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A Comparative Study of Gaussian TFA Learning and Statistical Tests on the Factor Number in APT

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Abstract— **The recently developed TFA model is found to be useful for determining factor number k in classical financial APT analysis. In this paper, comparisons of factor number determination using different techniques will be shown. Results reveal that TFA is superior to MLFA as well as eigenvalue analysis.**

I. Introduction

Well-known in the finance literature [1], the Arbitrage Pricing Theory (APT) assumes that cross-sectional expected returns of securities follow a multi-factor model which is characterized by their sensitivities, usually called factor loadings, to k unknown economic factors. Pursuit to the original model by Ross, returns are generated under an *exact factor structure* in which the residual component of returns not explained by the factors is uncorrelated among securities, i.e. white noise. Conventional factor analytic approaches such as Maximum Likelihood Factor Analysis (MLFA) have been applied to recover both the factors and factor loadings and subsequent goodness-of-fit hypothesis test such as the Likelihood Ratio (LR) test is carried out to ascertain the minimum number of factors required to fit the model.

Since the key requirement for the APT that nonfactor risk be approximately eliminated through diversification can still be achieved without the assumption of a strict factor structure, Chamberlain and Rothschild [2] extend the exact factor structure to include the so-called *approximate factor structure*. The main difference arises from the residual component being no longer uncorrelated. For the approximate factor structure, weak correlation within the residual component is possible.

Empirical evidence that the minimum number of factors k accepted by LR test tends to increase with the number of cross-sectional securities p used creates doubts on the validity of exact factor structure assumption. Rather, this result seems to support the approximate factor structure hypothesis. The reason why k increases with p may be explained by the higher probability of including securities with correlated idiosyncratic returns as p increase. Moreover, the use of an approximate factor model is intuitively more appealing because it makes it probable that there exists some factor pertinent to specific industry rather than to the whole market.

Assuming an approximate factor structure, the LR statistic is no longer useful for the factor number identification purpose. On the other hand, analysis of eigenvalues of population covariance matrix has been proved in [2] to be a suitable criterion. If k eigenvalues of the population covariance matrix increase without bound as the number of securities in the population increases, then the elements of the corresponding k eigenvectors of the covariance matrix can be used as factor sensitivities. Moreover, it has been shown in [3] this conclusion holds for the sample covariance matrix as well. In spite of this, Brown [4] spots that empirically the criterion typically biased towards too few factors and the result consistent with one factor may be equally consistent with k equally weighted factors that are priced. The reason is due to rotation of the original factors that minimizes the apparent number of priced factors.

Recently, the development of Temporal Baysian Ying-Yang (TBYY) Theory proposed by Xu [5], [6] leads to the inception of a new factor analytic technique called Temporal Factor Analysis (TFA). TFA can be seen as an extension to MLFA with the strength to overcome rotation indeterminacy as well as to provide an appropriate answer to the number of hidden factors via its model selection ability. As a result, it may serve as an alternative tool for traditional APT analysis. In this paper, results of a comparative study on factor number determination using typical approaches will be presented.

The rest of the paper is divided into four sections. Section II gives an overview of APT. Section III reviews the TFA model and highlight its benefits in the APT analysis. Hypothetical experiments and statistical tests results will be presented in section IV, which is followed by hypothesis testing on APT using real financial data in section V. Section VI will be devoted to concluding remarks.

II. The Arbitrage Pricing Theory

The APT begins with the assumption that the $n \times 1$ vector of asset returns, $\tilde{R}_t$, is generated by a linear stochastic process with k factors [1], [7], [8]:

$$\tilde{R}_t = \bar{R} + Af_t + e_t \tag{1}$$

where f_t is the $k \times 1$ vector of realizations of k common factors, A is the $n \times k$ matrix of factor weights or loadings, and e_t is a $n \times 1$ vector of asset-specific risks. It is assumed that f_t and e_t have zero expected values so that $\bar{R}$ is the $n \times 1$ vector of mean returns.

