Week 6 Tutorial

CSCI2100 Teaching Team 2021

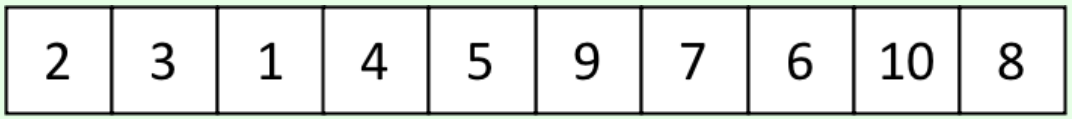
Department of Computer Science and Engineering

The Chinese University of HongKong

Pivot Selection

Input: An array A of n integers in arbitrary order.
Output: An element in A whose rank is between $\frac{n}{10}$ and $\frac{9n}{10}$.

Example:



A

Valid answers: any number from 2 to 9.

Pivot Selection

Algorithm

1. Randomly pick an integer v from A.
2. Get the rank r of v.
3. If r is not in $[n/10, 9n/10]$, repeat from 1.
4. Otherwise, output v.

Cost Analysis

How many times do we have to repeat Step 1 and 2?

Each run finds a valid answer v with probability $4/5$. Thus, we need to repeat $5/4$ times in expectation.

Hence, our algorithm finishes in $O(n)$ expected time.

Think: If we use the pivot picked in the above manner for k-selection, what is the expected cost of the k-selection algorithm discussed in the lecture?

Pivot Selection

Think: what if

> **Input:** An array A of n integers in arbitrary order.
> **Output:** An element in A whose rank is between $0.4999n$ and $0.5n$?

The next few slides will introduce you to some basic ideas behind generating a random number. As you will see, all we need is the ability to generate a random bit.

Coin Game 1

Given a fair coin, how do you generate a number from 1 to 4 uniformly at random?

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Solution: Flip the coin twice. Assign numbers as follows:

- (Head, Head): 1
- (Head, Tail): 2
- (Tail, Head): 3
- (Tail, Tail): 4

Coin Game 2

Given a fair coin, how do you generate a number from 1 to 3 uniformly at random?

Hint: Use the previous algorithm as a black box.

Coin Game 1

Given a fair coin, how do you generate a number from 1 to 3 uniformly at random?

Solution: Run the algorithm in Coin Game 1. If the algorithm returns 4, ignore it and run again.

Cost: The number of repeats is $O(1)$.

Coin Game 3

Given a fair coin, how do you generate a number from 1 to n uniformly at random?

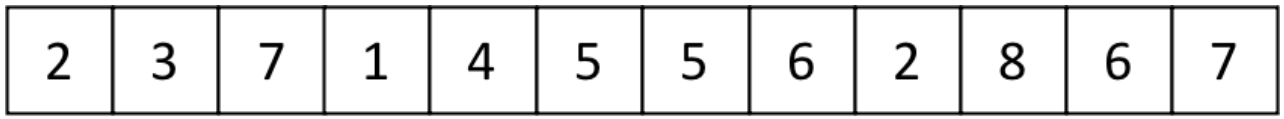
Solution: See a regular exercise.

In the last part of the tutorial, we will modify merge sort to sort a **multi-set**.

Sorting a Multi-Set

So far we have assumed the input to sorting is a set S of integers.

How to sort a multi-set A, i.e. a collection of integers which may contain duplicates?

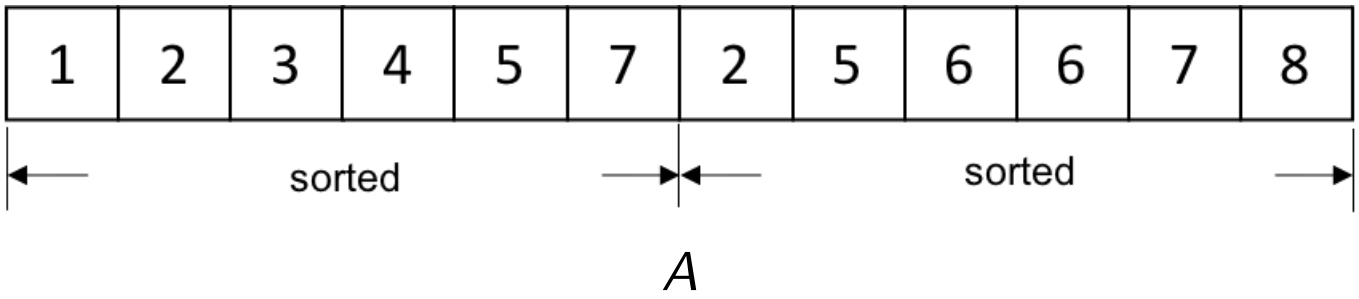


A

Merge Sort

1. Sort the first half of the array A.
2. Sort the second half of the array A.
3. Consider both subproblems solved and merge the two halves of the array into the final sorted sequence.

We only need to modify Step 3.



Merging

At the beginning, set $i = j = 1$.

Repeat until $i > n/2$ or $j > n/2$:

1. If $A_1[i]$ (i.e., the i-th integer of A_1) is smaller or equal to $A_2[j]$, append $A_1[i]$ to A, and increase $i$by 1.
2. Otherwise, append $A2[j]$ to A, and increase j by 1.