

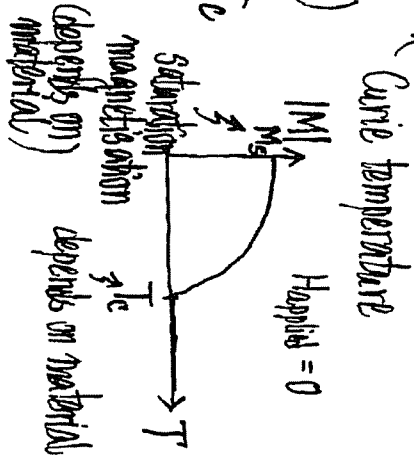
XVI. Ferromagnetism

A. Key Features

spontaneous magnetization

- No external applied field

$$\begin{cases} M \neq 0 & \text{for } T < T_c \\ \text{"spontaneous"} & \\ \text{(:: No applied field)} & \\ M = 0 & \text{for } T > T_c \end{cases}$$



Saturation Magnetization M_s , Curie Temperature T_c , and Effective Number of Bohr Magnetons p for Ferromagnetic Elements

Material	M_s (10^6 A/m)	T_c (K)	p
Iron	1.75	1043	2.219
Cobalt	1.45	1404	1.715
Nickel	0.512	631	0.604
Gadolinium	2.00	289	7.12
Terbium	1.44	230	4.95
Dysprosium	2.01	85	6.84
Holmium	2.55	20	8.54

Sources: American Institute of Physics Handbook (D. W. Gray, Ed.) (New York: McGraw-Hill, 1963).

"Universal behavior" after proper normalization of data

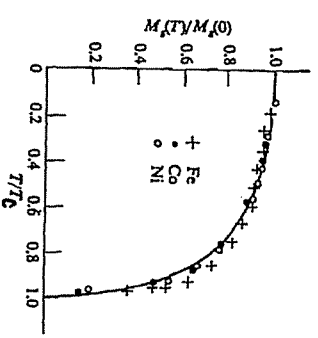
- Ferromagnetic material A: Measure $M(T)$
 $\Rightarrow T_c^{(A)}, M_s^{(A)}$

Ferromagnetic material B: Measure $M(T)$

$\Rightarrow T_c^{(B)}, M_s^{(B)}$

$$\frac{M(T)}{M_s} \text{ vs } \frac{T}{T_c}$$

- Nearly same curve for different materials
- particularly for $T \approx T_c$



Data collapse onto the same curve

$M \sim (T_c - T)^p$ for $T \lesssim T_c$

and same value of p for many different materials

$M =$ "order parameter" of the problem of ferromagnetism

"continuous phase transition" at $T = T_c$:

$$\begin{matrix} T > T_c & M = 0 \text{ phase} \\ T < T_c & M \neq 0 \text{ phase} \end{matrix}$$

and M changes continuously at T_c .

Paramagnetic behavior at $T > T_c$

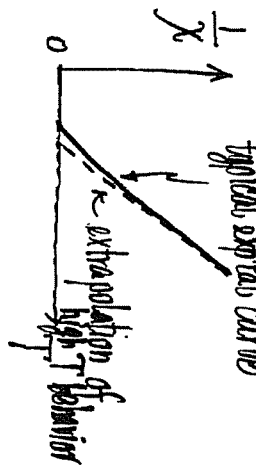
$T > T_c$, $M = 0$ if no applied field

With applied field,

$M \neq 0$ and χ is positive (paramagnetic)

χ follows the form

$$\chi = \frac{C}{T - T_c}$$



Recall: Paramagnetic: $\chi \sim \frac{C}{T}$

B. Hints from theory of paramagnetism

- ferromagnetic materials
 - atoms/ions with permanent magnetic dipole moment

$M \neq 0$ for $T < T_c$ without applied field

Key idea: there must be some internal field acting on each dipole moment

- internal field comes from interaction between a dipole with neighboring dipole moments
- kT competes with internal field

