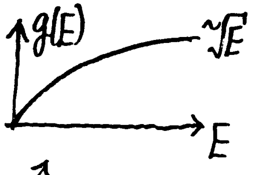
H. Electronic Contribution to heat capacity

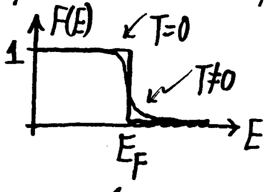
- Recall: Phonon contribution $\sim T^3$ (Debye)
- For conductors, there is a $\sim T$ (linear T) part that shows up clearly in low-temperature data (see figure on next page)

Qualitative understanding of $\sim T$ term:

- $T=0$, all states up to E_F are occupied
-

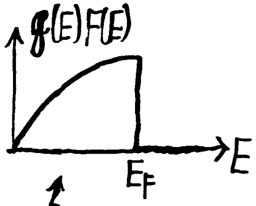


gives how many states are there at energy E (no information on occupancy of states)



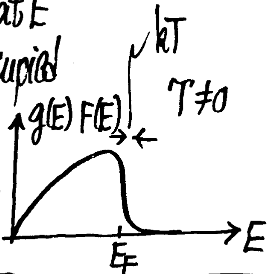
gives information on probability of a state at E being occupied

T=0



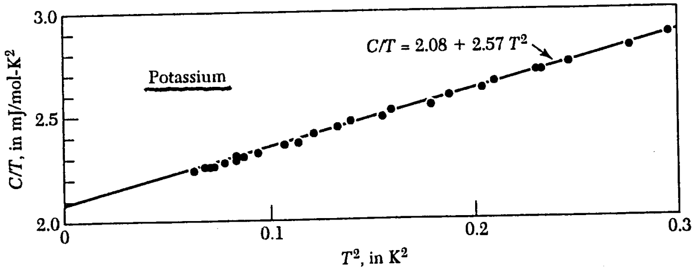
gives the occupied states at $T=0$ (Area gives number density of conduction electron)

← compare →



differ only by a small bit in a range of $\sim kT$ near E_F

Exp'tal Facts



Exp'tal results of Potassium at very low temperatures

$\frac{C}{T}$ vs T^2 gives a straight line (metals)

$$\Rightarrow C = \gamma T + \underbrace{aT^3}$$

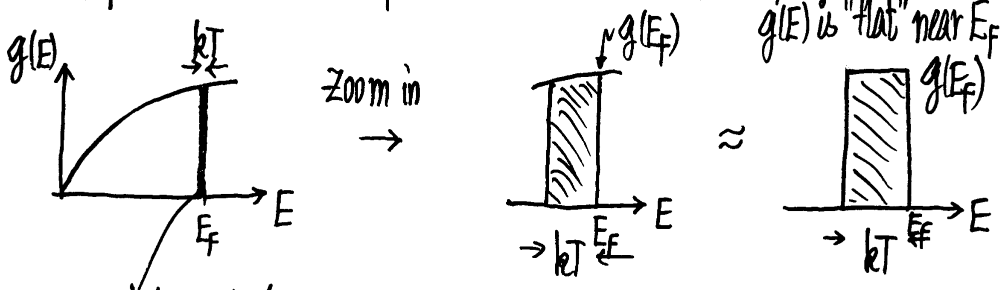
- this term appears in metals only
- it is apparant only at very low temperatures
- comes from the electrons (ideal Fermi Gas) at $T \ll T_F$

- due to lattice vibrations (phonons)
- this term appears in both metals and insulators
- this term becomes smaller at low temperatures because kT cannot excite most of the normal modes (oscillators)

- $C = \frac{\partial U}{\partial T}$
- $U(T) - U(T=0)$ = change in energy due to electrons near E_F

$$\approx \left(g(E_F)\, kT\right) \cdot kT$$

$kT \sim 0.01\,\text{eV}$, $E_F \sim$ a few eV $\quad (kT \ll E_F)$



these electrons may be affected by temperature as there are unoccupied states in their vicinity

Area = $g(E_F) \cdot kT$
= # electrons affected by thermal energy kT

$\therefore\ C \approx 2\, g(E_F)\, k^2 T \sim T$ (<u>as observed</u>)

A full calculation (statistical physics) gives

$$C = \frac{\pi^2}{3} g(E_F)\, k^2 T$$

∴ Hand-waving argument gives very good result!
[since it captures the key physics – Pauli Exclusion Principle]

$\therefore$ Measuring $C \Rightarrow \gamma$ (metals)
$\Rightarrow g(E_F)$ (DOS at E_F)

