 3

$$(0+1)(0+1)$$

strings of length 2

$$(0+1)(0+1)(0+1)$$

strings of length 3

Understanding regular expressions

What is the following language?

$$((0+1)(0+1)+(0+1)(0+1)(0+1))^*$$

strings that can be broken into blocks, where each block has length 2 or 3

Which are in the language?

ε	1	01	011	00110	011010110

Understanding regular expressions

What is the following language?

$$((0+1)(0+1)+(0+1)(0+1)(0+1))^*$$

strings that can be broken into blocks, where each block has length 2 or 3

Which are in the language?

ε	1	01	011	00110	011010110
✓	✗	✓	✓	✓	✓

The regular expression represents all strings except 0 and 1

Understanding regular expressions

What is the following language?

$$(1 + 01 + 001)^* \, (\varepsilon + 0 + 00)$$

Understanding regular expressions

What is the following language?

$$(1 + 01 + 001)^* \overbrace{(\varepsilon + 0 + 00)}^{\text{ends in at most two 0s}}$$

Understanding regular expressions

What is the following language?

$$\underbrace{(1 + 01 + 001)^*}_{\text{at most two 0s between two consecutive 1s}} \overbrace{(\varepsilon + 0 + 00)}^{\text{ends in at most two 0s}}$$

Never three consecutive 0s

The regular expression represents strings not containing 000

Examples:

ε 00 0110010110 0010010

Writing regular expressions

Write a regular expression for all strings with two consecutive 0s

Writing regular expressions

Write a regular expression for all strings with two consecutive 0s

(anything)00(anything)

$$(0+1)^*00(0+1)^*$$