Recall: Paramagnetism

$$M = n g \mu_B J \cdot B_J \left(\frac{g \mu_B J B}{kT} \right)$$

Brillouin function

B-field acting on the independent (non-interacting) magnetic moments

Each dipole moment $\vec{\mu}_i$ interacts with applied B-field

\therefore Hamiltonian is:

$$H_{para} = \sum_i (-\vec{\mu}_i \cdot \vec{B}_0)$$

due to external applied field

C. Approaching the ferromagnetic problem

Moments are interacting (no longer independent)

usually short range (nearest neighbors)

$$H_{ferro} = \sum_{\langle ij \rangle} -J_{ij} \vec{\mu}_i \cdot \vec{\mu}_j + \sum_i (-\vec{\mu}_i \cdot \vec{B}_0)$$

describes interaction between $\vec{\mu}_i$ and $\vec{\mu}_j$ (" $-ve$ " when $J_{ij} > 0$)

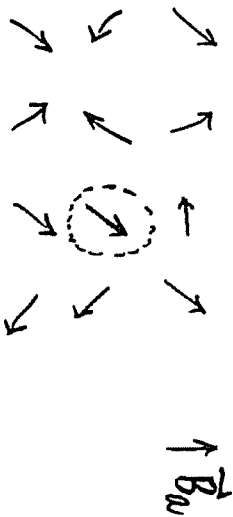
energy is lowest when $\vec{\mu}_i \parallel \vec{\mu}_j$

i.e., ferromagnetic interaction

describes effect of an external applied field (if it is there)

Then, one can do statistical mechanics on H_{ferro} .

D. The Weiss molecular field theory: Mean field theory.



① feels \vec{B}_a + a field due to moment-moment interaction with the neighboring moments

(i) approximate as



feels \vec{B}_a + a mean field from the environment that depends on the average behavior of the moments

(ii) $\vec{B}_{local} = \vec{B}_{mean field} + \vec{B}_a$
 ① is nothing special

⇒ same mean field on all moments

"ignore fluctuations": ignore differences in the real local fields at different moments

(iii) set up self-consistency (自洽)

- if moments are randomly oriented (M is small), expect mean field is weak
- if moments are aligned (M is large), expect mean field is strong

∴ $\vec{B}_{mean field} \propto \vec{M}$

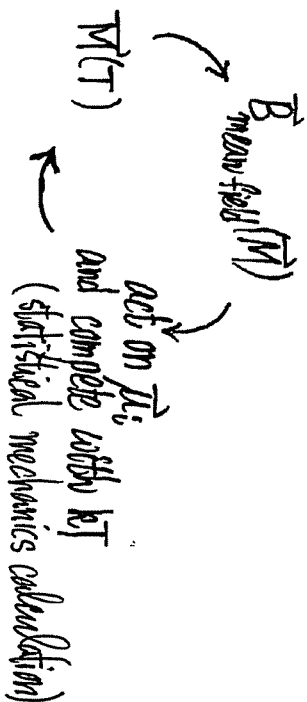
write $\vec{B}_{mean field} = \gamma \mu_0 \vec{M}$

where constant: a parameter that characterizes the strength of mean field (related to strength J_{ij} of interaction)

∴ $\vec{B}_{local} = \gamma \mu_0 \vec{M} + \vec{B}_a$

• but $\vec{M}(T)$ is what we want to calculate!

Self-consistent calculation



• Within mean field approximation, the problem becomes:

- independent magnetic moments in \vec{B}_{local}

turned into a problem studied in paramagnetism!

Apply result in paramagnetism:

$$M = N_s \cdot B_T \left(\frac{g\mu_B J}{kT} B_{\text{local}} \right)$$

$\vec{B}_a + \vec{B}_{\text{mean field}}$

$$M = N_s \cdot B_T \left(\frac{g\mu_B J}{kT} (\gamma\mu_0 M + B_a) \right)$$

Mean field theory

• the key equation

• a self-consistent equation for $M(T)$ or $M(T, B_a)$ if $B_a \neq 0$

⁺ Recall: $B_T(x) \equiv \frac{2J+1}{2J} \coth\left(\frac{2J+1}{2J} \cdot x\right) - \frac{1}{2J} \coth\left(\frac{x}{2J}\right)$

