The Higgs Boson

1. The Higgs Mechanism – extremely simplified
2. Significance of the recent discovery
3. The detectors (John Leung)
4. The data (Martin Kwok)

Chu Ming-chung, Department of Physics, CUHK
mcchu@phy.cuhk.edu.hk

The "God particle"

http://press.web.cern.ch/press/PressReleases/Releases2012/PR17.12E.html

蘋果日報:「它是所有物質的質量之源，是促成宇宙形成的重要粒子, …」http://www.youtube.com/watch?v=av_hWBQ7C_8

都市日報:「有一種說法認為，找到「上帝粒子」，就找到萬物之源。」
http://www.metrohk.com.hk/pda/pda_detail.php?id=189963&selectedDate=2012-07-05&categoryID=all

LA Times: "…the so-called God particle that theorists believe gives all other particles mass."

Nownews:「『上帝粒子』被認為是宇宙中所有基本粒子的質量之源，使得物質得以形成、凝聚、演化。」
http://www.nownews.com/2012/07/05/91-2831211.htm

Higgs Field (popular science level)

Particle mass: inertia against acceleration

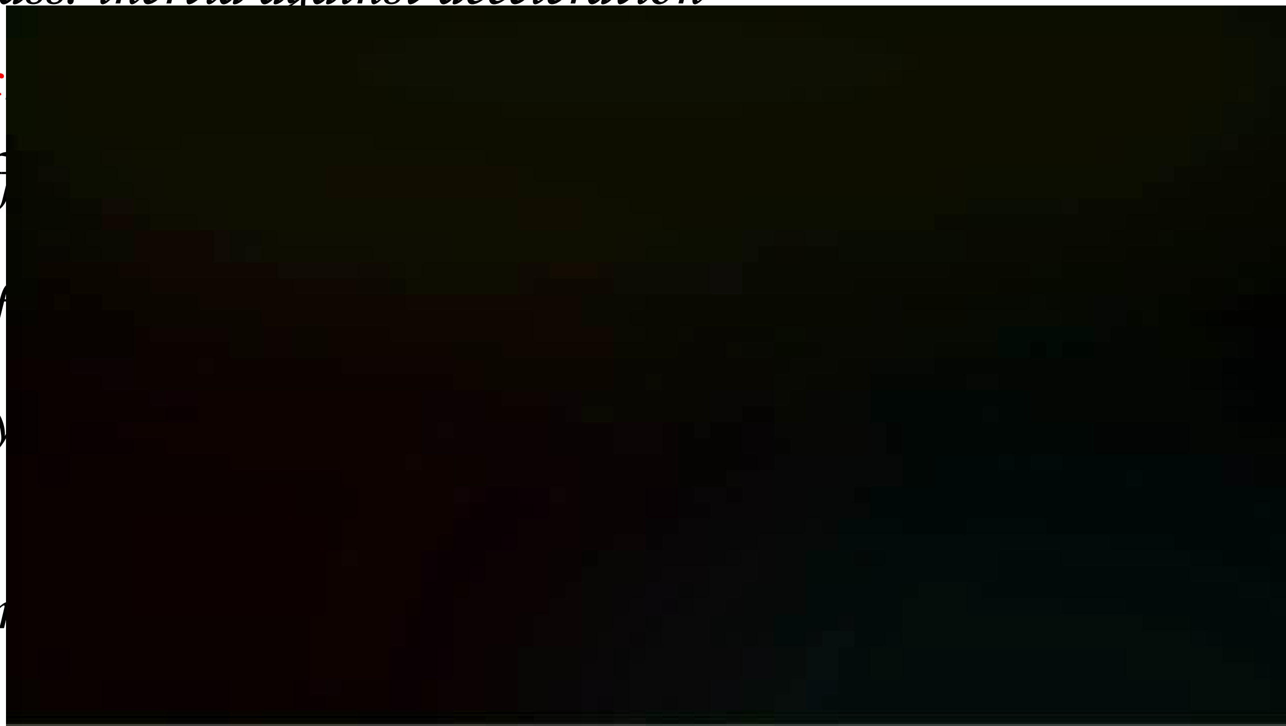
Higgs mec

Eg.: the ef

vacuum = f

Elementary

Different i



Electron has
arger effective
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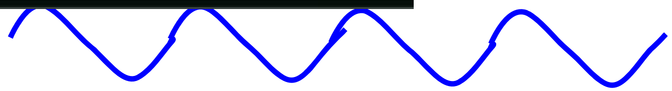
ttice provides
otential ~
ggs field