<u>What if we don't know QM and Pauli Exclusion Principle?</u>

"Free electrons" $\Rightarrow \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{p_z^2}{2m}$
for each particle

Classical statistical mechanics $\Rightarrow \frac{1}{2}kT$ for each term

$$\therefore\ U = N \cdot 3 \cdot \tfrac{1}{2}kT$$

$$\Rightarrow C_{\text{classical}} = \frac{3}{2}Nk \qquad \text{(wrong!)}$$

This should be compared with

$$C = \frac{\pi^2}{3} g(E_F)\, k^2 T$$
$$= \frac{\pi^2}{2} Nk \cdot \left(\frac{kT}{E_F}\right)$$
$$\sim C_{\text{classical}} \cdot \underbrace{\left(\frac{kT}{E_F}\right)}_{\ll 1 \ (\sim 10^{-2})}$$

$\therefore\ C_{\text{actual}} \ll C_{\text{classical}}$
(since most electrons can't contribute due to Pauli Exclusion Principle)

I. A review on basics of (pre-quantum) transport theory

(a) R, G, ρ, σ (Resistance, Conductance, resistivity, conductivity)

$$R = \frac{V}{I}$$

V ← Voltage across a piece of material (Volts)

I ← current (Amperes)

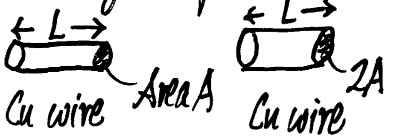
R: resistance (Ohm)

the equation defines R — depends on material and dimensions (size) of sample

$$G = \frac{1}{R}$$

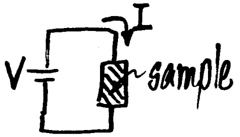
defines G

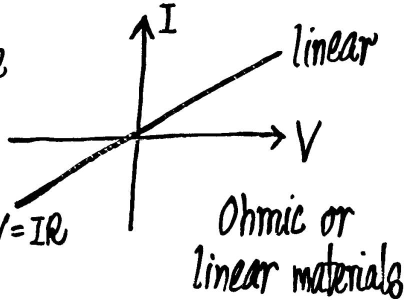
G: conductance



Ohm's law:

R = constant



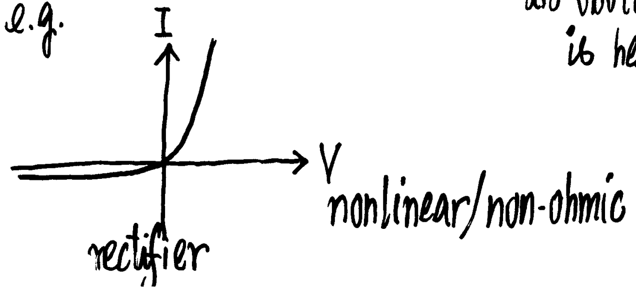


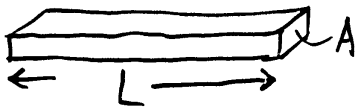
Note: Many (useful) materials show non-ohmic I-V.

I(V) such that $\frac{V}{I} \neq$ constant

↖ an obvious/simple reason is heating/temperature effect

e.g.





$$R = \rho \frac{L}{A}$$

ρ: resistivity (a material's property) (Ohm-m)

$$\vec{J} = \sigma \vec{E}$$

$\vec{J}$: current density

$\vec{E}$: electric field

σ: conductivity $= \frac{1}{\rho}$ [units: $(\text{Ohm-m})^{-1}$ or $\frac{S}{m}$] ("siemens")

Ohmic (linear) materials:

σ is independent of $\vec{E}$

Isotropic material: (各向同性)

$$\vec{J} = \sigma \vec{E}$$

σ: a scalar quantity

$\vec{J}$ follows the direction of $\vec{E}$

Anisotropic material:

$$\vec{J} = \overleftrightarrow{\sigma} \vec{E}$$

$\overleftrightarrow{\sigma}$: tensor (3x3 matrix)

$$\begin{pmatrix} J_x \\ J_y \\ J_z \end{pmatrix} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

- Real materials show a very wide range of ρ or σ
- can differ by ~ 10^{24}!

