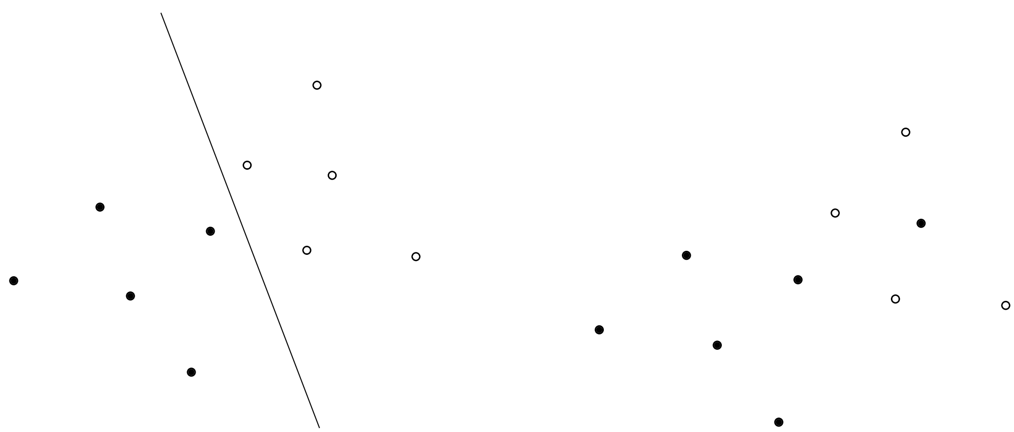
Exercises

Problem 1. Consider a set L of lines in $\mathbb{R}^2$ and an arbitrary line $\ell^* \in L$. Construct a set of halfplanes as follows: for each line $\ell \in L$ and $\ell \neq \ell^*$, add to H a halfplane h that covers all the points in $\mathbb{R}^2$ on or below ℓ. Denote by $\boldsymbol{v}$ the unit normal vector of ℓ^* pointing up. Prove:

- The linear programming (LP) instance defined by H and $\boldsymbol{v}$ (i.e., find a point p maximizing $\boldsymbol{p} \cdot \boldsymbol{v}$ subject to the constraint that p falls in all the halfplanes of H) must have a bounded solution.
- Let p be an optimal solution of the above LP instance. Prove: p is above ℓ^* if and only if ℓ^* is on the boundary of the lower envelope of L.

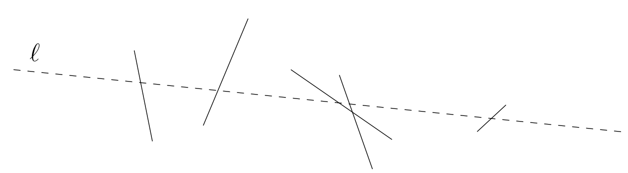
Problem 2. Let P be a set of n points in $\mathbb{R}^d$, where the dimensionality d is a fixed constant. Each point is colored in either black or white. Determine whether there exists a line ℓ that separates the black points from the white ones. Your algorithm must finish in $O(n)$ expected time.



The answer is yes for the dataset in the left figure (a separation line is shown), while the answer is no for the right figure.

Problem 3. Give an $O(n^2 \log n)$ time algorithm to compute the line arrangement of n lines in $\mathbb{R}^2$.

Problem 4* (textbook exercise 8.16). Let S be a set of n line segments segments in $\mathbb{R}^2$. Decide in $O(n^2 \log n)$ time whether there exists a line intersecting all the segments in S.



In the above example, S consists of the 5 (solid) line segments, and the dashed line ℓ is what we look for.

Problem 5*. Let P be a set of n points in $\mathbb{R}^2$. Find in $O(n \log n)$ time the line of the maximum slope that passes two points in P.