CSCI5020 External Memory Data Structures: Exercise List 4

In the following problems, B is the block size, and M is the memory capacity.

Problem 1. Assuming $M \geq B^2$, describe an algorithm to construct an external interval tree on N intervals in $O(\frac{N}{B} \log_B N)$ I/Os.

Problem 2 (Ray Shooting on Rays). Let S be a set of N horizontal rays in $\mathbb{R}^2$ shooting towards right, i.e., each ray in S has the form $[x, \infty) \times y$. Given a point q in $\mathbb{R}^2$, a *ray shooting* query finds the first ray that is hit by a vertical ray shooting upwards from q. Describe a structure that uses $O(N/B)$ space and answers a ray shooting query in $O(\log_B N)$ I/Os. Make your structure fully dynamic such that each insertion and deletion can be supported in $O(\log_B N)$ I/Os.

Problem 3. Let $L = \{\ell_1, ..., \ell_l\}$ be a set of l vertical lines in $\mathbb{R}^2$, where $l = \sqrt{B}$. Let S be a set of N horizontal segments such that each segment in S has its endpoints on two different lines in L. Given a vertical ray r shooting downwards from a point, a query reports all the segments in S intersecting r. Give a structure on S that consumes $O(N/B)$ space, and answers a query in $O(1 + K/B)$ I/Os, where K is the number of segments reported. Your structure also needs to support an insertion and a deletion in $O(\log_B N)$ I/Os amortized, assuming $M \geq B^2$.

Problem 4 (Ray Intersecting Segments). Let S be a set of N horizontal segments in $\mathbb{R}^2$. Given a vertical ray r shooting downwards from a point, a query reports all the segments in S intersecting r. Describe a structure on S that consumes $O(N/B)$ space, and answers a query in $O(\log_B^2 N + K/B)$ I/Os, where K is the number of segments reported. Your structure also needs to support an insertion and a deletion in $O(\log_B N)$ I/Os amortized, assuming $M \geq B^2$.