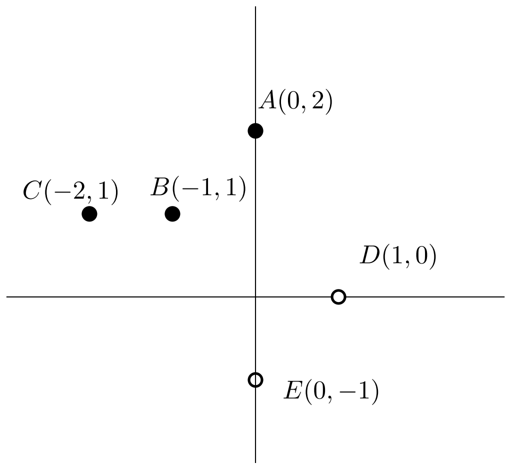
CMSC5724: Quiz 2

Name: **Student ID:**

The following figure shows a set P of 5 points where black points have lable 1 and white ones have lable -1. Answer Problems 1 and 2 based on P.



Problem 1 (30%). Use the Perceptron algorithm to find a line that (i) crosses the origin and (ii) separates the black points from the white ones. Recall that Perceptron starts with a vector $\boldsymbol{w} = \boldsymbol{0}$ and iteratively adjusts it using a violation point. You need to show the value of $\boldsymbol{w}$ after every adjustment.

Answer: At the beginning, $\boldsymbol{w} = (0, 0)$. We use $\boldsymbol{A}$ to denote the vector form of A. Define $\boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D}$ and $\boldsymbol{E}$ similarly.

Iteration 1. Since $\boldsymbol{A}$ does not satisfy $\boldsymbol{w} \cdot \boldsymbol{A} > 0$, update $\boldsymbol{w}$ to $\boldsymbol{w} + \boldsymbol{A} = (0, 0) + (0, 2) = (0, 2)$.

Iteration 2. Since $\boldsymbol{D}$ does not satisfy $\boldsymbol{w} \cdot \boldsymbol{D} < 0$, update $\boldsymbol{w}$ to $\boldsymbol{w} - \boldsymbol{D} = (0, 2) - (1, 0) = (-1, 2)$.

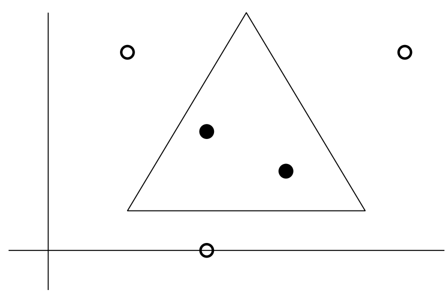
Iteration 3. No more violation points. So we have found a separation line $-x + 2y = 0$.

Problem 2 (30%). Give an instance of quadratic programming to find an origin-passing separation plane with the maximum margin.

Answer: Minimizing $w_1^2 + w_2^2$ subject to the following constraints:

- $w_2 \geq 1$
- $-w_1 + w_2 \geq 1$
- $-2w_1 + w_2 \geq 1$
- $w_1 \leq -1$
- $-w_2 \leq -1$

Problem 3 (40%). A triangle classifier h in $\mathbb{R}^2$ is described by a triangle Δ in the plane. Given a point $p \in \mathbb{R}^2$, $h(p)$ equals 1 if p is covered by Δ, or -1 otherwise. For example, the figure below gives a triangle classifier which maps each black point to 1 and each white point to -1. Let $\mathcal{H}$ be the set of all the triangle classifiers. Prove: the VC-dimension of $\mathcal{H}$ on $\mathbb{R}^2$ is at least 5.



Answer: Consider the following 5 points on a circle. It can be easily verified that for any subset $S \subseteq \{A, B, C, D, E\}$, there is a triangle classifier h such that $h(p) = 1$ if $p \in S$, or $h(p) = -1$ otherwise. Thus, the VC-dimension of $\mathcal{H}$ on $\mathbb{R}^2$ is at least 5.