Short experimental projects I
Physics Department
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Topic: Wilberforce pendulum

designed by

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Topics you should know first:

1. Theory of **coupled oscillators**.
2. Normal modes & normal coordinates.
3. Torsional pendulum, spring-mass system (with spring mass correction) & calculation of moment of inertia.

Objectives:

1. Learn Wilberforce pendulum theory.
2. Learn energy transfer in coupled oscillator.
3. Learn **Fourier Transform** & use Fast Fourier Transform to find the normal mode frequencies.
4. Learn **MathCad**.

Check-list for the project:

1. Learn physics of the Wilberforce pendulum (Ref. 1-3)
2. Learn principle of Fourier Transform.(Ref. 4) You need to learn some MathCad operations. (Ref. 5)
3. Set up the Wilberforce pendulum. Use PASCO Motion Sensor (an ultrasound distance sensor) to measure the vertical displacement. (See Appendix on p. 2 & Ref. 6)
4. Measure the spring parameters: outer diameter, wire diameter, relaxed length & spring constant.
5. Measure pendulum bob parameters: dimensions and masses of disk, arm & nuts.
6. Measure frequency of vertical motion.
7. Measure frequency of torsional pendulum as a function of moment of inertia.
8. Select a moment of inertia (I) & initial vertical displacement (z_0) & record the pendulum bob oscillation (i.e., vertical displacement $z(t)$ as a function of time). Measure also the beat rate.

   Do a Fast Fourier Transform to find the normal mode frequencies f_1 & f_2 (by method described in the Appendix or Ref. 5).

   Compare $|f_1 - f_2|$ with the beat rate.
9. Find normal mode frequencies f_1 & f_2 as a function of moment of inertia. (Ref. 2)

   Note: $\left(f_1^2 - f_2^2\right)^2 = \left[\frac{\delta^2}{16\pi^4}\right] I^{-2} + \left[\frac{\varepsilon^2 - 2\delta k}{16\pi^4 m}\right] I^{-1} + \frac{k^2}{16\pi^4 m^2}$ [1]

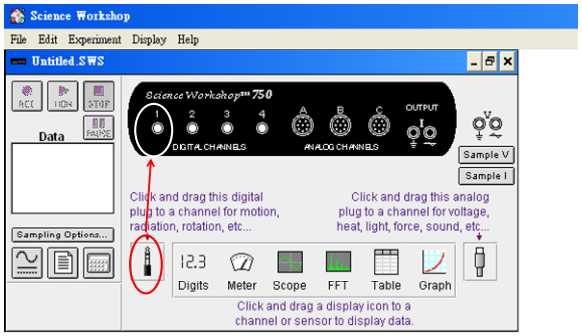
   where we have used the symbols in Ref. 1-2.
10. In the resonance case, demonstrate the complete energy transfer. Use the initial conditions described in equations [28] & [29] of Ref. 1.
11. Plot $\left(f_1^2 - f_2^2\right)^2$ as a function of I^{-1}. Fit data by equation [1] & find the Wilberforce pendulum parameters δ, ε & k..

Appendix: Instructions of setting up PASCO position sensor

(C.K. Lee, 2011-6-23)

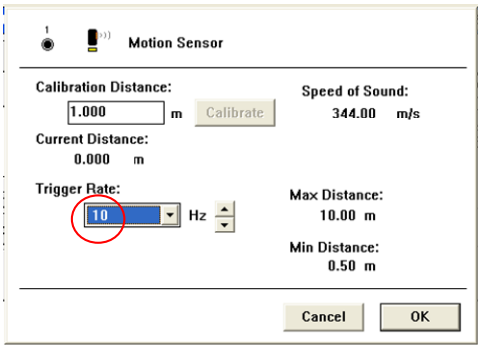
Insert yellow plug to **channel 1** & black plug to **channel 2**.

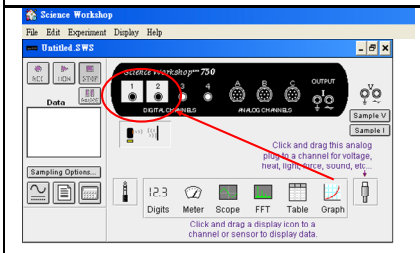
Double-click the icon **Science Workshop** on PC desktop to start the program.



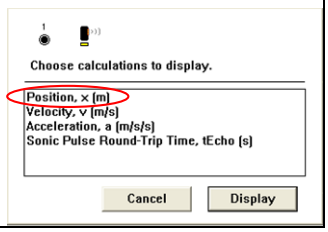
Click the **digital plug** & drag it to **channel 1**.
Select **Motion Sensor** and press **OK**.