10^{-8} Ω-m
good conductors

ρ Electrical Resistivity (ohm-m)	(log)		σ Electrical Conductivity (S/m)
	−8	8	copper, silver
intercalated graphite	−7	7	iron
	−6	6	stainless steel, metallic glass
graphite (in-plane) graphite (out of plane)	−5	5	$YBa_2Cu_3O_7$ (ab plane)
polyacetylene (doped)	−4	4	$YBa_2Cu_3O_7$ (c-axis)
	−3	3	silicon (doped)
TTF-TCNQ	−2	2	ZnO (doped) seawater
	−1	1	Fe_3O_4
	0	0	germanium
	1	−1	
	2	−2	silicon
	3	−3	InSb
	4	−4	
	5	−5	water AgCl
polyacetylene (undoped)	6	−6	
	7	−7	
	8	−8	ZnO (undoped)
	9	−9	
	10	−10	
Bakelite	11	−11	NaCl
polypyrrole	12	−12	
	13	−13	
Lucite (PMMA)	14	−14	Al_2O_3
polyvinyl chloride	15	−15	mica silica
polyethylene, teflon	16	−16	diamond

10^8 S/m (10^8 $Ω^{-1}m^{-1}$)

steel: alloyed iron

} high Tc SC (anisotropic)

semiconductors

insulators
10^{16} Ω-m

10^{-16} S/m

Resistivity and conductivity of a variety of materials at room temperature. (Note: values are approximate, and for insulators and semiconductors, *very* sensitive to impurities.)

[From J.D. Livingston, "Electronic Properties of Engineering Materials"

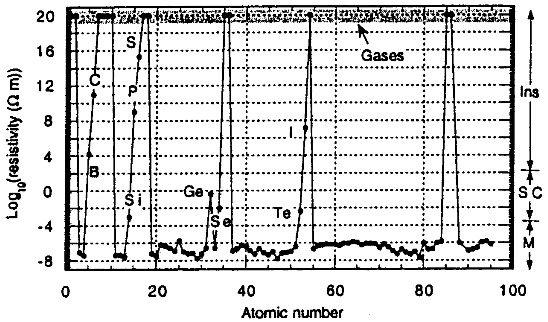
ρ and σ of pure elements at room temperature