III. Temporal Factor Analysis

A. An Overview of TFA

Suppose the relationship between a state $y_t \in \mathbb{R}^k$ and an observation $x_t \in \mathbb{R}^d$ are described by the first-order state-space equations as follows [5], [6]:

$$\begin{aligned} y_t &= By_{t-1} + \varepsilon_t, && (2)\\ x_t &= Ay_t + e_t, \qquad t = 1, 2, \ldots, N. && (3) \end{aligned}$$

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where ε_t and e_t are mutually independent zero-mean white noises with $E(\varepsilon_i \varepsilon_j) = \Sigma_\varepsilon \delta_{ij}$, $E(e_i e_j) = \Sigma_e \delta_{ij}$, $E(\varepsilon_i e_j) = 0$, and δ_{ij} is the Kronecker delta function:

$$\delta_{ij} = \begin{cases} 1, & \text{if } i = j, \\ 0, & \text{otherwise.} \end{cases} \tag{4}$$

We call ε_t driving noise upon the fact that it drives the source process over time. Similarly, e_t is called measurement noise because it happens to be there during measurement. The above model is generally referred to as the TFA model.

In the context of APT analysis, (1) can be obtained from (3) by substituting $(\tilde{R}_t - \bar{R})$ for x_t and f_t for y_t. The only difference between the APT model and the TFA model is the added (2) for modelling temporal relation of each factor. The added equation represents the factor series $y = \{y_t\}_{t=1}^T$ in a multi-channel auto-regressive process, driven by an i.i.d. noise series $\{\varepsilon_t\}_{t=1}^T$ that are independent of both y_{t-1} and e_t. Specifically, it is assumed that ε_t is Gaussian distributed. Moreover, TFA is defined such that the k sources $y_t^{(1)}, y_t^{(2)}, \ldots, y_t^{(k)}$ in this state-space model are statistically independent. The objective of TFA [5], [6] is to estimate the sequence of y_t's with unknown model parameters $\Theta = \{A, B, \Sigma_\varepsilon, \Sigma_e\}$ through available observations.

In implementation, an adaptive algorithm has been suggested. At each time unit, factor loadings are estimated by cross-sectional regression and factor scores are estimated by maximum likelihood learning. Xu proposes a simplified version of the algorithm in [6] and is as shown below.

Assume $G(\varepsilon_t|0, \mathrm{I})$ and $G(e_t|0, \Sigma)$.

- **Step 1** Fix A, B and Σ, estimate the hidden factors y_t by

$$\hat{y}_t = [I + A^T \Sigma^{-1} A]^{-1} (A^T \Sigma^{-1} \bar{x}_t + B \hat{y}_{t-1}), \tag{5}$$
$$\varepsilon_t = \hat{y}_t - B \hat{y}_{t-1}, \tag{6}$$
$$e_t = \bar{x}_t - A \hat{y}_t, \tag{7}$$

- **Step 2** Fix, $\hat{y}_t$, update B, A and Σ_e by gradient descent method as follows:

$$B^{\text{new}} = B^{\text{old}} + \eta \, \text{diag}[\varepsilon_\text{t} \hat{y}_{\text{t}-1}^\text{T}], \tag{8}$$
$$A^{\text{new}} = A^{\text{old}} + \eta e_t \hat{y}_t^T, \tag{9}$$
$$\Sigma^{\text{new}} = (1 - \eta) \Sigma^{\text{old}} + \eta e_t e_t^T. \tag{10}$$

B. Model Selection vs Appropriate Number of Factors

Central to the discussion in the paper about the number of factors in APT, TFA is superior to MLFA in view of its model selection ability. In the context of APT analysis, the scale or complexity of the model is equivalent to the number of hidden factors in the original factor structure. As a result, model selection refers to deciding the appropriate number of factors in APT. We can achieve the aim of model selection by enumerating the cost function $J(k)$ with k incrementally and then select an appropriate k by [6], [9], [10]

$$\min_k J(k) = \frac{1}{2}[k \ln(2\pi) + k + \ln|\Sigma|] \tag{11}$$

where Σ is the covariance matrix of measurement noise.

