

Collaborating on homework is encouraged, but you must write your own solutions in your own words and list your collaborators. Copying someone else's solution will be considered plagiarism and may result in failing the whole course.

Please answer clearly and concisely. Explain your answers. Unexplained answers will get lower scores or even no credits.

(1) (18 points) Briefly justify (in about 3-5 sentences) the following statements.

(a) If  $L$  is regular, then

$$L' = \{w \mid w \text{ is a string in } L \text{ and } w \text{ begins and ends with the same symbol}\}$$

is also regular.

(b) There are irregular languages  $L_1$  and  $L_2$  such that  $L_1 \cup L_2$  is regular.

(c) A string  $u$  is an anagram of a string  $w$  if  $u$  is obtained from  $w$  by possibly rearranging the symbols. For example, `listen` is an anagram of `silent`. Formally, if  $w = w_1w_2 \dots w_n$ , then  $u$  is its anagram if  $u = w_{\sigma(1)}w_{\sigma(2)} \dots w_{\sigma(n)}$  for some permutation  $\sigma$ .

There is a regular  $L$  such that  $L^A = \{u \mid u \text{ is an anagram of } w \in L\}$  is not regular.

(2) (32 points) Which of the following languages are regular, and which are not? To show a language is regular, give a DFA, NFA, or regular expression for it. To show a language is not regular, prove it using the pumping lemma or using pairwise distinguishable strings.

(a)  $L_1 = \{1^n1^n \mid n \geq 0\}$

(b)  $L_2 = \{1^m01^n \mid m \geq 0, n \geq 0, m > n\}$

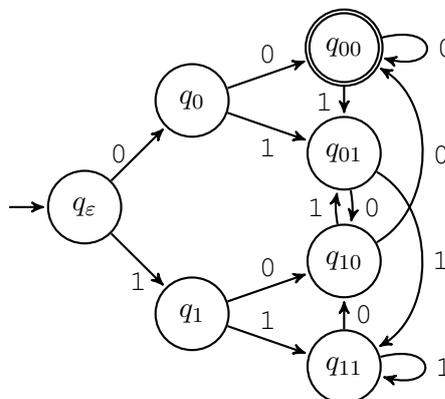
(c)  $L_3 = \{w \in \{0, 1\}^* \mid w \text{ has the same number of occurrences of } 00 \text{ and } 11\}$

Note: `000` has two occurrences of `00`

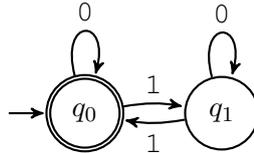
(d)  $L_4 = \{w \in \{0, 1\}^* \mid w \text{ has the same number of occurrences of } 010 \text{ and } 101\}$

Note: `01010` has two occurrences of `010` and one occurrence of `101`

(3) (30 points) Consider the following DFA:



- (a) Run the minimization algorithm on this DFA. Show the table of pairs of distinguishable states at the end of the algorithm (see page 25 of Lecture 7). Also draw the minimized DFA.
- (b) Show that every pair of distinct states in the minimized DFA from part (a) is distinguishable (by giving a string to distinguish them, similar to Page 11 of Lecture 7).
- (c) Convert the following DFA into a regular expression using the conversion algorithm from class. Show the preprocessing step and how the NFA changes after each state is eliminated.



- (4) (20 points) This is the only problem in this course where we deal with POSIX-style regular expressions, as opposed to regular expressions in formal language theory.

The file `propernames` contains a partial list of first names. Each name consists of one or more English letters in upper or lower case. To search for each of the following information in the file, write a `grep` command of the form

```
grep -iE 'regex' propernames
```

Also give a short explanation (1-3 sentences) how your regex works in each case.

Recall that the option `-i` ignores distinction between upper and lower case. You can test your commands with the file `http://www.cse.cuhk.edu.hk/~siuon/csci3130/other/propernames`.

- (a) Any name that has exactly four consonants, such as **Sandra**. `y` is regarded as a vowel.
- (b) Any name without consecutive consonants, so **Audrey** is excluded.
- (c) Any name that doesn't end with `nn`, so **Glenn** is excluded.
- (d) Any name with a substring `s` of length two such that `s` appears at least twice, such as **George**.

For part (d) you may want to use the backreference feature of `grep`.

Your solution should not use `-v` flag of `grep` which swaps non-matches with matches.