Z	Element	ρ(Ω m)	σ(S m⁻¹)	Z	Element	ρ(Ω m)	σ(S m⁻¹)
1	Hydrogen, H	—	—	49	Indium, In	8.37×10^{-8}	1.19×10^{7}
2	Helium, He	—	—	50	Tin, Sn	1.1×10^{-7}	9.1×10^{6}
3	Lithium , Li	8.55×10^{-8}	1.17×10^{7}	51	Antimony, Sb	3.9×10^{-7}	2.56×10^{6}
4	Beryllium, Be	4×10^{-8}	2.5×10^{7}	52	Tellurium, Te	0.00436	229
5	Boron, B	18000	5.56×10^{5}	53	Iodine, I	1.37×10^{-7}	7.30×10^{6}
6	Carbon (diamond), C	10^{11}	10^{-11}	54	Xenon, Xe	—	—
7	Nitrogen, N	—	—	55	Caesium, Cs	2×10^{-7}	5×10^{6}
8	Oxygen, O	—	—	56	Barium, Ba	5×10^{-7}	2×10^{6}
9	Fluorine, F	—	—	57	Lanthanum, La	5.7×10^{-7}	1.75×10^{6}
10	Neon, Ne	—	—	58	Cerium, Ce	7.3×10^{-7}	1.37×10^{6}
11	Sodium, Na	4.2×10^{-8}	2.38×10^{7}	59	Praseodymium, Pr	6.8×10^{-7}	1.47×10^{6}
12	Magnesium, Mg	4.38×10^{-8}	2.28×10^{7}	60	Neodymium, Ne	6.4×10^{-7}	1.56×10^{6}
13	Aluminium, Al	2.66×10^{-8}	3.77×10^{7}	61	Promethium, Pm	5×10^{-7}	2×10^{6}
14	Silicon, Si	0.001	1000	62	Samarium, Sm	9.4×10^{-7}	1.06×10^{6}
15	Phosphorus, P	1×10^{-9}	1×10^{9}	63	Europium, Eu	9×10^{-7}	1.11×10^{6}
16	Sulphur, S	2×10^{15}	5×10^{-16}	64	Gadolinium, Gd	1.34×10^{-6}	7.46×10^{5}
17	Chlorine, Cl	—	—	65	Terbium, Tb	1.14×10^{-6}	8.77×10^{5}
18	Argon, Ar	—	—	66	Dysprosium, Dy	5.7×10^{-7}	1.75×10^{6}
19	Potassium, K	6.15×10^{-8}	1.63×10^{7}	67	Holmium, Ho	8.7×10^{-7}	1.15×10^{6}
20	Calcium, Ca	3.43×10^{-8}	2.92×10^{7}	68	Erbium, Er	8.7×10^{-7}	1.15×10^{6}
21	Scandium, Sc	6.1×10^{-7}	1.64×10^{6}	69	Thulium, Th	7.9×10^{-7}	1.27×10^{6}
22	Titanium, Ti	4.2×10^{-7}	2.38×10^{6}	70	Ytterbium, Yb	2.9×10^{-7}	3.45×10^{6}
23	Vanadium, V	2.48×10^{-7}	4.03×10^{6}	71	Lutetium, Lu	7.9×10^{-7}	1.27×10^{6}
24	Chromium, Cr	1.27×10^{-7}	7.87×10^{6}	72	Hafnium, Hf	3.51×10^{-7}	2.85×10^{6}
25	Manganese, Mn	1.85×10^{-6}	5.41×10^{5}	73	Tantalum, Ta	1.25×10^{-7}	8.03×10^{6}
26	Iron, Fe	9.71×10^{-8}	1.03×10^{7}	74	Tungsten, W	5.65×10^{-8}	1.77×10^{7}
27	Cobalt, Co	6.24×10^{-8}	1.60×10^{7}	75	Rhenium, Re	1.93×10^{-7}	5.18×10^{6}
28	Nickel, Ni	6.84×10^{-8}	1.46×10^{7}	76	Osmium, Os	8.12×10^{-8}	1.23×10^{7}
29	Copper, Cu	1.67×10^{-8}	5.98×10^{7}	77	Iridium, Ir	5.3×10^{-8}	1.89×10^{7}
30	Zinc, Zn	5.92×10^{-8}	1.69×10^{7}	78	Platinum, Pt	1.06×10^{-7}	9.43×10^{6}
31	Gallium, Ga	2.7×10^{-7}	3.70×10^{6}	79	Gold, Au	2.35×10^{-8}	4.26×10^{7}
32	Germanium, Ge	0.46	2.1739	80	Mercury, Hg	9.41×10^{-7}	1.06×10^{6}
33	Arsenic, As	2.6×10^{-7}	3.85×10^{6}	81	Thallium, Th	1.8×10^{-7}	5.56×10^{6}
34	Selenium, Se	0.01	100	82	Lead, Pb	2.07×10^{-7}	4.84×10^{6}
35	Bromine, Br	—	—	83	Bismuth, Bi	1.068×10^{-6}	9.36×10^{5}
36	Krypton, Kr	—	—	84	Polonium, Po	1.4×10^{-6}	7.14×10^{5}
37	Rubidium, Rb	1.25×10^{-7}	8×10^{6}	85	Astatine, At	—	—
38	Strontium, Sr	2.3×10^{-7}	4.35×10^{6}	86	Radon, Rn	—	—
39	Yttrium, Y	5.7×10^{-7}	1.75×10^{6}	87	Francium, Fr	–	–
40	Zirconium, Zr	4.21×10^{-7}	2.37×10^{6}	88	Radium, Ra	1×10^{-6}	1×10^{6}
41	Niobium, Nb	1.25×10^{-7}	8×10^{6}	89	Actinium, Ac	–	–
42	Molybdenum, Mo	5.2×10^{-8}	1.92×10^{7}	90	Thorium, Th	1.3×10^{-7}	7.69×10^{6}
43	Technetium, Tc	2.26×10^{-7}	4.42×10^{6}	91	Protractinium, Pa	1.77×10^{-7}	5.65×10^{6}
44	Ruthenium, Ru	7.6×10^{-8}	1.32×10^{7}	92	Uranium, U	3.08×10^{-7}	3.25×10^{6}
45	Rhodium, Rh	4.51×10^{-8}	2.22×10^{7}	93	Neptunium, Np	1.22×10^{-6}	8.20×10^{5}
46	Palladium, Pd	1.08×10^{-7}	9.26×10^{6}	94	Plutonium, Pu	1.46×10^{-6}	6.85×10^{5}
47	Silver, Ag	1.59×10^{-8}	6.29×10^{7}	95	Americium, Am	6.8×10^{-7}	1.4706×10^{6}
48	Cadmium, Cd	6.83×10^{-8}	1.46×10^{7}				

- Most are conductors (elemental solids)
- Only a few are exceptional bad conductors, e.g. C, S, Si, Ge, Se.

