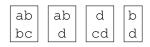Collaborating on homework is encouraged, but you must write your own solutions in your own words and list your collaborators. Copying someone else's solution will be considered plagiarism and may result in failing the whole course.

Please answer clearly and concisely. Explain your answers. Unexplained answers will get lower scores or even no credits.

(1) (12 points) Prove that the following languages are decidable.

    (a) $L_1 = \{\langle R \rangle \mid$ all strings of even length can be generated by $R\}$
        Here, R is a regular expression over alphabet $\{\texttt{a}, \texttt{b}\}$.

    (b) $L_2 = \{\langle D \rangle \mid D$ is a DFA that accepts an infinite number of strings$\}$

    (c) $L_1 = \{\langle G \rangle \mid G$ is a CFG that generates an infinite number of strings$\}$

(2) (12 points) For each of the following languages, say whether it is decidable. Justify your answer.

    (a) $L_1 = \{\langle M \rangle \mid$ Turing machine M accepts all palindromes (and perhaps others)$\}$
        Recall that a string w is a palindrome if it reads the same forward and backward

    (b) $L_2 = \{\langle D, G \rangle \mid D$ is a DFA and G is a CFG such that $L(D) = L(G)\}$

    (c) $L_3 = \{\langle M_1, M_2 \rangle \mid M_1, M_2$ are Turing machines so that $L(M_1) \neq L(M_2)\}$

    (d) $L_4 = \{\langle M, t \rangle \mid$ Turing machine M accepts some input in at most t steps$\}$

(3) (8 points) For each of the following variants of the Post Correspondence Problem (PCP), say if it is decidable or not. Justify your answer by describing a decider, or by reducing from PCP (over arbitrary alphabet).

    (a) PCP_1: PCP over the alphabet $\Sigma = \{\texttt{1}\}$.

    (b) PCP_2: PCP over the alphabet $\Sigma = \{\texttt{0}, \texttt{1}\}$.

The *alphabet* of PCP is the set of symbols that are allowed to appear on the tiles. Here is an instance of PCP over alphabet $\Sigma = \{\texttt{a}, \texttt{b}, \texttt{c}, \texttt{d}\}$:

$$\boxed{\frac{\texttt{ab}}{\texttt{bc}}} \quad \boxed{\frac{\texttt{ab}}{\texttt{d}}} \quad \boxed{\frac{\texttt{d}}{\texttt{cd}}} \quad \boxed{\frac{\texttt{b}}{\texttt{d}}}$$

(4) (8 points) You just got hired by **Doodle**, the hottest software company of the 2010s. Your new boss has several project proposals for you. However, you suspect that some of her proposals may be a bit unrealistic. But you won't turn down a proposal just because you don't like it, or else you might get fired pretty soon. You have to give a reason why you think it is not going to work.

For each of these software projects, say if you think the project is feasible or not. If you think it is feasible, say how you would approach it. If you think it is infeasible, explain why. Your explanation may be about 2-3 sentences.

(a) Doodle just came up with a new programming language called **Plum**. However most of their old programs, and there are many of them, are written in Java. Write an application which converts all their Java programs into Plum.

(b) Doodle solicits applications from developers that it then sells in its Doodle Store. But some of these developers are very clumsy and their applications tend to crash. It would be nice to have a tool that detects these crashing applications so Doodle can take them out of their store. Write a program called **crash detector**, which looks at the code of an application (written in Plum) and figures out if the application will ever crash.

(c) Some programs take a very long time to run. It would be nice to know roughly how long a program is going to run for ahead of time, so if it takes a long time you can go out and get lunch. Write an application called **timer**, that looks at a computer program and gives an estimate of its running time (say within a factor of two, so that if the actual running time is 10 seconds, your program should output 5-20 seconds).