For small x ($x \ll 1$):

$$B_T(x) = \frac{J+1}{3J} x - \frac{(2J+1)^4 - 1}{720 J^4} x^3 + \dots$$

$$B_T(x) = \tanh x$$

E. Spontaneous Magnetization from MFI

• $\vec{B}_a = 0$

$$M = N_s \cdot B_T \left(\frac{g\mu_B J \gamma\mu_0 M}{kT} \right) \quad (1)$$

- Trivial solution: $M=0$ since $B_T(0)=0$

• Are there solutions with $M \neq 0$?
conditions for such solutions?

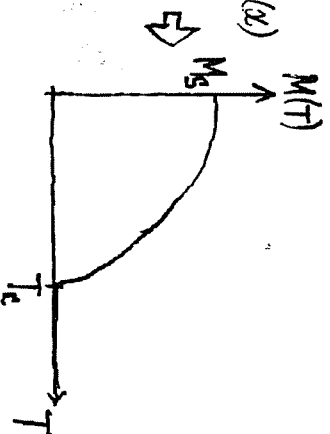
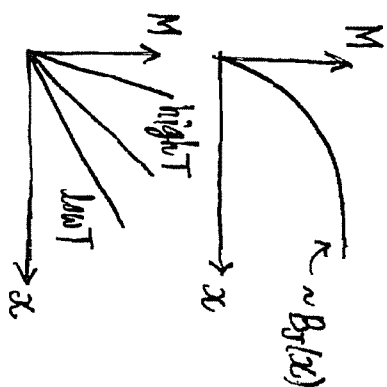
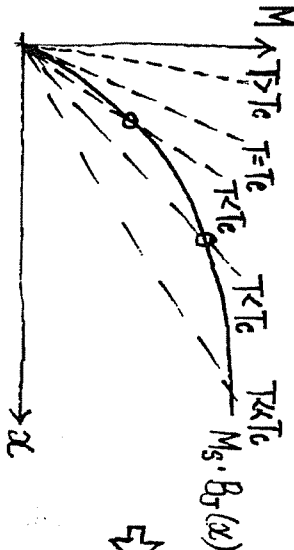
Think graphically

$$x \equiv \frac{g\mu_B J \gamma\mu_0 M}{kT}$$

$\therefore M = N_s \cdot B_T(x)$ looks like
RHS of (1)

but $M = \frac{kT x}{g\mu_B J \gamma\mu_0}$ gives

\therefore Solutions to (1) are intersections:



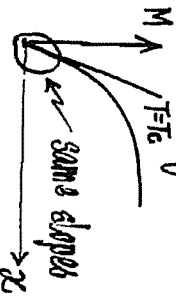
Determining T_c :

- At T_c or $T \approx T_c$, M only starts to grow

$\Rightarrow M$ is small $\Rightarrow x \ll 1$

$B_J(x) \approx \frac{J+1}{3J} x$ (also a straight line)

T_c is determined by:



$$\frac{kT_c}{g\mu_B J \gamma \mu_B} = M \cdot \frac{J+1}{3J}$$

$$\Rightarrow T_c = \frac{M_0 \mu_0 g \mu_B (J+1)}{3k} \gamma$$

Weiss constant
(strength of mean field)
(strength of interaction)

Implications:

- stronger moment-moment interactions \Rightarrow higher T_c
- Experimentally, measure T_c and use (2) to get γ for a measure of strength of interaction.
- Since $M_0 = n_0 g \mu_B J$,

$$T_c = \frac{n_0 \mu_0 (g \mu_B)^2 J (J+1)}{3k} \quad \text{or} \quad \gamma = \frac{3k T_c}{n_0 \mu_0 (g \mu_B)^2 J (J+1)} = \frac{3k T_c}{n_0 \mu_0 \mu_B^2 J^2}$$

[for $T_c \sim 10^3 K$, $\gamma \sim 10^4$]