Excited states of Higgs field = Higgs particles



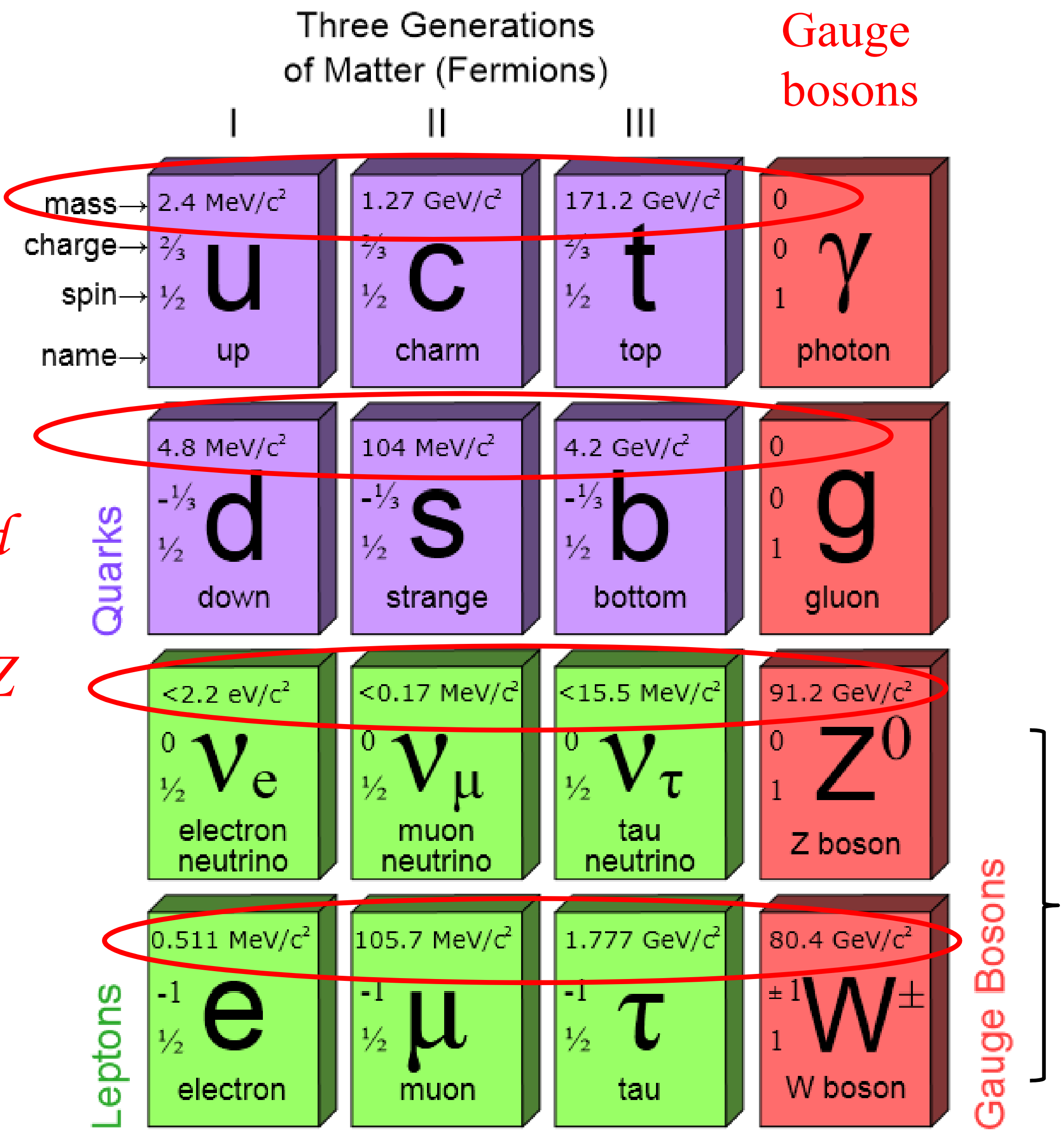
Vacuum = lowest energy state, could be full of particles/energy

Animation from CERN

Elementary particles

Why are the masses so different?

Why are γ and g massless while W and Z massive?



