PDA and CFG conversions

CSCI 3130 Formal Languages and Automata Theory

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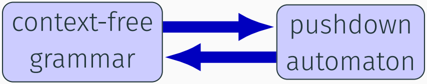
CFGs and PDAs

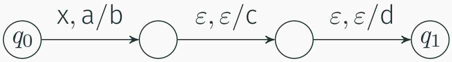
L has a context-free grammar if and only if it is accepted by some pushdown automaton.



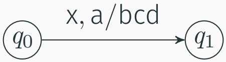
Will first convert CFG to PDA

Convention

A sequence of transitions like



will be abbreviated as



replace a by bcd on stack

Converting a CFG to a PDA

Idea: Use PDA to simulate derivations

$A \to 0A1$

$A \to B$

$B \to \#$

Example:

$A \Rightarrow 0A1 \Rightarrow 00A11 \Rightarrow 00B11 \Rightarrow 00\#11$

Rules:

1. Write the start symbol A onto the stack
2. Rewrite variable on top of stack (in reverse) according to production

PDA control		stack	input
write start variable	$\varepsilon, \varepsilon / A$	$\$A$	00#11
replace by production in reverse	$\varepsilon, A / 1A0$	$\$1A0$	00#11

Converting a CFG to a PDA

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Example:

$A \Rightarrow 0A1 \Rightarrow 00A11 \Rightarrow 00B11 \Rightarrow 00\#11$

Rules:

1. Write the start symbol A onto the stack
2. Rewrite variable on top of stack (in reverse) according to production
3. Pop top terminal if it matches input

PDA control		stack	input
write start variable	$\varepsilon, \varepsilon / A$	$\$A$	00#11
replace by production in reverse	$\varepsilon, A / 1A0$	$\$1A0$	00#11
pop terminal and match	$0, 0 / \varepsilon$	$\$1A$	0#11
replace by production in reverse	$\varepsilon, A / 1A0$	$\$11A0$	0#11

$\vdots$

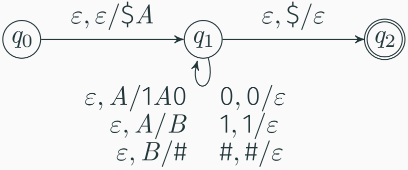
Converting a CFG to a PDA

CFG

$A \to 0A1$

$A \to B$

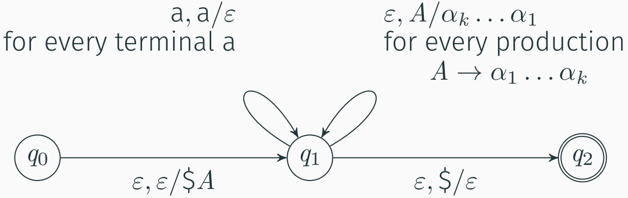
$B \to \#$



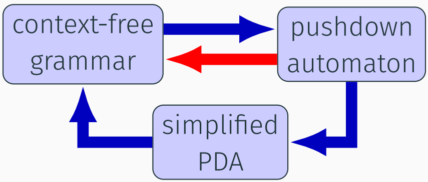
input	00#11	00#11	00#11	00#11	00#11
stack	\$$A$	\$1A0	\$1A	\$11A0	\$11A
input	00#11	00#11	00#11	00#11	00#11
stack	\$11B	\$11#	\$11	\$1	\$

$$A \Rightarrow 0A1 \Rightarrow 00A11 \Rightarrow 00B11 \Rightarrow 00\#11$$

General CFG to PDA conversion



From PDAs to CFGs

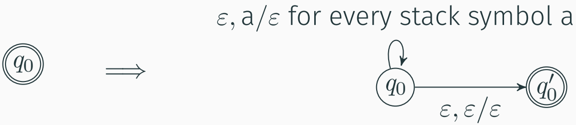


Simplified pushdown automaton:

- Has a single accepting state
- Empties its stack before accepting
- Each transition is either a push, or a pop, but not both

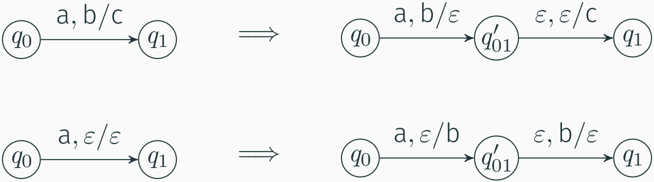
Single accepting state

Empties its stack before accepting



Simplifying the PDA

Each transition either pushes or pops, but not both



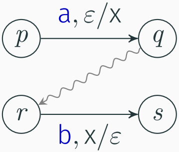
Simplified PDA to CFG

For every pair (q, r) of states in PDA, introduce variable A_{qr} in CFG

Intention:

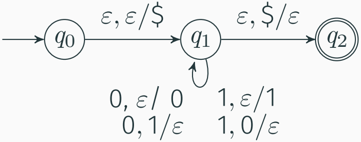
A_{qr} generates all strings that allow the PDA to go from q to r

(with empty stack both at q and at r)

PDA	CFG
	$A_{qq} \to \varepsilon$
	$A_{pr} \to A_{pq}A_{qr}$
	$A_{ps} \to \mathsf{a}A_{qr}\mathsf{b}$ a $= \varepsilon$ or b $= \varepsilon$ allowed

Start variable: A_{pq} (initial state p, accepting state q)

Example: Simplified PDA to CFG

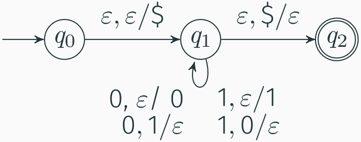


productions:

variables:

start variable:

Example: Simplified PDA to CFG



variables: $A_{00}, A_{11}, A_{22},$
A_{01}, A_{02}, A_{12}

start variable: A_{02}

productions:

$A_{02} \to A_{01} A_{12}$

$A_{01} \to A_{01} A_{11}$

$A_{12} \to A_{11} A_{12}$

$A_{11} \to A_{11} A_{11}$

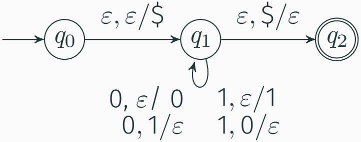
$A_{11} \to 0 A_{11} 1$

$A_{11} \to 1 A_{11} 0$

$A_{02} \to A_{11}$

$A_{00} \to \varepsilon$, $A_{11} \to \varepsilon$,
$A_{22} \to \varepsilon$

Example: Simplified PDA to CFG



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$A_{11} \to A_{11} A_{11}$

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$A_{02} \to A_{11}$

$A_{00} \to \varepsilon,\ A_{11} \to \varepsilon,$
$A_{22} \to \varepsilon$