- $\frac{M}{M_0}$ vs $\frac{T}{T_c}$ same for different materials

Using (1) and $T_c = \frac{M_0 \mu_0 g \mu_B (J+1)}{3k} \gamma$,

(1) can be written as:

$$\frac{M}{M_0} = B_J \left(\frac{3J}{J+1} \cdot \frac{T}{T_c} \cdot \frac{M}{M_0} \right) \quad (3) \quad \text{(equivalent to (1))}$$

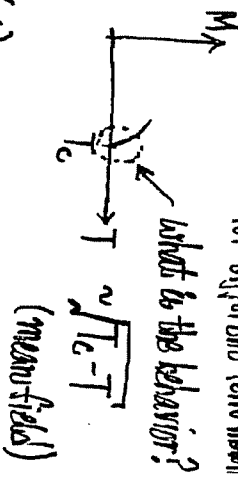
only the ratios $\frac{M}{M_0}$ and $\frac{T}{T_c}$ appear! $\left(\frac{3J}{J+1}\right)$ does not \Rightarrow an equation for $\frac{M}{M_0} \left(\frac{T}{T_c}\right)$

Behavior for $T \approx T_c$

- $T \rightarrow T_c$

M is small $\Rightarrow x$ is small

Keep 2 terms in $B_J(x)$



Using (3):

$$\frac{M}{M_0} = \left(\frac{T}{T_c}\right) \frac{M}{M_0} - \frac{(2J+1)^4 - 1}{240 J^4} \cdot \left(\frac{3J}{J+1}\right)^3 \left(\frac{T}{T_c}\right)^3 \left(\frac{M}{M_0}\right)^3$$

$$\Rightarrow \left(\frac{M}{M_0}\right)^2 \sim \left(\frac{T}{T_c}\right)^3 \left(\frac{T_c - T}{T_c}\right) \sim \frac{1}{T_c} (T_c - T) \quad (T \approx T_c)$$

$$\Rightarrow M \sim (T_c - T)^{1/2}$$

Writing $M \sim (T_c - T)^\beta$, $\beta = 1/2$ mean field theory

Table 8.3 Values of the saturation magnetization at 300 K, shown as M_s in units of $\text{J T}^{-1} \text{m}^{-3}$ and as $\mu_0 M_s$ in units of T, and the Curie temperature, T_c , for a selection of ferromagnetic and ferrimagnetic materials.

	M_s ($\times 10^5 \text{ J T}^{-1} \text{ m}^{-3}$)	$\mu_0 M_s$ (T)	T_c (K)
Iron	17.1	2.15	1043
Cobalt	14.0	1.76	1388
Nickel	4.85	0.61	627
Gadolinium *	20.6	2.60	292
CrO ₂	5.18	0.65	386
Fe ₃ O ₄	4.80	0.60	858
MnFe ₂ O ₄	4.10	0.52	573
NiFe ₂ O ₄	2.70	0.34	858

*Values of M_s and $\mu_0 M_s$ for gadolinium are at $T = 0 \text{ K}$ because $T_c < 300 \text{ K}$.

From Turtan, "The Physics of Solids"

F. Paramagnetic behavior for $T > T_c$

- $B_a = 0 \Rightarrow M = 0$
 - $B_a \neq 0$
- 

$$\frac{M}{M_s} = B_T \left(\frac{3T}{J+1} \cdot \frac{T_c}{T} \cdot \frac{M}{M_s} + \frac{q\mu_B J}{kT} B_a \right)$$

• Weak applied field, expect $M \ll M_s$
 and argument in B_T is small \Rightarrow expand B_T and keep one term

$$\begin{aligned} \frac{M}{M_s} &= \frac{J+1}{3T} \cdot \left(\frac{3T}{J+1} \cdot \frac{T_c}{T} \cdot \frac{M}{M_s} + \frac{q\mu_B J}{kT} B_a \right) \\ &= \frac{q(J+1)\mu_B B_a}{3kT} + \frac{T_c}{T} \frac{M}{M_s} \end{aligned}$$

$$\begin{aligned} \Rightarrow M &= M_s \cdot \frac{q\mu_B(J+1)}{3k(T-T_c)} B_a = \frac{M_s \cdot q\mu_B \mu_0(J+1)}{3k(T-T_c)} H_a \\ \chi &= \frac{C}{T-T_c} \sim (T-T_c)^{-1} \end{aligned}$$

$$C = \frac{M_s q\mu_B \mu_0(J+1)}{3k} = \frac{n(q\mu_B)^2 \mu_0 J(J+1)}{3k} = \frac{T_c}{\gamma}$$

