CSCI3160: Special Exercise Set 9

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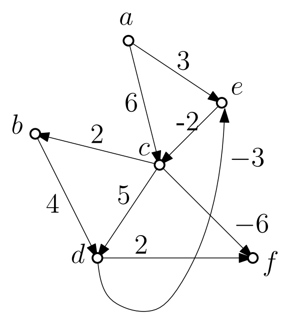
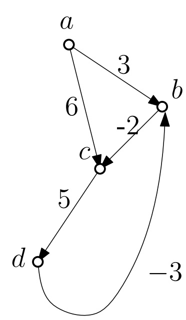
Problem 1. Consider the weighted directed graph $G = (V, E)$ below.



Suppose that we run Bellman-Ford's algorithm to find the shortest path distances from vertex a to all the other vertices. Recall that the algorithm performs $|V| - 1$ rounds of edge relaxations, and maintains a $dist(v)$ value for every vertex V. Give all the $dist(v)$ values after each round of edge relaxations.

Problem 2. Consider the weighted directed graph $G = (V, E)$ below.



Assign ids 1, 2, 3, and 4 to vertices a, b, c, and d, respectively. Suppose that we run the Floyd-Warshall algorithm to find the shortest path distance between vertex i and vertex j for all $i, j \in [1, 4]$. Recall that the algorithm needs to compute $spdist(i, j \mid\leq k)$ for all $i, j, k \in [1, 4]$. Give the value of $spdist(i, j \mid\leq k)$ for each possible combination of i, j, k.

Problem 3. Recall that the rationale behind the Floyd-Warshall algorithm is the following recursive function:

$$spdist(i, j \mid\leq k) = \min \begin{cases} spdist(i, j \mid\leq k-1) \\ spdist(i, k \mid\leq k-1) + spdist(k, j \mid\leq k-1) \end{cases}$$

Give a proof of the above function's correctness.

Problem 4. When we discussed Bellman-Ford's algorithm in the lecture, we described how to compute the shortest path distances from the source vertex s to the other vertices. Augment our description to produce also the shortest paths from s to the other vertices. The final algorithm should still run in $O(|V||E|)$ time.