Linked Lists, Stacks, and Queues

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A **data structure** stores a set of elements and supports certain operations on those elements.

The only data structure in our discussion so far is the **array**.

In this lecture, we will first discuss a new data structure, the **linked list**, and then utilize it to design two other structures: the **stack** and the **queue**.

Linked List

A **linked list** is a sequence of **nodes** where:

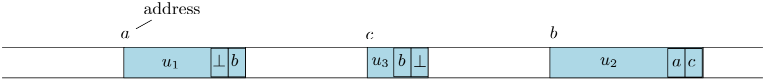
- a node is an array;
- a node's **address** is its array's starting memory address;
- each node stores in its array
  - a **back-pointer** to its preceding node (if it exists);
  - a **next-pointer** to its succeeding node (if it exists).

Recall that a "pointer" is a memory address.

In a linked list, the first node is called the **head** and the last node is called the **tail**.

Linked List

The figure below illustrates a linked list of three nodes u_1, u_2, and u_3, whose addresses are a, b, and c, respectively.



The back-pointer of node u_1 (the head) is `nil`, denoted by $\perp$. The next-pointer of u_3 (the tail) is also `nil`.

Example:

A linked list storing a set of integers $\{14, 65, 78, 33, 82\}$:

	b				a				d				e			c			
	78	a	c		65	$\perp$	b		82	c	e		14	d	$\perp$	33	b	d	

Conceptually, we can think of the sequence $(65, 78, 33, 82, 14)$ in the linked list as:

$$65 \longleftrightarrow 78 \longleftrightarrow 33 \longleftrightarrow 82 \longleftrightarrow 14$$

Two (Simple) Facts

Suppose that we use a linked list to store a set S of n integers (one node per integer).

Fact 1: The linked list uses $O(n)$ space, namely, $O(n)$ memory cells.

Fact 2: Starting from the head node, we can enumerate all the integers in S in $O(n)$ time.

A linked list storing a set S supports **updates**:

- **insertion**: add a new element to S;
- **deletion**: remove an existing element from S.

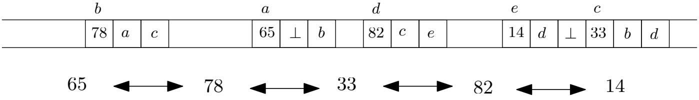
Insertion in a Linked List

To insert a new element e, append e to the linked list:

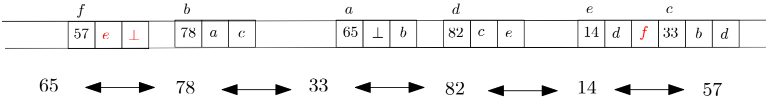
1. Identify the tail node u.
2. Create a new node u_{new} to store e.
3. Set the next-pointer of u to the address of u_{new}.
4. Set the back-pointer of u_{new} to the address of u.

$O(1)$ time.

Example



After inserting 57:



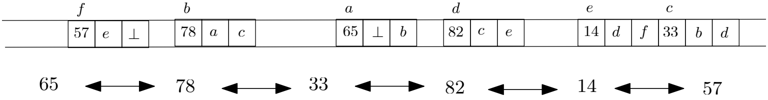
Deletion from a Linked List

Given a pointer to a node u in the linked list, we can delete the node as follows:

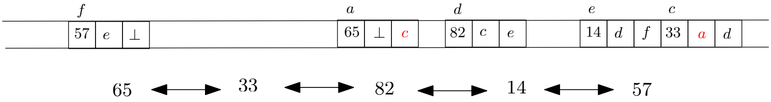
1. Identify the preceding node u_{prec} of u.
2. Identify the succeeding node u_{succ} of u.
3. Set the next-pointer of u_{prec} to the address of u_{succ}.
4. Set the back-pointer of u_{succ} to the address of u_{prec}.
5. Free up the memory of u.

$O(1)$ time

Example



After deleting 78:



Next, we will deploy the linked list to implement two data structures: stack and queue.

Stack

A **stack** manages a set S of elements and supports two operations:

- `push`(e): insert a new element e into S.
- `pop`: remove the **most recently inserted** element from S and returns it.

First-In-Last-Out (FILO).

Example

Consider the following sequence of operations on an empty stack:

- Push(35): $S = \{35\}$.
- Push(23): $S = \{35, 23\}$.
- Push(79): $S = \{35, 23, 79\}$.
- Pop: return 79 after removing it from S. Now $S = \{35, 23\}$.
- Pop: return 23 after removing it from S. Now $S = \{35\}$.
- Push(47): $S = \{35, 47\}$.
- Pop: return 47 after removing it from S. Now $S = \{35\}$.

Linked-List implementation of a Stack

Store the elements of S in a linked list L.

Push(e): insert e at the end of L.
Pop: delete the tail node of L and return the element therein.

At all times, keep track of a pointer to the tail node.

Guarantees:

- $O(n)$ space where $n = |S|$ (assuming that each element in S occupies $O(1)$ memory).
- Push in $O(1)$ time.
- Pop in $O(1)$ time.

Queue

A **queue** stores a set S of elements and supports two operations:

- `en-queue`(e): inserts an element e into S.
- `de-queue`: removes the **least recently inserted** element from S and returns it.

First-In-First-Out (FIFO).

Example

Consider the following sequence of operations on an initially empty queue:

- `En-queue`(35): $S = \{35\}$.
- `En-queue`(23): $S = \{35, 23\}$.
- `En-queue`(79): $S = \{35, 23, 79\}$.
- `De-queue`: return 35 after removing it from S. Now $S = \{23, 79\}$.
- `De-queue`: return 23 after removing it from S. Now $S = \{79\}$.
- `En-queue`(47): $S = \{79, 47\}$.
- `De-queue`: return 79 after removing it from S. Now $S = \{47\}$.

Linked-List Implementation of a Queue

Store the elements of S in a linked list L.

`En-queue`(e): insert e at the end of L.
`De-queue`: delete the head node of L and return the element therein.

At all times, keep track of the addresses of the head and the tail.

Guarantees:

- $O(n)$ space, where $n = |S|$ (assuming each element in S occupies $O(1)$ memory).
- `En-queue` in $O(1)$ time.
- `De-queue` in $O(1)$ time.