Writing $\chi \sim (T-T_c)^{-\gamma}$, $\gamma = 1$ mean field theory

G. Mean Field Theory is an approximation

- The theory captures all the key features!
- Quantitatively, it predicts

$$M \sim (T_c - T)^{1/2} \text{ and } \chi \sim (T_c - T)^{-1/2}$$

for the behavior $T \rightarrow T_c$ from below and above.

Does it work quantitatively?

Expts:

Fe	$\beta = 0.34$, $\gamma = 1.33$
Ni	$\beta = 0.42$, $\gamma = 1.35$
CrBr ₃	$\beta = 0.39$, $\gamma = 1.21$

Mean field $\beta = 1/2$, $\gamma = 1$

- slightly off!
- There are many (analytic[†] and numerical) techniques to go beyond mean field theory Theory of critical phenomena

[†] 1981 Nobel Physics Prize was awarded to work on accurate calculations of the critical exponents.

H. Origin of internal field

- Internal field is strong

$$B_{\text{mean field}} = \gamma \mu_B M$$

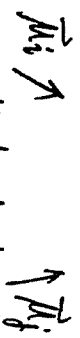
$$\Rightarrow \text{strongest } B_{\text{mean field}} \sim \gamma \mu_B M_g \sim \frac{g(J+1)\mu_B}{4\pi r^3}$$

$$M_g \sim 9.27 \times 10^{-24} \text{ J} \cdot \text{Tesla}^{-1}, \quad g \approx 2, \quad J \approx 1$$

$$T_e \sim 10^3 \text{ K (iron)}$$

$$\text{Strongest } B_{\text{mean field}} \sim 10^3 \text{ Tesla (Very strong)} \\ (\sim 10^7 \text{ G})$$

(i) Classical thinking



dipole-dipole interaction (EM)

dipole field of a moment $\sim \frac{\mu_0 \mu_g}{4\pi r^3}$ ← typical strength of dipole

← dipole field drops as $\frac{1}{r^3}$ (not quite short range)

$$\text{classical EM dipole-dipole energy} \sim \frac{\mu_0 \mu_g^2}{4\pi r^3} \sim 10^{-5} \text{ eV } (\sim 2-3 \text{ \AA})$$

Impossible to lead to $T_e \sim 1000 \text{ K}$

∴ The interaction is NOT of magnetic origin!

(ii) Quantum and Coulombic Origin

- $T_e \sim 10^2 - 10^3 \text{ K}$ indicates the range is that of Coulomb interaction of two charges about $\sim \text{few \AA}$ apart

• Qualitatively, internal interaction comes from

Pauli Exclusion principle
AND
Electrostatic interactions between charges in solids!

Depending on spin state of electrons, spatial state is restricted

↙ affects Coulomb interaction
"exchange interaction"

I. Basic idea of exchange interaction

XI-15

To illustrate the idea, consider a 2-electron system with spin-independent Hamiltonian

Then,

$$V_{\text{total}} = (\text{Spatial part}) \cdot (\text{spin part})$$

↳ must be antisymmetric w.r.t. interchanging the coordinates of the two electrons

$$V_{\text{total}}(\vec{r}_1, s_1; \vec{r}_2, s_2) = -V_{\text{total}}(\vec{r}_2, s_2; \vec{r}_1, s_1)$$

coordinates of electron 1 coordinates of electron 2

Spin Part

• Addition of two spin-1/2 angular momenta

$$\chi_S = \frac{1}{\sqrt{2}} (|\uparrow\rangle_1 |\downarrow\rangle_2 - |\downarrow\rangle_1 |\uparrow\rangle_2)$$

singlet state $S=0$ ($m_s=0$ only 1 value) antisymmetric w.r.t. $1 \leftrightarrow 2$

$$\chi_T = \begin{cases} |\uparrow\rangle_1 |\uparrow\rangle_2 & (m_s=+1) \\ \frac{1}{\sqrt{2}} (|\uparrow\rangle_1 |\downarrow\rangle_2 + |\downarrow\rangle_1 |\uparrow\rangle_2) & (m_s=0) \\ |\downarrow\rangle_1 |\downarrow\rangle_2 & (m_s=-1) \end{cases}$$