elements at room temperature

log(ρ) →



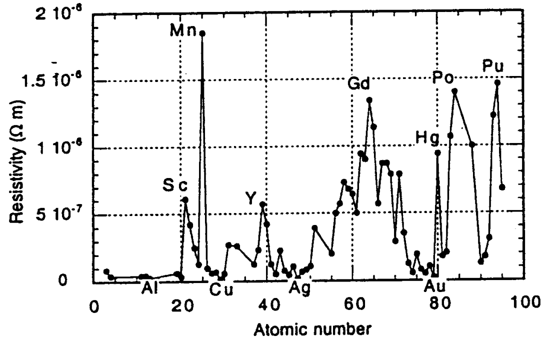
Insulators

semiconductors

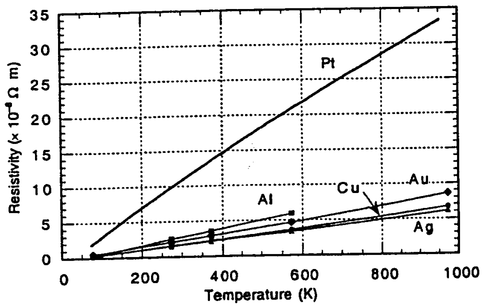
metals

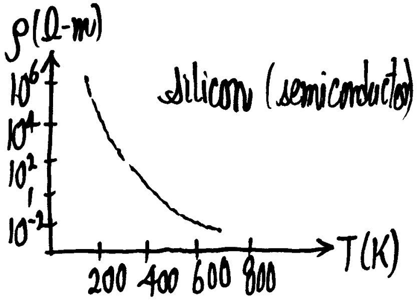
zoom in

linear scale →



ρ(T) for metals





Q: ρ(T) for metals? for semiconductors?

Q: Why do we have materials with ρ or σ differ by 10^{24}?

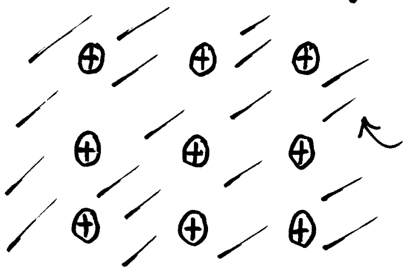
To answer these questions, we need to invoke ideas in quantum mechanic

(b) <u>Drude model of σ</u> (~1900)

- without knowing QM

$\underbrace{\text{classical}}_{\text{use non-quantum physics}}$ $\underbrace{\text{free}}_{\text{interactions between } e^- \text{ and } e^- \text{ and } e^- \text{ with ions are neglected between collisions}}$-$\underbrace{\text{electron}}_{\text{discovered in 1897 by Thomson}}$ theory of $\overbrace{\text{metals}}^{\text{good conductors}}$ [model conductors for theory: Na, K]

Metal: ions + sea of free electrons



Drude: This sea of electrons is responsible for conduction.

Drude:

Do the electrons move around for $\vec{E}=0$? (without applied field)

Yes, thermal motion

→ but electrons move around in all directions
(no preferred direction)

⇒ No net charge through a unit area per unit time

⇒ $\vec{J}=0$

⌜ An estimate of v_{thermal}: (classical theory)

$$\frac{1}{2}mv_{th}^2 = \frac{3}{2}kT \Rightarrow v_{th} = \sqrt{\frac{3kT}{\underset{\nwarrow \text{ mass of } e^-}{m}}}$$

For $T \sim 300\text{K}$,

$$v_{th} \sim \underbrace{10^5 \text{ ms}^{-1}}_{\text{high speed}}$$

⌟

How about $\vec{E} \neq 0$? (with applied field)

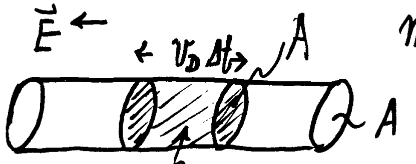
- We know $\vec{E}\neq 0$, $\vec{J}\neq 0$

⇒ $\vec{E}$ gives an <u>average drift velocity</u> $\vec{v}_D$ to the electrons
($v_D \ll v_{th}$)

In terms of $\vec{v}_D$, $\boxed{\vec{J} = -e\underset{\uparrow}{n}\vec{v}_D}$ (1)

charge carrier number density

⌜



charges in this volume $Av_D\cdot\Delta t$ will pass through a cross section A in time Δt due to average drift velocity v_D

\# charges in volume $Av_D\cdot\Delta t = (-e)\,n\cdot Av_D\cdot\Delta t$

$$J = \frac{\#\text{ charges}}{\text{area}\cdot\text{time}}$$

$$\vec{J} = \frac{(-e)\,n\cdot A\cdot\vec{v}_D\cdot\Delta t}{A\cdot\Delta t} = -en\vec{v}_D$$

$\vec{E}\neq 0$ gives $\vec{v}_D \neq 0$ and hence $\vec{J}\neq 0$ ⌟

A related quantity is $\underbrace{\text{mobility } \mu_e}$;
in semiconductor handbooks, one can find data on mobility.

