

PHYS3061 Homework2

Electrostatics and partial differential
equations

Introduction

- This homework prepares you for Lab 6
- It contains three problems (all of them are theoretical calculations)
 - First: Solving 2D Laplace equation in Cartesian coordinates
 - Second: Analyze the dimension of 2D Poisson equation
 - Third: Derive the Gauss Law in 2D
- In the lab, you will
 - Solve 2D Laplace and Poisson equation numerically, and compare with the theoretical results

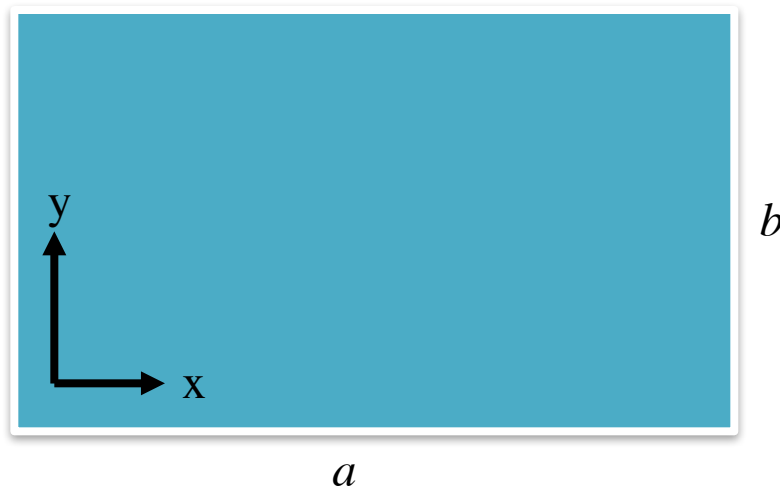
Problem 1 Solve the 2D Laplace equation

Consider a rectangular domain, with two sides a and b

(Figure 1). Solve for the general solution of two-dimensional Laplace equation

$$\nabla^2 \phi(x, y) = 0$$

by separation of variables (details in the PDF homework file).



Problem 2 Dimensions in 2D Poisson equation

Consider Poisson equation (or Gauss Law) in 2D

$$-\nabla^2 \phi = \frac{\rho(x, y)}{\epsilon_0}$$

the dimension (unit) of ϕ in the 3D case is “voltage”. However, it is no longer the case for 2D. Assuming ρ has the dimension of “charge per area”, derive the unit of ϕ in 2D.

Problem 3 Gauss Law in 2D

Using the differential form of the Gauss Law in question2 and the divergence theorem (Green's Theorem), derive the integral form

$$\oint \vec{E} \cdot d\vec{l} = \frac{q}{\epsilon_0}.$$

Namely, a close surface integral of the electric field will yield the charge enclosed, check the dimension you derived in the last problem.