CMSC5724: Exercise List 2

Answer the following questions based on the training set below

A	B	C	**class**
0	2	1	+
1	2	2	+
1	1	0	+
2	0	2	+
1	1	1	+
2	2	2	−
0	1	0	−
0	0	1	−
2	1	0	−
1	0	0	−

Problem 1. Estimate $\mathbf{Pr}[class = +]$ and $\mathbf{Pr}[C = 2 \mid class = +]$.

Answer. Since 5 out of 10 records belong to the class +, we estimate $\mathbf{Pr}[class = +]$ to be 1/2. Among the 5 records of class +, 2 of them have $C = 2$. Hence, we estimate $\mathbf{Pr}[C = 2 \mid class = +]$ to be 2/5.

Problem 2. Let us make the following *conditional independence assumption*: conditioned on a specific class, attributes A, B, C are independent. Estimate $\mathbf{Pr}[A = 1, B = 2 \mid class = +]$.

Answer. Under the assumption we have:

$$\mathbf{Pr}[A = 1, B = 2 \mid class = +] = \mathbf{Pr}[A = 1 \mid class = +] \cdot \mathbf{Pr}[B = 2 \mid class = +]$$

We estimate $\mathbf{Pr}[A = 1 \mid class = +] = 3/5$ and $\mathbf{Pr}[B = 2 \mid class = +] = 2/5$. Therefore, our estimate of $\mathbf{Pr}[A = 1, B = 2 \mid class = +]$ is 6/25.

Problem 3. Under the conditional independence assumption, decide the larger probability between $\mathbf{Pr}[class = + \mid A = 1, B = 2, C = 0]$ and $\mathbf{Pr}[class = - \mid A = 1, B = 2, C = 0]$.

Answer. By Bayes Theorem

$$\mathbf{Pr}[class = + \mid A = 1, B = 2, C = 0] = \frac{\mathbf{Pr}[A = 1, B = 2, C = 0 \mid class = +] \cdot \mathbf{Pr}[class = +]}{\mathbf{Pr}[A = 1, B = 2, C = 0]}$$

and

$$\mathbf{Pr}[class = - \mid A = 1, B = 2, C = 0] = \frac{\mathbf{Pr}[A = 1, B = 2, C = 0 \mid class = -] \cdot \mathbf{Pr}[class = -]}{\mathbf{Pr}[A = 1, B = 2, C = 0]}$$

To compare the two, we only need to estimate the numerators of the above fractions. Towards this purpose, we have:

$$\begin{aligned} & \mathbf{Pr}[A = 1, B = 2, C = 0 \mid class = +] \cdot \mathbf{Pr}[class = +] \\ = \; & \mathbf{Pr}[A = 1 \mid class = +] \cdot \mathbf{Pr}[B = 2 \mid class = +] \cdot \mathbf{Pr}[C = 0 \mid class = +] \cdot \mathbf{Pr}[class = +] \\ \text{(estimate)} \; = \; & \frac{3}{5} \cdot \frac{2}{5} \cdot \frac{1}{5} \cdot \frac{1}{2} \\ = \; & 6/250 \end{aligned}$$

$$\begin{aligned}
& \mathbf{Pr}[A=1,B=2,C=0 \mid \mathit{class}=-]\cdot\mathbf{Pr}[\mathit{class}=-] \\
= \;& \mathbf{Pr}[A=1 \mid \mathit{class}=-]\cdot\mathbf{Pr}[B=2 \mid \mathit{class}=-]\cdot\mathbf{Pr}[C=0 \mid \mathit{class}=-]\cdot\mathbf{Pr}[\mathit{class}=-] \\
\text{(estimate)} = \;& \frac{1}{5}\cdot\frac{1}{5}\cdot\frac{3}{5}\cdot\frac{1}{2} \\
= \;& 3/250
\end{aligned}$$

We therefore conclude that $\mathbf{Pr}[\mathit{class}=+ \mid A=1,B=2,C=0] > \mathbf{Pr}[\mathit{class}=- \mid A=1,B=2,C=0]$.

Problem 4. Suppose that we make an alternative conditional independence assumptions: A and C are independent, conditioned on a class label and a value of B. Decide the larger probability between $\mathbf{Pr}[\mathit{class}=+ \mid A=1,B=2,C=0]$ and $\mathbf{Pr}[\mathit{class}=- \mid A=1,B=2,C=0]$.

Answer. By Bayes Theorem

$$\begin{aligned}
& \mathbf{Pr}[\mathit{class}=+ \mid A=1,B=2,C=0] \\
= \;& \frac{\mathbf{Pr}[A=1,B=2,C=0 \mid \mathit{class}=+]\cdot\mathbf{Pr}[\mathit{class}=+]}{\mathbf{Pr}[A=1,B=2,C=0]} \\
= \;& \frac{\mathbf{Pr}[A=1,C=0 \mid \mathit{class}=+,B=2]\cdot\mathbf{Pr}[B=2 \mid \mathit{class}=+]\cdot\mathbf{Pr}[\mathit{class}=+]}{\mathbf{Pr}[A=1,B=2,C=0]} \\
= \;& \frac{\mathbf{Pr}[A=1 \mid \mathit{class}=+,B=2]\cdot\mathbf{Pr}[C=0 \mid \mathit{class}=+,B=2]\cdot\mathbf{Pr}[B=2 \mid \mathit{class}=+]\cdot\mathbf{Pr}[\mathit{class}=+]}{\mathbf{Pr}[A=1,B=2,C=0]}
\end{aligned}$$

where the last step used the given conditional independence assumption. Likewise,

$$\begin{aligned}
& \mathbf{Pr}[\mathit{class}=- \mid A=1,B=2,C=0] \\
= \;& \frac{\mathbf{Pr}[A=1 \mid \mathit{class}=-,B=2]\cdot\mathbf{Pr}[C=0 \mid \mathit{class}=-,B=2]\cdot\mathbf{Pr}[B=2 \mid \mathit{class}=-]\cdot\mathbf{Pr}[\mathit{class}=-]}{\mathbf{Pr}[A=1,B=2,C=0]}
\end{aligned}$$