C. Grounds and Benefits for Using TFA in APT Analysis

Firstly, we believe that factors has Gaussian distributions. There is a consensus that the noisy component in most econometric and statistical models being Gaussian distributed. The rationale comes from the central limit theorem which implies that the compounding of a large number of unknown distributions will be approximately normal. Secondly, we believe that factors recovered must be independent of each other. Although economic factors are seldom independent, it is helpful to discover statistically independent factors for the purpose of analysis because the restriction of independence will rule out many possible solutions which contain redundant elements. Furthermore, economic interpretation of factors recovered can be easily achieved by appropriate combination of those independent factors. Thirdly, we believe there exists temporal relation between factors. Equation (2) of the TFA model is nothing more than an AR(1) time series model. The reason why an AR model of order more than 1 is not required can be attributed to the weak form of Efficient Market Hypothesis (EMH). Given the assumption of the weak form EMH is valid, stock price today is conditionally independent of all previous prices given the price of yesterday.

Compared with MLFA, TFA has at least three benefits. First, with the independence assumption in the derivation, the recovered factors are assured to be statistically independent. Second, it has been shown in [5] that taking into account temporal relation effectively removes rotation indeterminacy. As a result, the solution given by TFA is unique. Theorem 3 in [5] illustrates this point. Third, it can determine the number of hidden factors via its model selection ability. Moreover, it should be noted that MLFA is a special case of the model with $B = 0$ in (2).

IV. Comparisons Using Hypothetical Data

In this section, we aim to support our discussions above by hypothetical experiments. In the sequel, we will present the results of APT test using real stock return data.

A. Test Methodology

For hypothetical and real experiments, we will compare the sensitivity of different tests on identifying the number of factors k with regard to the number of securities used. As discussed above, under the assumption of exact factor structure we will first use LR test on the results of MLFA. Then we will analyze the eigenvalues of the sample covariance matrix, assuming an underlying approximate factor structure. Finally, the results will be compared with that found by TFA's model selection criterion.

B. LR Test Statistic

The LR statistic proposed in [11] and modified in [12] is given by

$$\begin{aligned} \text{LR} &= (N - \frac{2p + 4k + 11}{6})(\ln|AA' + \Sigma| - \ln|S|) \\ &\quad + (N - \frac{2p + 4k + 11}{6})(\mathrm{tr}[(\mathrm{AA}' + \Sigma)^{-1}\mathrm{S}] - \mathrm{p}), \end{aligned}$$

where N is the sample size, S is the sample return covariance matrix and p is the total number of securities. The first and second terms in the sum of LR refer to the variance and bias components, respectively, of the statistic [13]. It has been shown in [11] that the maximum likelihood factor estimates are unbiased, and consequently, the bias component will converge asymptotically to zero. As a result, the LR statistic measures the overall error in the factor estimates of the sample covariances by comparing the generalized variances. When the normality assumptions apply, general properties of the LR statistic establish that it has an asymptotic central χ^2 distribution with $[(p-k)^2-(p+k)]/2$ degrees of freedom. The minimum number of factors k can be inferred from the computed p value at a specific level of significance, which is 5% in this paper.

C. Eigenvalues Analysis

Eigenvalues of the sample correlation or covariance matrix can be obtained either by direct calculation or indirectly via performing PCA. According to [2], for an approximate k-factor structure, the first k eigenvalues of the covariance matrix of returns grow without limit as the number of securities, p, increases, while the remaining $p-k$ stay constant. However, for limited number of securities, we can only determine the factor number heuristically via counting the number of relatively large eigenvalues.

D. Experimental Illustrations

In this experiment, we assume returns of 30, 60 and 90 securities being generated randomly via a fixed number of factors 5 by the TFA model in (2) & (3). The parameters used to generate $N=1000$ data points are predetermined as follows:

A: a $p\times 5$ matrix where p = 30, 60 or 90,

B: $$B=\begin{pmatrix}0.12 & 0 & 0 & 0 & 0\\ 0 & 0.21 & 0 & 0 & 0\\ 0 & 0 & -0.15 & 0 & 0\\ 0 & 0 & 0 & 0.24 & 0\\ 0 & 0 & 0 & 0 & -0.13\end{pmatrix},$$

$\varepsilon_{\mathbf{t}}$: randomly generated with pdf $G(\varepsilon_t|0,\mathrm{I})$ where I is an identity matrix of order 5,

$\mathbf{e_t}$: randomly generated with pdf $G(e_t|0,\Sigma)$ where Σ is a $p\times p$ matrix with diagonal elements σ_{ii} and off-diagonal elements σ_{ij} and $\sigma_{ii}\sim U(0.1,0.25)$, $\sigma_{ij}\sim U(0,0.01)$, where $U(p,q)$ denotes uniform distribution in the interval $[a,b]$,

$\mathbf{y_0}$: $y_0=\begin{pmatrix}0 & 0 & 0 & 0 & 0\end{pmatrix}^T$.

