BMEG3120: Exercise List 8

Consider the set F of the following functional dependencies on attributes A, B, C, D, E, F:

$$\begin{aligned} A &\rightarrow B \\ A &\rightarrow C \\ CD &\rightarrow E \\ CD &\rightarrow F \\ B &\rightarrow E \end{aligned}$$

Answer the following questions.

Problem 1. Prove $CD \rightarrow EF$ by applying Armstrong's Axioms, i.e., you can use only reflexivity, transitivity, and augmentation.

Answer. Applying augmentation to $CD \rightarrow E$ gives $CD \rightarrow ECD$. Applying augmentation to $CD \rightarrow F$ gives $ECD \rightarrow EF$. Now, by transitivity on $CD \rightarrow ECD$ and $ECD \rightarrow EF$, we get $CD \rightarrow EF$.

Problem 2. Prove $AD \rightarrow EF$ by repeatedly applying Armstrong's Axioms.

Answer. Applying augmentation to $A \rightarrow C$ gives $AD \rightarrow CD$. Applying transitivity on $AD \rightarrow CD$ and $CD \rightarrow E$ gives $AD \rightarrow E$. Similarly, by transitivity on $AD \rightarrow CD$ and $CD \rightarrow F$, we get $AD \rightarrow F$. Given $AD \rightarrow E$ and $AD \rightarrow F$, we can get $AD \rightarrow EF$ following derivation similar to the one in Problem 1.

Problem 3. Prove that $BC \rightarrow F$ cannot be derived from F.

Answer. We utilize the fact that the algorithm discussed in the class for computing the closure of an attribute set has been proven to be correct (by the database people). With the algorithm we get $BC^+ = \{B, C, E\}$. Since F is not in the closure, $BC \rightarrow F$ is wrong.

Problem 4. Is AD a candidate key of the table $R(A, B, C, D, E)$?

Answer. Yes, because AD^+ includes all attributes, but neither $A^+ = \{A, B, C, E\}$ nor $D^+ = \{D\}$ does.

Problem 5. Is AB a candidate key of the table $R(A, B)$?

Answer. No, because $A^+ = \{A, B\}$.