Triplet state $S=1$ χ_T is symmetric w.r.t. $1 \leftrightarrow 2$

XI-16

Total Spin $\hat{S}_1 = \hat{S}_1 + \hat{S}_2$

$$\begin{cases} \hat{S}_1^2 \chi_S = 0 & (S=0, \text{ so } S(S+1)=0) \\ \hat{S}_1^2 \chi_T = 2\hbar^2 \chi_T & (S=1, \text{ so } S(S+1)\hbar^2 = 2\hbar^2) \end{cases}$$

$$\hat{S}_1^2 \chi_S = \frac{3}{4}\hbar^2 \chi_S \quad \left\{ \begin{array}{l} \text{(since "1" is a spin-1/2 electron)} \\ \hat{S}_1^2 \chi_T = \frac{3}{4}\hbar^2 \chi_T \end{array} \right.$$

$$\hat{S}_2^2 \chi_S = \frac{3}{4}\hbar^2 \chi_S \quad \left\{ \begin{array}{l} \text{(since "2" is a spin-1/2 electron)} \\ \hat{S}_2^2 \chi_T = \frac{3}{4}\hbar^2 \chi_T \end{array} \right.$$

$$\hat{S}_2^2 = (\hat{S}_1 + \hat{S}_2)^2 = \hat{S}_1^2 + \hat{S}_2^2 + 2\hat{S}_1 \cdot \hat{S}_2$$

$$\Rightarrow \boxed{\hat{S}_1 \cdot \hat{S}_2 = \frac{1}{2} (\hat{S}_2^2 - \hat{S}_1^2 - \hat{S}_2^2)}$$

relates to $\vec{r}_1 \cdot \vec{r}_2$ for 2 electrons

$$(\hat{S}_1 \cdot \hat{S}_2) \chi_S = -\frac{3}{4}\hbar^2 \chi_S$$

$$(\hat{S}_1 \cdot \hat{S}_2) \chi_T = +\frac{1}{4}\hbar^2 \chi_T$$

- χ_S and χ_T are eigenstates of $(\hat{S}_1 \cdot \hat{S}_2)$ with different eigenvalue
- this is why we work with χ_S and χ_T , instead of $|\uparrow\rangle_1, |\downarrow\rangle_1, |\uparrow\rangle_2, |\downarrow\rangle_2$.

Spatial Part

- Solving the Schrodinger equation
 - single particle states $\psi_A(\vec{r}_1), \psi_B(\vec{r}_1), \dots$
- 2-particle:

$$\psi_A(\vec{r}_1) \psi_B(\vec{r}_2) + \psi_A(\vec{r}_2) \psi_B(\vec{r}_1) \leftarrow \text{symmetric}$$

$$\psi_A(\vec{r}_1) \psi_B(\vec{r}_2) - \psi_A(\vec{r}_2) \psi_B(\vec{r}_1) \leftarrow \text{antisymmetric}$$

Total wavefunctions

$$\Psi_{\text{singlet}} = \frac{1}{\sqrt{2}} [\underbrace{\psi_A(\vec{r}_1) \psi_B(\vec{r}_2) + \psi_A(\vec{r}_2) \psi_B(\vec{r}_1)}_{\text{symmetric}}] \cdot \underbrace{\chi_S}_{\text{antisymmetric}}$$

$$\Psi_{\text{triplet}} = \frac{1}{\sqrt{2}} [\underbrace{\psi_A(\vec{r}_1) \psi_B(\vec{r}_2) - \psi_A(\vec{r}_2) \psi_B(\vec{r}_1)}_{\text{antisymmetric}}] \cdot \underbrace{\chi_T}_{\text{3 choices symmetric}}$$

This is how the Pauli Exclusion Principle controls the total wavefunction.