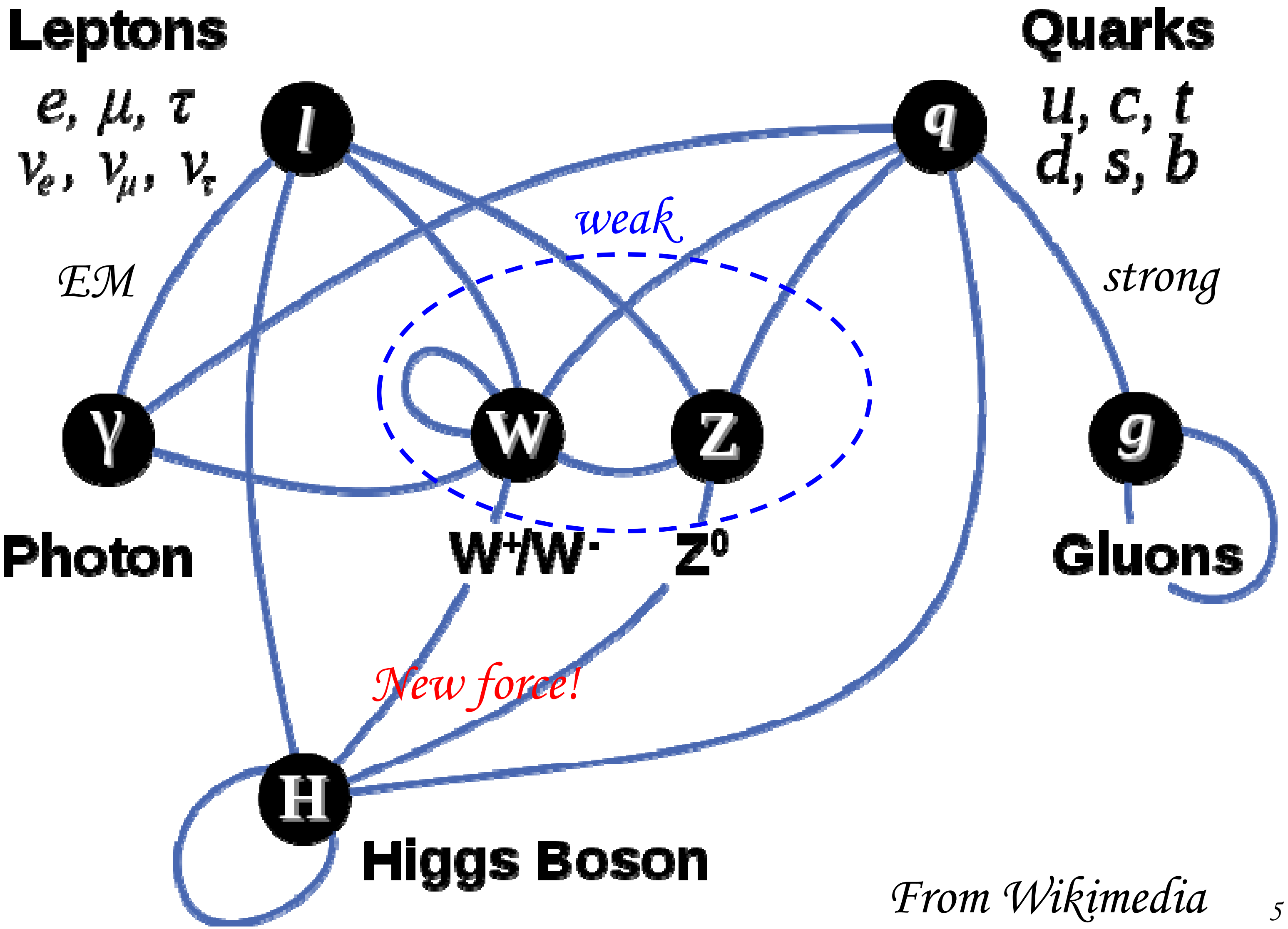
Carrier of EM force

Carrier of strong force

Carriers of weak force

Figure from Wikimedia

Higgs Boson



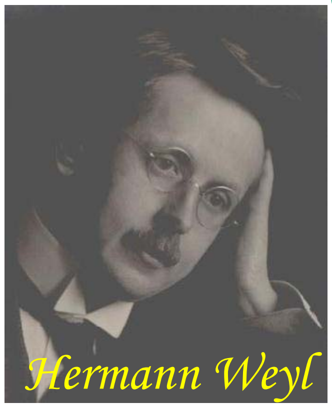
From Wikimedia

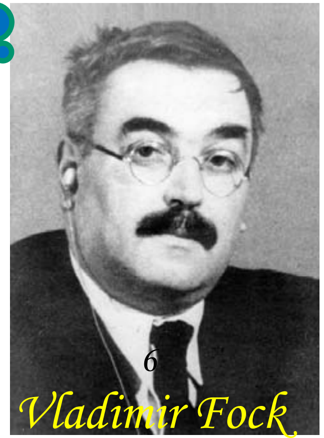
History

Gauge fields must be massless! Can't get weak and nuclear forces.