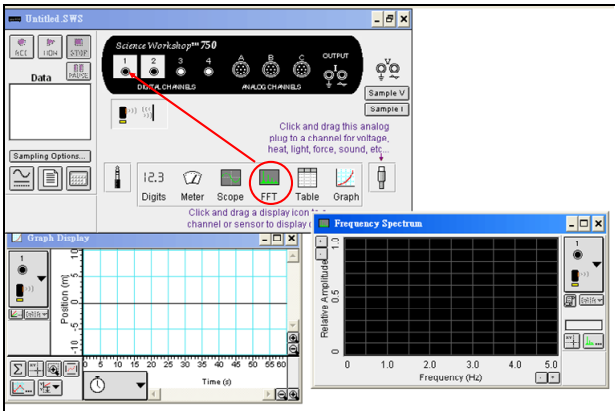
Set **Trigger Rate** = 10 Hz.



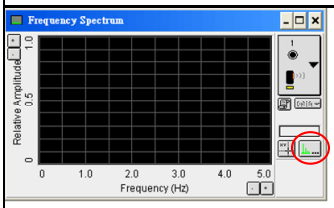


1. Drag the **Graph** icon to **channel 1**.
2. Select **Position, x (m)**, & press **Display**.

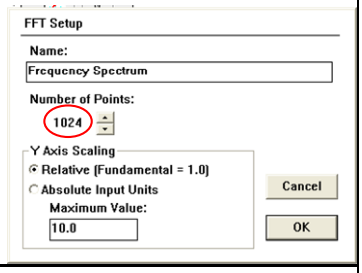




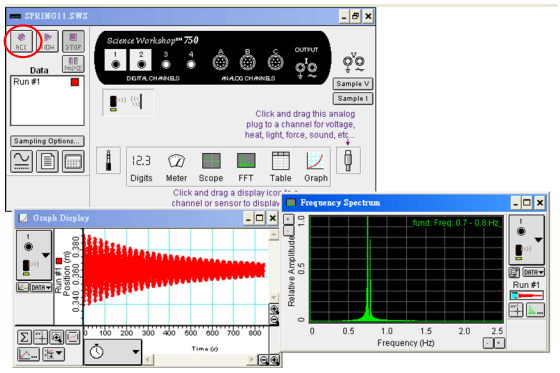
Similarly, drag the icon **FFT** to **channel 1** to turn on the **Fast Fourier Transform** display.

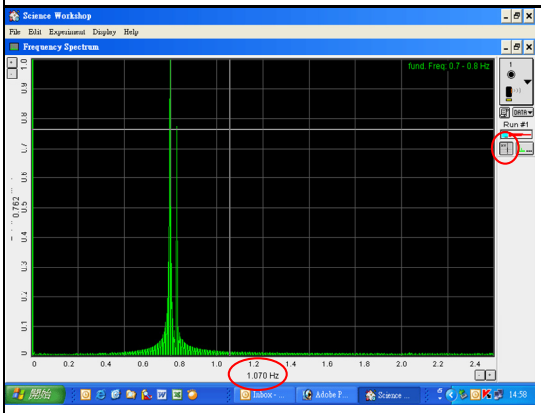


1. Double-click the icon .
2. Set **Number of Points** to **1024**.



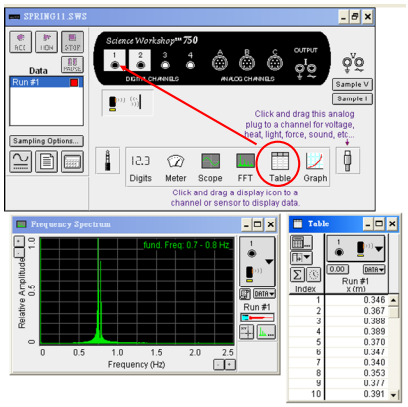
Click **REC** to start the data acquisition.



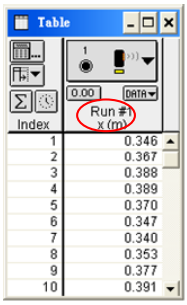


1. Display the cursor by clicking the **cursor** icon.
2. The position of cursor is displayed at the bottom.

To display data, drag the **Table** icon to channel 1.



1. Click the indicated area on the table to highlight the data column.
2. Select **Copy** from **Edit** of the pull-down menu to copy & paste data to Excel.



References (I have a folder of references for this project.)

1. R. E. Berg & T. S. Marshall, "Wilberforce pendulum oscillations and normal modes", American Journal of Physics, vol.59, pp.32-38 (1991).
2. F. G. Karioris, "Wilberforce pendulum, demonstration size", The Physics Teacher, vol. 31, pp.314-315 (1993).
3. E. Debowska et al., "Computer visualization of the beating of a Wilberforce pendulum", European Journal of Physics, vol.20, pp.89-95 (1999).
4. For an introduction of Fourier transform, read S. Van Bramer, "An Introduction to the Fourier Transform", J. Chem. Educ., vol.76, p.286 (1999).
5. MathCad 2001 software is available in UL: **(QA76.95.M38 2001 CD).**
6. Instruction sheet for PASCO Motion Sensor II model Cl-6742.
7. H.K. Wong & C.K. Lee, "MathCad Program (Fourier Transform) for Wilberforce pendulum experiment" (available on course webpage).