- 2 electrons will interact with each other and with ions

$U(\vec{r}_1, \vec{r}_2)$ = potential energy due to interactions

$$= \underbrace{V_{e_1}(|\vec{r}_1 - \vec{r}_2|)}_{\text{Coulomb repulsion } > 0} + \underbrace{V_{e_1 i_1}(\vec{r}_1)}_{\text{attractive } < 0} + \underbrace{V_{e_1 i_2}(\vec{r}_2)}_{\text{attractive } < 0}$$

- Want to evaluate $\langle U \rangle_{\text{singlet}}$ and $\langle U \rangle_{\text{triplet}}$

if $\langle U \rangle_{\text{triplet}} < \langle U \rangle_{\text{singlet}}$

$\Rightarrow \chi_T$ ($S=1$) is preferred \Rightarrow two spins tend to align!

(ferromagnetic interaction)

$$\begin{aligned} \langle U \rangle_{\text{singlet}} &= \langle \Psi_{\text{singlet}} | U | \Psi_{\text{singlet}} \rangle \\ &= \int d^3r_1 d^3r_2 |\psi_A(\vec{r}_1)|^2 U(\vec{r}_1, \vec{r}_2) |\psi_B(\vec{r}_2)|^2 \leftarrow \text{"direct" term} \\ &\quad + \int d^3r_1 d^3r_2 \psi_A^*(\vec{r}_1) \psi_B^*(\vec{r}_2) U(\vec{r}_1, \vec{r}_2) \psi_B(\vec{r}_1) \psi_A(\vec{r}_2) \\ &\equiv U_0 + J_{\text{ex}} \end{aligned}$$

exchange integral (can be positive or negative)

• Similarly, $\langle U \rangle_{\text{triplet}} = \langle \Psi_{\text{triplet}} | U | \Psi_{\text{triplet}} \rangle$
 $= U_0 - J_{\text{ex}}$

If $J_{\text{e}} > 0$,

$\langle U \rangle_{\text{singlet}} > \langle U \rangle_{\text{triplet}}$

- $\Rightarrow \chi_{\text{T}}$ spin state preferred
- \Rightarrow parallel alignment of spins preferred
- \Rightarrow ferromagnetic interaction

If $J_{\text{e}} < 0$,

$\langle U \rangle_{\text{singlet}} < \langle U \rangle_{\text{triplet}}$

- $\Rightarrow \chi_{\text{S}}$ spin state preferred
- \Rightarrow anti-parallel alignment of spins preferred
- \Rightarrow anti-ferromagnetic interaction

Note: $\langle U \rangle_{\text{singlet}} - \langle U \rangle_{\text{triplet}} = 2J_{\text{e}}$

This is how BM and Coulomb interaction govern the internal interaction of neighboring magnetic moments.

J. Effective spin-dependent interaction

- Write down an effective Hamiltonian that gives

$\langle H \rangle_{\text{singlet}} - \langle H \rangle_{\text{triplet}} = 2J_{\text{ex}}$

$\hat{H}_{\text{spin}} = U_0 - \frac{2J_{\text{ex}}}{\hbar^2} \hat{S}_1 \cdot \hat{S}_2$ (*)

\hat{H}_{spin} effective Hamiltonian

$\frac{2J_{\text{ex}}}{\hbar^2}$ a constant

an effective spin-spin interaction that mimics the effect of BM + Coulombic interaction

Check: $\langle \chi_{\text{S}} | \hat{H}_{\text{spin}} | \chi_{\text{S}} \rangle = U_0 - \frac{2J_{\text{ex}}}{\hbar^2} \langle \chi_{\text{S}} | \hat{S}_1 \cdot \hat{S}_2 | \chi_{\text{S}} \rangle$

spin & singlet state ($S=0$)

$= U_0 - \frac{2J_{\text{ex}}}{\hbar^2} \left(-\frac{3}{4} \hbar^2 \right)$ (see VII-16)