1954: Yang-Mills Gauge theory $SU(2)$ $\rightarrow$ nuclear force

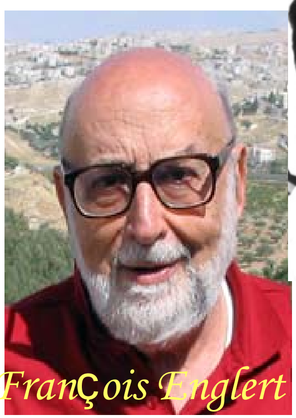
1926: Gauge theory symmetries $\rightarrow$ forces $U(1) \rightarrow EM$

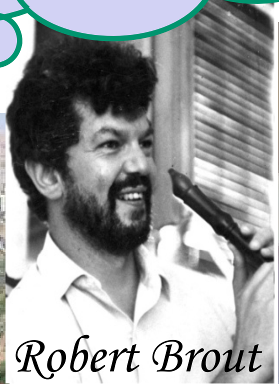


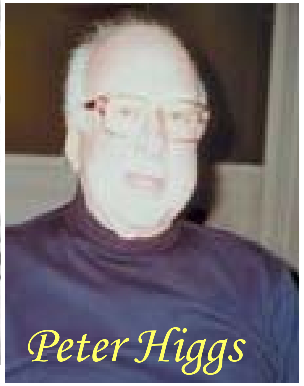


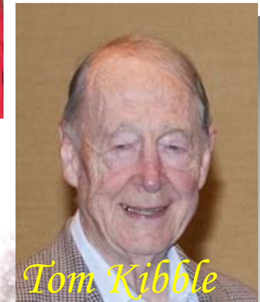
History

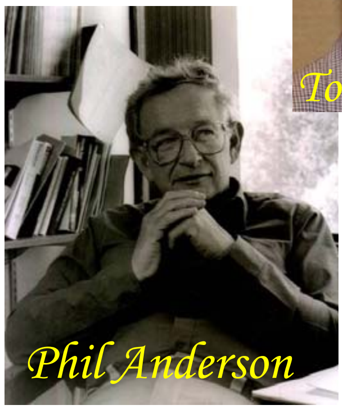
1963/4: 'Higgs mechanism' → massive gauge bosons



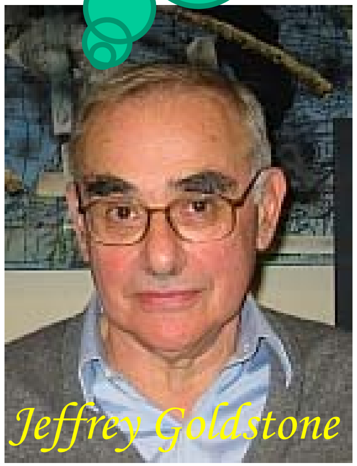








1961,62: SSB for relativistic field theory

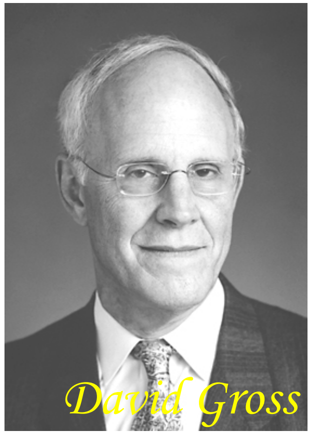


1960: Spontaneous Symmetry Breaking in superconductor → gap (massive photons), Meissner effect, etc. Nobel Prize 2008

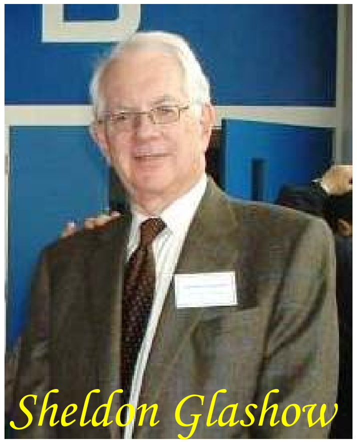
http://www.scholarpedia.org/article/Englert-Brout-Higgs-Guralnik-Hagen Kibble_mechanism_%2
http://nobelprize.org/nobel_prizes/physics/laureates/2008/phyadv08.pdf