Now we compare the numerators of the two fractions:

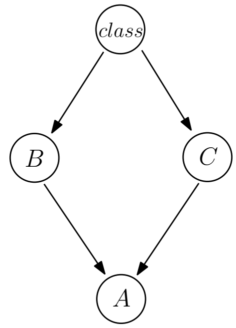
$$\begin{aligned}
& \mathbf{Pr}[A=1 \mid \mathit{class}=+,B=2]\cdot\mathbf{Pr}[C=0 \mid \mathit{class}=+,B=2]\cdot \\
& \mathbf{Pr}[B=2 \mid \mathit{class}=+]\cdot\mathbf{Pr}[\mathit{class}=+] \\
\text{(estimate)} \quad = \;& \frac{1}{2}\cdot\gamma\cdot\frac{2}{5}\cdot\frac{1}{2}=\frac{\gamma}{10}
\end{aligned}$$

where $\gamma > 0$ is a very small value introduced to avoid probability 0.

$$\begin{aligned}
& \mathbf{Pr}[A=1 \mid \mathit{class}=-,B=2]\cdot\mathbf{Pr}[C=0 \mid \mathit{class}=-,B=2]\cdot \\
& \mathbf{Pr}[B=2 \mid \mathit{class}=-]\cdot\mathbf{Pr}[\mathit{class}=-] \\
\text{(estimate)} \quad = \;& \gamma\cdot\gamma\cdot\frac{1}{5}\cdot\frac{1}{2}=\frac{\gamma^2}{10}
\end{aligned}$$

We thus conclude that $\mathbf{Pr}[\mathit{class}=+ \mid A=1,B=2,C=0] > \mathbf{Pr}[\mathit{class}=- \mid A=1,B=2,C=0]$.

Problem 5. Based on the following Bayesian network, decide the larger probability between $\mathbf{Pr}[\mathit{class}=+ \mid A=1,B=1,C=0]$ and $\mathbf{Pr}[\mathit{class}=- \mid A=1,B=1,C=0]$.



Answer.

$$
\begin{aligned}
& \mathbf{Pr}[class = + \mid A = 1, B = 1, C = 0] \\
= \quad & \frac{\mathbf{Pr}[A = 1, B = 1, C = 0 \mid class = +] \cdot \mathbf{Pr}[class = +]}{\mathbf{Pr}[A = 1, B = 1, C = 0]} \\
= \quad & \frac{\mathbf{Pr}[A = 1 \mid B = 1, C = 0] \cdot \mathbf{Pr}[B = 1 \mid class = +] \cdot \mathbf{Pr}[C = 0 \mid class = +] \cdot \mathbf{Pr}[class = +]}{\mathbf{Pr}[A = 1, B = 1, C = 0]}
\end{aligned}
$$

where the last step used the conditional independence assumptions implied by the Bayesian network. Likewise,

$$
\begin{aligned}
& \mathbf{Pr}[class = - \mid A = 1, B = 1, C = 0] \\
= \quad & \frac{\mathbf{Pr}[A = 1 \mid B = 1, C = 0] \cdot \mathbf{Pr}[B = 1 \mid class = -] \cdot \mathbf{Pr}[C = 0 \mid class = -] \cdot \mathbf{Pr}[class = -]}{\mathbf{Pr}[A = 1, B = 1, C = 0]}
\end{aligned}
$$

It remains to compare the numerators of the two fractions:

$$
\begin{aligned}
& \mathbf{Pr}[A = 1 \mid B = 1, C = 0] \cdot \mathbf{Pr}[B = 1 \mid class = +] \cdot \\
& \mathbf{Pr}[C = 0 \mid class = +] \cdot \mathbf{Pr}[class = +] \\
\text{(estimate)} \quad = \quad & \frac{1}{3} \cdot \frac{2}{5} \cdot \frac{1}{5} \cdot \frac{1}{2} = \frac{1}{75}
\end{aligned}
$$

$$
\begin{aligned}
& \mathbf{Pr}[A = 1 \mid B = 1, C = 0] \cdot \mathbf{Pr}[B = 1 \mid class = -] \cdot \\
& \mathbf{Pr}[C = 0 \mid class = -] \cdot \mathbf{Pr}[class = -] \\
\text{(estimate)} \quad = \quad & \frac{1}{3} \cdot \frac{2}{5} \cdot \frac{3}{5} \cdot \frac{1}{2} = \frac{1}{25}
\end{aligned}
$$

We thus conclude that $\mathbf{Pr}[class = + \mid A = 1, B = 1, C = 0] < \mathbf{Pr}[class = - \mid A = 1, B = 1, C = 0]$.