Mobility μ_e:

Ohm's law : σ is field independent $(\vec{J} = \sigma\vec{E})$

$$\vec{J} = -en\vec{v}_D$$

The two equations are consistent if $\vec{v}_D \propto \vec{E}$

write as $\vec{v}_D = -\mu_e \vec{E}$

$-$: due to (-e) charge of electron

μ_e : mobility [Units: $m^2 V^{-1} s^{-1}$]

$$\vec{J} = -en\vec{v}_D$$
$$= -en(-\mu_e\vec{E})$$
$$= en\mu_e\vec{E}$$
$$= \sigma\vec{E}$$

$$\therefore \boxed{\sigma = en\mu_e} \quad (2)$$

σ : conductivity (material's property)

n : carrier number density (material's property)

μ_e : mobility (material's property)

RHS : isolate the 2 factors affecting σ

Remark: (1) and (2) are general relations.

Effect of $\vec{E} \neq 0$

- leads to $\vec{v}_D$
- but $\vec{J}$ does not keep on increasing with time!

$\Rightarrow$ the simple idea of $\vec{E}$-field accelerates the electron:

$$m\frac{d\vec{v}}{dt} = -e\vec{E} \Rightarrow \vec{v} = \frac{-e}{m}\vec{E}\,t$$

$$\Rightarrow \vec{J} \propto t$$

does NOT work!

- Experimentally, after the circuit is closed, the ammeter reading increases and then eventually becomes stationary.

Analogy: (a) Why ain't you hurt when raindrops keep falling on your head?

(b) Millikan's oil drop experiment

Idea: Terminal Velocity!

How can the electrons attain a terminal velocity in the presence of $\vec{E}$?

Need a frictional or damping term

Drude: come from electrons' frequent collisions with ions

Equation of motion / Hydrodynamic description of frictional force (drag) due to collisions

- Collisions serve as a source of frictional or viscous drag force

Viscous force: ▪ oppose motion
▪ $\propto v$

Eq. of motion:

$$m\frac{d\vec{v}_D}{dt} = \underset{\substack{\uparrow \\ \text{sum of} \\ \text{all forces}}}{\vec{F}} = \underbrace{-e\vec{E}}_{\substack{\uparrow \\ \text{driving} \\ \text{force}}} \underbrace{- \frac{m\vec{v}_D}{\tau}}_{\substack{\text{frictional drag} \\ \text{due to collisions}}} + \text{other possible forces}$$

$$\therefore \boxed{m\frac{d\vec{v}_D}{dt} \approx -\frac{m\vec{v}_D}{\tau} - e\vec{E}} \quad (*)$$

τ = a parameter characterizes the importance of collisions[+]
$\cong$ average time between collisions
= collision time or relaxation time

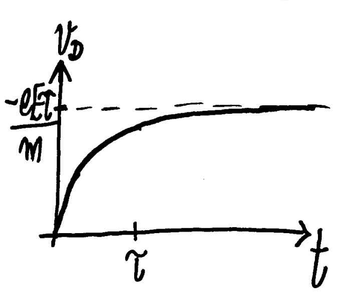
Noting $\vec{p}$ = momentum = $m\vec{v}_D$

$$\frac{d\vec{p}}{dt} = -\frac{\vec{p}}{\tau} - e\vec{E}$$

[+] Thus, the Drude model amounts to saying that the electrons are accelerated by the E-field only for the duration τ between collisions.

(*) can be solved to give:

$$v_D = -\frac{eE\tau}{m}\left[1 - e^{-t/\tau}\right]$$



For $t \gg \tau$, v_D becomes saturated
(terminal velocity)
$\Downarrow$
steady state

In steady state, $\frac{d}{dt} = 0 \Rightarrow -\frac{m\vec{v}_D}{\tau} = e\vec{E}$

$$\Rightarrow \vec{v}_D = \underbrace{-\frac{e\tau}{m}}_{\mu_e = \text{mobility}}\vec{E}$$

$\therefore$ Collisions $\rightarrow$ frictional drag $\rightarrow$ a steady $\vec{v}_D$

$$\vec{J} = -en\vec{v}_D = \frac{ne^2\tau}{m}\vec{E} = \sigma\vec{E}$$

$$\therefore \boxed{\sigma = \frac{ne^2\tau}{m}} \quad \text{Key result}^{+}$$

<u>Note</u>: The condition $t \gg \tau$ means that there are many collisions in the time duration t.