$= U_0 + \frac{3}{2} J_{\text{ex}}$

$\langle \chi_{\text{T}} | \hat{H}_{\text{spin}} | \chi_{\text{T}} \rangle = U_0 - \frac{2J_{\text{ex}}}{\hbar^2} \langle \chi_{\text{T}} | \hat{S}_1 \cdot \hat{S}_2 | \chi_{\text{T}} \rangle$

$= U_0 + \frac{1}{2} J_{\text{ex}}$

Thus $\langle \hat{H}_{\text{spin}} \rangle_{\text{singlet}} - \langle \hat{H}_{\text{spin}} \rangle_{\text{triplet}} = 2J_{\text{ex}}$ as required.

- This justifies the use of

$$H_{\text{Hem}} = \sum_{ij} -J_{ij} \vec{\mu}_i \cdot \vec{\mu}_j + \sum_i (-\vec{\mu}_i \cdot \vec{B}_a)$$

as the starting point in exploring ferromagnetism. But the physical origin of the first term is the exchange interaction.

- (*) is the starting point of studying phase transitions and critical phenomena (since there are interactions)

$J_{\text{ex}} > 0$ ferromagnetic interaction

$J_{\text{ex}} < 0$ anti-ferromagnetic interaction

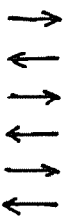
- short range interaction (e.g. between nearest neighbors) can lead to long range order!

Variations:

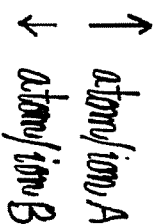
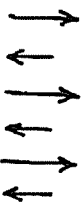
- spins in chain (1D), lattices (2D, 3D, ...)
- spins (or moments): classical or quantum (operator) components of spin
- lead to Ising model, X-Y model, Heisenberg model

K. Other interesting topics in magnetic properties

- Anti-ferromagnetism



- Ferri magnetism

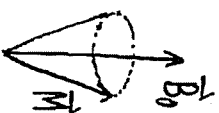


- Ferromagnetism in metals
 - role of conduction electrons (Stoner model)

- Domain walls and domains, hysteresis

- Paramagnetic resonance

$$\frac{d\vec{M}}{dt} = -\frac{q\mu_B}{\hbar} \vec{M} \times \vec{B}$$



probe the characteristic frequency

- Relaxation

- how \vec{M} gives up some energy to achieve alignment with field?
 - e.g. to lattice vibration
- how neighboring \vec{M} 's lose correlation in response to an applied field?

• NMR

• Ferromagnetic resonance

• Spin waves

Phonon

Ground state (no lattice vibrations)

$\omega \sim \text{vibrations}$ $T=0$

Low energy excitations

Lattice vibrational waves (collective excitations)

- restoring forces provided by bonding (springs)

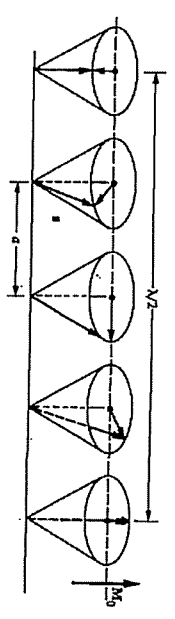
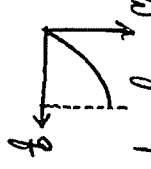
(quanta: phonon)

Ferromagnetism

Ground state (all aligned)

$\uparrow \uparrow \uparrow \uparrow \uparrow$ $T=0$

Low energy excitations

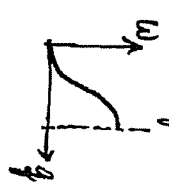


Spin waves (collective excitations)

- restoring forces come from

(quanta: magnon)

$$\vec{\mu}_i \cdot \vec{\mu}_j \sim \cos \theta_{ij} \sim -\theta_{ij}^2$$



Refs:

• Kittel: Ch. 11, 12, 13

• Christmas: Ch. 11, 12

• For more detailed discussions on magnetic properties:

S. Blundell, "Magnetism in Condensed Matter" (physicist's approach)

N.A. Spaldin, "Magnetic materials: fundamentals and device applications" (more applied approach)