Experimental results showing the number of factors identified by LR statistics are shown in Table I and that by eigenvalues analysis and TFA are shown in Table II.

As shown in Table I, the minimum number of factors k identified by MLFA and LR statistics is very sensitive to the number of securities p and increases progressively with p. The acceptable k at 5% level of significance is 8, 15 and 22 for 30, 60 and 90 securities respectively. On the other hand, evidence in Table II based on eigenvalues of sample covariance matrices seems to support the one-factor structure. Clearly, decision based on LR statistics tends to overestimate k while that based on eigenvalues tends to understate the same. The cost function $J(k)$ is not only insensitive to p, but also estimates k correctly in all three cases. This can be seen from Table II that all minima of $J(k)$ occur at $k=5$. Fig. 1 plots the values of $J(k)$ against the number of factors for different number of securities.

TABLE I

SENSITIVITIES OF LR STATISTIC TO THE NUMBER OF SECURITIES FOR FACTOR NUMBER DETERMINATION.

30 Securities			
k	D.f.	LR Stat.	p-Value
1	405	22382.70	0.0000
2	376	16301.92	0.0000
3	348	9892.19	0.0000
4	321	3629.01	0.0000
5	295	413.43	0.0000
6	270	354.16	0.0004
7	246	301.05	0.0095
8	223	245.44	0.1445
9	201	205.01	0.4083
10	180	170.33	0.6857
11	160	136.31	0.9128
12	141	111.52	0.9682
13	123	85.43	0.9960
14	106	68.12	0.9984
15	90	52.90	0.9994
16	75	36.95	0.9999

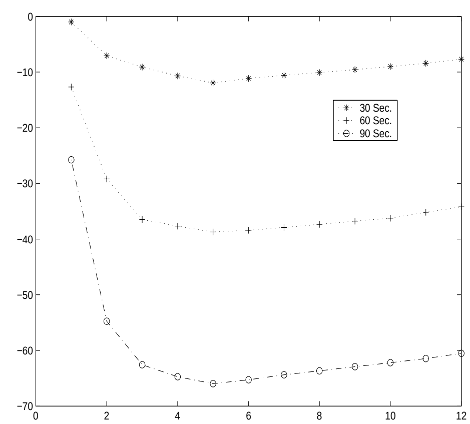

Fig. 1. $J(k)$ for different number of securities using hypothetical data.

V. COMPARISONS USING REAL FINANCIAL DATA

In this section, similar methodology discussed in the last section will be applied to historical stock data for the analysis of APT.

A. Data Considerations

We have carried out our analysis using past stock price and return data of Hong Kong. Daily closing prices of 86 actively trading stocks covering the period from January 1, 1998 to December 31, 1999 are used. The number of trading days throughout this period is 522. These stocks can be subdivided into three main categories according to different indices they constitute. Of the 86 equities, 30 of them belongs to the Hang Seng Index (HSI) constituents, 32 are Hang Seng China-Affiliated Corpora-

TABLE I
CONTINUED.

60 Securities			
k	D.f.	LR Stat.	p-Value
1	1710	56457.41	0.0000
2	1651	40846.49	0.0000
3	1593	27085.83	0.0000
4	1536	11633.35	0.0000
5	1480	2039.07	0.0000
6	1425	1881.16	0.0000
7	1371	1761.94	0.0000
8	1318	1649.73	0.0000
9	1266	1541.83	0.0000
10	1215	1446.79	0.0000
11	1165	1362.35	0.0000
12	1116	1275.88	0.0006
13	1068	1200.28	0.0028
14	1021	1118.80	0.0173
15	975	1036.56	0.0837
16	930	964.72	0.2088
17	886	898.52	0.3776
18	843	833.85	0.5822
19	801	773.62	0.7503
20	760	714.91	0.8776
21	720	657.58	0.9533
22	681	603.74	0.9846
23	643	559.14	0.9924
24	606	515.95	0.9966
25	570	470.37	0.9991

TABLE I
CONTINUED.

