Exercises

Problem 1. Let H be a set of halfspaces in $\mathbb{R}^2$. Prove: H is infeasible for LP if and only if H has an infeasible subset of size 3 (i.e., three halfspaces in H have an empty intersection).

Problem 2. In the lecture, we proved that our LP algorithm runs in $O(n)$ expected time when the input set H is feasible. Extend the proof to the case where H is infeasible. You can still make the general position assumption.

(Hint: Use the result of Problem 2 in backward analysis.)

Problem 3. Prove: our LP algorithm runs in $O(n)$ expected time even if the boundary lines of 3 (or more) halfspaces pass the same point. You can still make the assumption of no horizontal boundary lines.

Problem 4*. Give an algorithm to solve LP in $O(n)$ expected time in $\mathbb{R}^d$ for any constant d.

(Hint: Reduce the d-dimensional problem to $(d-1)$-dimensional.)

Problem 5 (textbook exercise 4.15). A polygon is *star-shaped* if there is a point p inside the polygon that is visible to all the vertices of the polygon (two points in a polygon are visible to each other if the segment connecting the two points is completely inside the polygon). In the figure below, the left polygon is star-shaped but the right one is not. Given a polygon of n vertices, determine in $O(n)$ expected time whether it is star-shaped.