CSCI2100/ESTR2102: Quiz 1

Name: Student ID:

Problem 1 (25%). Prove: $10n + n^{1/3} = O(n)$.

Solution. $10n + n^{1/3} \leq 11n$ for all $n \geq 1$.

Problem 2 (25%). Prove: n^2 is not $O(100 \cdot n)$.

Solution. Assume, for contradiction purposes, that $n^2 = O(100n)$, namely, there exist constants c_1, c_2 such that $n^2 \leq c_1 100n$ for all $n \geq c_2$. This means $n \leq 100c_1$ for all $n \geq c_2$, which is impossible and, hence, gives a contradiction.

Problem 3 (35%). You are given: (i) an array A which contains n integers sorted in ascending order, and (ii) an integer q. Design an algorithm to find how many integers in A are larger than or equal to q. For example, if $A = (2, 4, 10, 18, 20, 22)$ and $q = 17$, then the answer is 3. Your algorithm must finish in $O(\log n)$ time.

Solution. We discuss only the case where q is not in A (the opposite case is similar and omitted). Perform binary search to find the predecessor of q in A. If the predecessor is the i-th element of A, then return $n - i$.

Problem 4 (15%). You are given: (i) an array A which contains n integers in an **arbitrary** order, and (ii) an integer q. Write an algorithm to find the predecessor of q in A. For example, if $A = (10, 8, 4, 6, 12, 2)$ and $q = 9$, then the answer is 8. Your algorithm must finish in $O(n)$ time.

Solution. Scan A and, at any moment, keep the maximum of all the elements that are already seen and are less than or equal to A.