90 Securities			
k	D.f.	LR Stat.	p-Value
1	3915	87345.03	0.0000
2	3826	66935.65	0.0000
3	3738	47084.75	0.0000
4	3651	24081.84	0.0000
5	3565	4753.52	0.0000
6	3480	4527.27	0.0000
7	3396	4336.92	0.0000
8	3313	4172.11	0.0000
9	3231	4005.57	0.0000
10	3150	3851.76	0.0000
11	3070	3693.61	0.0000
12	2991	3541.37	0.0000
13	2913	3400.69	0.0000
14	2836	3265.36	0.0000
15	2760	3139.81	0.0000
16	2685	3008.03	0.0000
17	2611	2882.60	0.0001
18	2538	2766.56	0.0009
19	2466	2656.09	0.0040
20	2395	2551.48	0.0131
21	2325	2440.66	0.0466
22	2256	2341.54	0.1025
23	2188	2254.77	0.1564
24	2121	2157.80	0.2837
25	2055	2056.26	0.4880
26	1990	1961.68	0.6702
27	1926	1863.31	0.8439
28	1863	1783.27	0.9056
29	1801	1701.73	0.9530
30	1740	1611.15	0.9871
31	1680	1538.08	0.9940
32	1621	1463.44	0.9978

tions Index (HSCCI) constituents and the remaining 24 are Hang Seng China Enterprises Index (HSCEI) constituents.

B. Data Preprocessing

Before carrying out the analysis, the stock prices must be converted to stationary stock returns. The transformation applied can be described in four steps as shown below.

- **Step 1** Transform the raw prices to returns by $R_t = \frac{p_t - p_{t-1}}{p_{t-1}}$.
- **Step 2** Calculate the mean return $\bar{R}$ by $\frac{1}{N}\sum_{t=1}^{N} R_t$.
- **Step 3** Subtract $\bar{R}$ from R_t to get the zero-mean return.
- **Step 4** Let the result of above transformation be the adjusted return $\widetilde{R}_t$.

C. Empirical Test Results

The aim of our experiment is examine the relationship between the number of factors affecting stocks of various indices as well as the whole market. Table VII gives an overview of the final results based on the details shown in Table III,IV,V and VI. From Table VII, we can see that the number of factors k determined based on the methodology of MLFA increases progressively with the number of securities included in a particular group. According to MLFA, there are 11 factors for HSI constituents, 12 for HSCCI constituents, 9 for HSCEI constituents and 33 for all market securities as a whole. On the other hand, the number of factors as revealed through the analysis of eigenvalues of sample covariance matrix is 1 irrespective of indices. The findings by the previous two methods can be contrasted with that discovered by the model selection criterion of TFA. Since the unique k associated with the minimum value of the cost function $J(k)$ corresponds to the appropriate factor number in APT, the factor numbers are 4 for both HSI and HSCEI, 3 for HSCCI and 5 for all securities. Fig. 2 shows a plot of $J(k)$ against the factor number k for different index constituents.

D. Result Interpretation

The correct determination of factor number is critical for the test of APT. However, the issue of the appropriate number of factors has been the subject of some controversy in the literature [7], [14], [15], [16], [13], [4]. Although Roll and Ross [7] are keen on the belief that the number of factors is not more than five based on some empirical research findings, it is still far from conclusive because the tool on which the financial APT analy-

TABLE II

SENSITIVITIES OF EIGENVALUE AND $J(k)$ TO THE NUMBER OF SECURITIES FOR FACTOR NUMBER DETERMINATION.

	Eigenvalue			Values of $J(k)$		
k	30 Sec	60 Sec	90 Sec	30 Sec	60 Sec	90 Sec
1	35.97	79.00	120.03	-1.00	-12.69	-25.76
2	4.51	8.86	11.31	-7.09	-29.19	-54.75
3	3.39	6.06	8.81	-9.10	-36.48	-62.57
4	2.81	5.91	8.16	-10.69	-37.67	-64.72
5	1.46	3.08	5.58	-11.96	-38.73	-65.97
6	0.28	0.31	0.34	-11.15	-38.41	-65.27
7	0.26	0.30	0.33	-10.57	-37.91	-64.38
8	0.24	0.28	0.32	-10.09	-37.36	-63.67
9	0.23	0.27	0.31	-9.56	-36.77	-62.92
10	0.22	0.27	0.29	-9.03	-36.24	-62.21
11	0.21	0.25	0.29	-8.42	-35.19	-61.45
12	0.20	0.24	0.28	-7.69	-34.21	-60.52