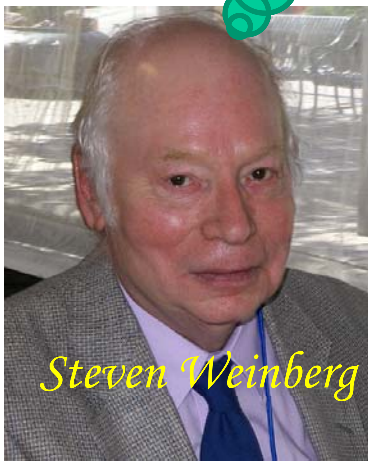
History

1973: asymptotic freedom,
only for gauge theories!
Nobel Prize 2004



1967: electroweak
unification SU(2)xU(1)
gauge theory + SSB
Nobel Prize 1979





Basic concepts

- Gauge symmetry and gauge interaction
- Why must gauge fields be massless?
- Spontaneous Symmetry Breaking
- Higgs Mechanism: preserving gauge symmetry, yet massive gauge fields

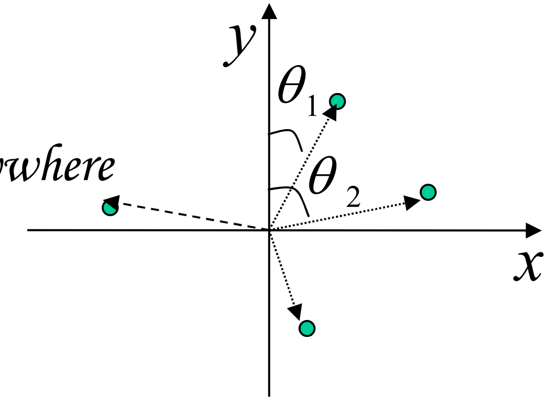
Gauge Symmetry

- ***Global symmetry***: *a transformation independent of* (x, t) *that leaves the system unchanged. Eg. rotating the angle coordinates of all particles*

$$\theta_i \rightarrow \theta_i + \theta_o$$

constant, same for all particles anywhere

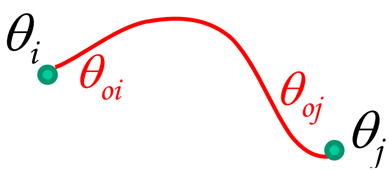
$$\Delta\theta_{ij} \equiv \theta_i - \theta_j \ \text{independent of } \theta_o$$



- *Can we make the symmetry local?*

$\theta_o(x,t)$ or $\theta_{oi}(t)$? *local gauge symmetry*

- *Yes, as long as each particle also carries* $\theta_{oi}(t)$ *and lets others know: imagine putting in a string between each pair of particles carrying the information* $\theta_{oi}(t)$ *and* $\theta_{oj}(t)$.



$$\Delta\theta_{ij} \equiv \theta_i - \theta_j \underbrace{+\theta_{oi} - \theta_{oj}}$$

~ *a gradient* $\nabla\theta$

Need a vector field to carry the info. $\rightarrow$ *gauge field*

EM vector potential $\mathbf{A} \rightarrow \mathbf{A} + \nabla\theta$

Gauge theory: add a vector field (gauge field) to restore the local gauge symmetry

Free particle Schrödinger Equation:

$$i\hbar\frac{\partial}{\partial t}\psi(x,t) = -\frac{\hbar^2}{2m}\nabla^2\psi(x,t)$$

$$\psi = |\psi| e^{i\theta(x,\, t)}$$

Clearly have global gauge symmetry $U(1)$: $\theta \rightarrow \theta + \phi$

Local $U(1)$ *Gauge Symmetry?*

$$\theta(x,t) \rightarrow \theta(x,t) + \phi(x) \qquad \psi \rightarrow \psi' = \psi e^{i\phi(x)}$$

Schrödinger Equation becomes:

$$\vec{\nabla}\psi' = \vec{\nabla}\left[\psi e^{i\phi(x)}\right] = \left[\vec{\nabla}\psi\right] e^{i\phi(x)} + i\left[\vec{\nabla}\phi\right]\psi e^{i\phi(x)}$$

$$\Rightarrow i\hbar\partial_t\psi' = -(\hbar^2/2m)[\nabla - i\nabla\phi]^2\,\psi'$$

Extra term $\sim \theta_{oi} - \theta_{oj}$

Free Schrödinger Equation does not obey local $U(1)$ *symmetry!*

Rescuing Local Gauge Symmetry

What if the particle is not free, but coupled to EM field?

$$\boldsymbol{p} \rightarrow \mathbf{p} + q\boldsymbol{A} \qquad \textit{(Griffiths Ch. 7)}$$

EM Vector potential

Q.M.: $\boldsymbol{p} \rightarrow -i\hbar\nabla \Rightarrow \nabla \rightarrow \nabla + (i/\hbar)q\boldsymbol{A}$

$\boldsymbol{A} \rightarrow \boldsymbol{A} + (\hbar/q)\nabla\phi$ *gauge freedom:*

choose any $\phi(x)$ without affecting physics!

