Exercises: Path Independence

For Problems 1-4, first decide whether the line integral is path independent. If so, calculate the integral on a piecewise smooth arc from point $(0,0)$ to point $(1,1)$ in 2d, or from point $(0,0,0)$ to point $(1,1,1)$ in 3d.

Problem 1. $\int_C 2e^{x^2}(x\cos(2y)\,dx - \sin(2y)\,dy)$.

Problem 2. $\int_C (x^2y\,dx - 4xy^2\,dy + 8z^2x\,dz)$.

Problem 3. $\int_C (e^y\,dx + (xe^y - e^z)\,dy - ye^z\,dz)$.

Problem 4. $\int_C (4y\,dx + (4x+z)\,dy + (y-2z)\,dz)$.

Solve Problems 5-8 by resorting to path independence.

Problem 5. Calculate $\int_C d\boldsymbol{r} = \int_C dx + \int_C dy$ where C is a smooth curve from point $p = (1,2)$ to $q = (3,4)$.

Problem 6. Calculate $\int_C 2xy\,dx + \int_C x^2\,dy$ where C is a smooth curve from point $p = (1,2)$ to $q = (3,4)$.

Problem 7. Calculate $\int_C yz\,dx + \int_C xz\,dy + \int_C xy\,dz$ where C is a smooth curve from point $p = (1,2,3)$ to $q = (3,4,5)$.

Problem 8. Calculate $\int_C yz\,dx + \int_C xz\,dy + \int_C xy\,dz$ where C is the curve given by $\boldsymbol{r}(t) = [\cos(t), \sin(t), 1]$ with $t \in [0, 2\pi]$.