TABLE III

EMPIRICAL RESULTS OF FACTOR NUMBER DETERMINATION USING REAL STOCK DATA: 30 HSI CONSTITUENTS.

k	D.f.	LR Stat.	p-Value	Eigen.	$J(k)$
1	405	1753.21	0.0000	0.0180	-30.0975
2	376	1372.65	0.0000	0.0020	-39.4636
3	348	1051.90	0.0000	0.0014	-40.8083
4	321	746.88	0.0000	0.0013	-42.4021
5	295	575.57	0.0000	0.0012	-41.4884
6	270	463.34	0.0000	0.0011	-40.5723
7	246	396.55	0.0000	0.0009	-39.6502
8	223	320.69	0.0000	0.0008	-38.4755
9	201	265.05	0.0016	0.0008	-37.3477
10	180	212.93	0.0470	0.0008	-36.2142
11	160	175.97	0.1836	0.0007	-35.0094
12	141	146.20	0.3649	0.0006	-33.7939
13	123	112.61	0.7387	0.0006	
14	106	91.02	0.8497	0.0006	
15	90	66.69	0.9689	0.0005	
16	75	50.61	0.9863	0.0005	
17	61	34.31	0.9977	0.0004	
18	48	23.10	0.9991	0.0004	

sis is based suffers from various indeterminacies discussed in the previous sections. Consequently, the determination of factor number can be described as being based on some heuristic approaches. Intuitively, we do not expect the factor number to grow with the number of securities used under a stable market structure, nor do we expect the number of factors to be one because both theoretically and empirically the multi-factor APT model is expected to be superior to the one-factor Capital Asset Pricing Model (CAPM). Interestingly, the factor number determined via the cost function $J(k)$ and TFA is quite reasonable and agrees with our intuition.

TABLE IV

EMPIRICAL RESULTS OF FACTOR NUMBER DETERMINATION USING REAL STOCK DATA: 32 HSCCI CONSTITUENTS.

k	D.f.	LR Stat.	p-Value	Eigen.	$J(k)$
1	464	2132.32	0.0000	0.0427	-33.9164
2	433	1597.10	0.0000	0.0048	-42.8971
3	403	1257.86	0.0000	0.0043	-47.8219
4	374	991.13	0.0000	0.0026	-46.9477
5	346	748.25	0.0000	0.0023	-46.0319
6	319	624.20	0.0000	0.0019	-45.0285
7	293	495.61	0.0000	0.0019	-43.9239
8	268	406.72	0.0000	0.0018	-42.7937
9	244	339.17	0.0001	0.0017	-41.6815
10	221	286.07	0.0021	0.0016	-40.5148
11	199	237.61	0.0318	0.0015	-39.2081
12	178	201.73	0.1074	0.0014	-37.9458
13	158	168.74	0.2649	0.0012	
14	139	141.90	0.4158	0.0012	
15	121	113.73	0.6678	0.0011	
16	104	88.18	0.8668	0.001	
17	88	75.11	0.8346	0.0009	
18	73	48.25	0.9888	0.0009	
19	59	33.05	0.9975	0.0008	
20	46	22.80	0.9984	0.0008	
21	34	14.20	0.9989	0.0007	

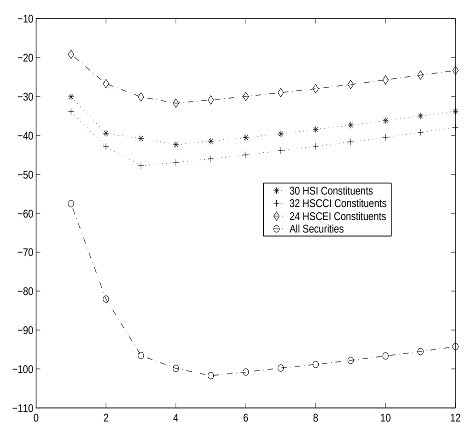

Fig. 2. Plot of $J(k)$ against k for different index constituents.