[+] Some of Drude's thoughts on what are causing τ is wrong. But the result $\sigma = \frac{ne^2\tau}{m}$ remains valid in quantum treatments.

$$\vec{v}_D = -\frac{e\tau}{m}\vec{E}$$ (not increasing with time, just like the terminal velocity)

$$\underbrace{m\vec{v}_D}_{} = -e\tau\vec{E}$$

each electron acquires such momentum

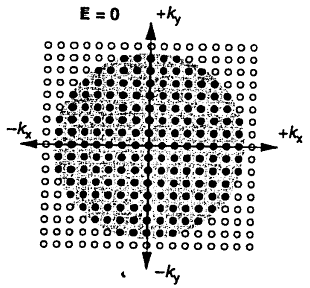
$$\underbrace{\frac{m\vec{v}_D}{\hbar}}_{} = \frac{-e\tau\vec{E}}{\hbar}$$

↖ each electron shifts by this amount $\Delta\vec{k}$ in $\vec{k}$-space

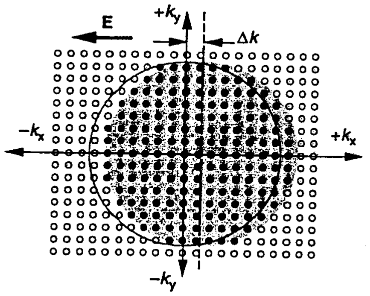
$\vec{E}=0$ Fermi sphere centered at $\vec{k}=0$

$\vec{E}\neq0$ Shifted Fermi sphere

$\vec{E}=0$



$\vec{E}\neq0$



- Shifted Fermi sphere in steady state
- Steady state is maintained by $\vec{E}$ and electron scattering processes

$$\boxed{\sigma = \frac{ne^2\tau}{m}}$$

- The most important formula in thinking about conductivity
- τ or $1/\tau$ is a parameter that models the effects of scatterings of electrons that lead to resistance
  - this parameter is not easy to calculate (needs QM perturbation theory)
  - usually this parameter can be extracted from data
- What kinds of scattering mechanism that would lead to resistance?
  - electron – regular array of ions? ×
  - electron - lattice vibrations (electron-phonon scattering) ✓
  - electron - impurity scattering ✓
  - electron - sample boundary scattering ✓
  - electron - electron scattering (Coulomb) ✓

⋮

Values of n for metals

FREE ELECTRON DENSITIES OF SELECTED METALLIC ELEMENTS[a]

ELEMENT	Z	n ($10^{22}/\text{cm}^3$)	r_s(Å)	r_s/a_0
Li (78 K)	1	4.70	1.72	3.25
Na (5 K)	1	2.65	2.08	3.93
K (5 K)	1	1.40	2.57	4.86
Rb (5 K)	1	1.15	2.75	5.20
Cs (5 K)	1	0.91	2.98	5.62
Cu	1	8.47	1.41	2.67
Ag	1	5.86	1.60	3.02
Au	1	5.90	1.59	3.01
Be	2	24.7	0.99	1.87
Mg	2	8.61	1.41	2.66
Ca	2	4.61	1.73	3.27
Sr	2	3.55	1.89	3.57
Ba	2	3.15	1.96	3.71
Nb	1	5.56	1.63	3.07
Fe	2	17.0	1.12	2.12
Mn (α)	2	16.5	1.13	2.14
Zn	2	13.2	1.22	2.30
Cd	2	9.27	1.37	2.59
Hg (78 K)	2	8.65	1.40	2.65
Al	3	18.1	1.10	2.07
Ga	3	15.4	1.16	2.19
In	3	11.5	1.27	2.41
Tl	3	10.5	1.31	2.48
Sn	4	14.8	1.17	2.22
Pb	4	13.2	1.22	2.30
Bi	5	14.1	1.19	2.25
Sb	5	16.5	1.13	2.14

[a] At room temperature (about 300 K) and atmospheric pressure, unless otherwise noted. The radius r_s of the free electron sphere is defined in Eq. (1.2). We have arbitrarily selected one value of Z for those elements that display more than one chemical valence. The Drude model gives no theoretical basis for the choice. Values of n are based on data from R. W. G. Wyckoff, *Crystal Structures*, 2nd ed., Interscience, New York, 1963.

From Ashcroft and Mermin: Solid State Physics

Note:

$n \sim 10^{22} - 10^{23}/\text{cm}^3$

Q: How can one obtain n from crystal structure? from density? from atomic volume (cm^3/mole)?

Remark: It is important to have some common sense on materials properties in studying solid state physics.

ρ = resistivity

[$\mu\Omega$-cm]

ELECTRICAL RESISTIVITIES OF SELECTED ELEMENTS[a]

ELEMENT	77 K	273 K	373 K	$\frac{(\rho/T)_{373\text{ K}}}{(\rho/T)_{273\text{ K}}}$
Li	1.04	8.55	12.4	1.06
Na	0.8	4.2	Melted	
K	1.38	6.1	Melted	
Rb	2.2	11.0	Melted	
Cs	4.5	18.8	Melted	
Cu	0.2	1.56	2.24	1.05
Ag	0.3	1.51	2.13	1.03
Au	0.5	2.04	2.84	1.02
Be		2.8	5.3	1.39
Mg	0.62	3.9	5.6	1.05
Ca		3.43	5.0	1.07
Sr	7	23		
Ba	17	60		
Nb	3.0	15.2	19.2	0.92
Fe	0.66	8.9	14.7	1.21
Zn	1.1	5.5	7.8	1.04
Cd	1.6	6.8		
Hg	5.8	Melted	Melted	
Al	0.3	2.45	3.55	1.06
Ga	2.75	13.6	Melted	
In	1.8	8.0	12.1	1.11
Tl	3.7	15	22.8	1.11
Sn	2.1	10.6	15.8	1.09
Pb	4.7	19.0	27.0	1.04
Bi	35	107	156	1.07
Sb	8	39	59	1.11

[a] Resistivities in microhm centimeters are given at 77 K (the boiling point of liquid nitrogen at atmospheric pressure), 273 K, and 373 K. The last column gives the ratio of ρ/T at 373 K and 273 K to display the approximate linear temperature dependence of the resistivity near room temperature.

Source: G. W. C. Kaye and T. H. Laby, *Table of Physical and Chemical Constants*, Longmans Green, London, 1966.

From Ashcroft and Mermin: Solid State Physics

$$\sigma = \frac{ne^2\tau}{m} \Rightarrow \tau = \frac{m\sigma}{ne^2} = \frac{m}{ne^2\rho}$$

DRUDE RELAXATION TIMES IN UNITS OF 10^{-14} SECOND[a]

$[10^{-14}s]$

ELEMENT	77 K	273 K	373 K
Li	7.3	0.88	0.61
Na	17	3.2	
K	18	4.1	
Rb	14	2.8	
Cs	8.6	2.1	
Cu	21	2.7	1.9
Ag	20	4.0	2.8
Au	12	3.0	2.1
Be		0.51	0.27
Mg	6.7	1.1	0.74
Ca		2.2	1.5
Sr	1.4	0.44	
Ba	0.66	0.19	
Nb	2.1	0.42	0.33
Fe	3.2	0.24	0.14
Zn	2.4	0.49	0.34
Cd	2.4	0.56	
Hg	0.71		
Al	6.5	0.80	0.55
Ga	0.84	0.17	
In	1.7	0.38	0.25
Tl	0.91	0.22	0.15
Sn	1.1	0.23	0.15
Pb	0.57	0.14	0.099
Bi	0.072	0.023	0.016
Sb	0.27	0.055	0.036

Ex: Obtain τ from data of ρ and n

Ex: obtain the mobility μ_e from data

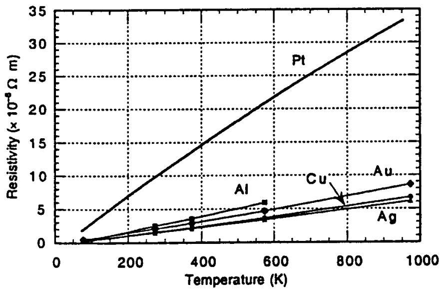
Take Ag at 77K $\quad \tau \sim 2\times10^{-13}s$

$l \sim$ mean free path $\sim v_F \cdot \tau \sim 10^5 ms^{-1} \cdot 10^{-13}s \sim 10^{-8}m$

Lattice constant $\sim 3Å = 3\times10^{-10}m$

$\Rightarrow$ electron can move through many lattice constants between scattering!

Why? (need energy band theory)



$\rho(T)$ for good metals increase linearly with T in a range of temperature near and above room temp.

Why? Electron-$\underbrace{\text{phonon}}_{\text{lattice vibrations}}$ scatterings dominate

$$\sigma = \frac{ne^2\tau}{m} \Rightarrow \rho = \frac{m}{ne^2}\cdot\frac{1}{\tau}$$

$\frac{1}{\tau}$ = "scattering rate" = # scatterings per second

$\frac{1}{\tau} \propto n_{ph}(T)$ = # phonons at temp. T

For $T \gg \theta_D$ (Debye temperature), $n_{ph}(T) \sim T$

$\therefore \frac{1}{\tau} \propto T \Rightarrow \rho \propto T$ (as observed)

Refs: Kittel Ch.6

Ashcroft and Mermin Ch.1