VI. CONCLUSION

In this paper, we have conducted a comparative study on factor number determination in financial APT analysis. It is found that LR test on results of MLFA is biased towards more factors while the identification via eigenvalues of sample covariance matrix tends to bias towards a smaller factor number. Empirical test results reveal that TFA can estimate the correct number of factors via a simple cost function and thus TFA can provide a reasonable answer to the number of existing factors in the stock market in Hong Kong.

TABLE V

EMPIRICAL RESULTS OF FACTOR NUMBER DETERMINATION USING REAL STOCK DATA: 24 HSCEI CONSTITUENTS.

k	D.f.	LR Stat.	p-Value	Eigen.	$J(k)$
1	252	1155.54	0.0000	0.0294	-19.1867
2	229	707.41	0.0000	0.0034	-26.7143
3	207	519.18	0.0000	0.0027	-30.1253
4	186	402.98	0.0000	0.0022	-31.7391
5	166	335.53	0.0000	0.0020	-30.9156
6	147	273.94	0.0000	0.0018	-30.0051
7	129	223.13	0.0000	0.0015	-29.0193
8	112	171.50	0.0003	0.0015	-28.0158
9	96	135.15	0.0052	0.0014	-26.9287
10	81	105.64	0.0344	0.0012	-25.7410
11	67	77.31	0.1826	0.0012	-24.5273
12	54	47.45	0.7234	0.0011	-23.3333
13	42	29.45	0.9280	0.0011	
14	31	16.74	0.9826	0.0010	

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TABLE VI

EMPIRICAL RESULTS OF FACTOR NUMBER DETERMINATION USING REAL STOCK DATA: ALL 86 SECURITIES.

k	D.f.	LR Stat.	p-Value	Eigen.	$J(k)$
1	70124	13836.27	0.0000	0.0794	-57.5487
2	69749	8974.26	0.0000	0.0090	-82.0707
3	69375	8013.02	0.0000	0.0055	-96.5370
4	69002	7322.74	0.0000	0.0049	-99.8654
5	68630	6748.97	0.0000	0.0042	-101.7367
6	68259	6206.89	0.0000	0.0033	-100.8063
7	67889	5734.00	0.0000	0.0030	-99.7729
8	67520	5386.65	0.0000	0.0028	-98.8376
9	67152	5013.30	0.0000	0.0024	-97.7964
10	66785	4715.36	0.0000	0.0022	-96.6583
11	66419	4477.92	0.0000	0.0021	-95.5152
12	66054	4246.48	0.0000	0.0021	-94.2694
13	65690	4027.50	0.0000	0.0020	
14	65327	3817.85	0.0000	0.0019	
15	64965	3637.25	0.0000	0.0018	
16	64604	3461.50	0.0000	0.0018	
17	64244	3294.01	0.0000	0.0017	
18	63885	3149.45	0.0000	0.0016	
19	63527	3001.69	0.0000	0.0016	
20	63170	2848.76	0.0000	0.0015	
21	62814	2708.99	0.0000	0.0014	
22	62459	2557.03	0.0000	0.0014	
23	62105	2425.27	0.0000	0.0013	
24	61752	2299.82	0.0000	0.0013	
25	61400	2184.25	0.0000	0.0013	
26	61049	2072.12	0.0000	0.0012	
27	60699	1973.43	0.0000	0.0012	
28	60350	1869.59	0.0000	0.0012	
29	60002	1774.60	0.0002	0.0011	
30	59655	1681.16	0.0013	0.0011	
31	59309	1586.46	0.0082	0.0011	
32	58964	1502.31	0.0275	0.0010	
33	58620	1418.03	0.0813	0.0010	
34	58277	1342.88	0.1584	0.0010	
35	57935	1270.46	0.2676	0.0009	
36	57594	1199.01	0.4136	0.0009	
37	57254	1138.35	0.4999	0.0009	
38	56915	1068.94	0.6699	0.0009	
39	56577	999.59	0.8231	0.0008	
40	56240	919.63	0.9572	0.0008	

TABLE VII

RESULTS SUMMARY OF FACTOR NUMBER k DETERMINED BASED ON REAL FINANCIAL DATA.

Stock index	Total number of securities	MLFA	Eigen.	$J(k)$
HSI	30	11	1	4
HSCCI	32	12	1	3
HSCEI	24	9	1	4
All Securities	86	33	1	5