The extra term in the free Schrödinger Equation can be cancelled by a gauge transformation in $\boldsymbol{A}$*!*

s.t. $i\hbar\partial_t\psi' = -(\hbar^2/2m)[\nabla + (i/\hbar)q\mathbf{A}]^2\psi'$

i.e., local $U(1)$ invariant! $\boldsymbol{A}$ *= gauge field*

EM coupling makes Schrödinger Equation local gauge invariant!

Requiring local gauge invariant gives rise to interaction!

symmetry $\longrightarrow$ *dynamics*

Gauge theories

U(1) $\rightarrow$ *EM is the simplest gauge theory.*

SU(2) $\rightarrow$ *Yang-Mills* (*θ becomes a* 2x2 *unitary matrix*)

SU(3) $\rightarrow$ *QCD* (*θ becomes a* 3x3 *unitary matrix*)

All interactions are believed to be generated by gauge theories.

Special relativity: global Lorentz covariance
General relativity: local Lorentz covariance
General covariance: physics is invariant w.r.t. coordinate choice

Asymptotic freedom: needed for consistency of field theory, only true for gauge theories!

But gauge fields must be massless!
Weak interaction: short-ranged $\rightarrow$ *massive gauge fields!*

Why must gauge fields be massless?

Euler-Lagrange Equation

- Eq. of motion can be 'derived' from a Lagrangian

$L = T - V = L(q_i, \dot{q}_i; t)$ via Euler-Lagrange Equation $T = \mathrm{KE}$, $V = \mathrm{PE}$

$$\boxed{\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) = \frac{\partial L}{\partial q_i} \qquad (1)}$$

- Generalization to field theory:

$$q_i \to \phi_i(x), \quad \dot{q}_i \to \partial_\mu \phi_i(x)$$

$$(1) \to \boxed{\partial_\mu \left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)}\right] = \frac{\partial \mathcal{L}}{\partial \phi_i}, \qquad (2)}$$

where $\mathcal{L}$ = Lagrangian density = functional of ϕ_i.

symmetries of Lagrangian → symmetries of equation of motion (eg. Lorentz and gauge invariant)

Mass of gauge fields

Euler-Lagrange Eq. $\rightarrow$ equation of motion ($F = ma$) from Lagrangian

$$\boxed{\partial_\mu \left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)} \right] = \frac{\partial \mathcal{L}}{\partial \phi_i},}$$

Can construct Lagrangian from known equations of motion

Can show that the mass of a particle (field) is given by the coefficient of the quadratic term in $\mathcal{L}$. Eg. $\mathcal{L} = ½\,[\partial_\mu\phi\,\partial^\mu\phi - m^2\phi^2]$ for a scalar field

Think of ϕ^2 as $\propto$ number density of particles, each contributing m^2 to energy density

Mass term for photons (if exists): $-m^2 A^\mu A_\mu$

But gauge transformation: $\mathbf{A} \rightarrow \mathbf{A} + \nabla\phi$

$\therefore$ Mass of photon (gauge fields) would violate local gauge symmetry!

Local $U(1)$ symmetry $\rightarrow$ photon has exactly zero mass!

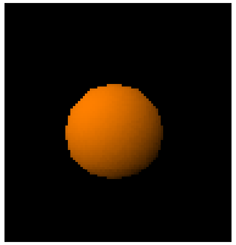
Good for EM, strong interactions, bad for weak (W, Z)!

Giving mass to gauge fields:
Spontaneous Symmetry Breaking

Spontaneous Symmetry Breaking (SSB)

Symmetries of the Lagrangian/Hamiltonian may not be realized in all states. Eg. H atom: rotational symmetry is observed in 1s, but not in some of the p states.

Not SSB: 1s (g.s.) obeys rotational symmetry



Eg. some 2p orbitals that 'break' rotational symmetry

SSB = ground state does not obey a symmetry of the Lagrangian.

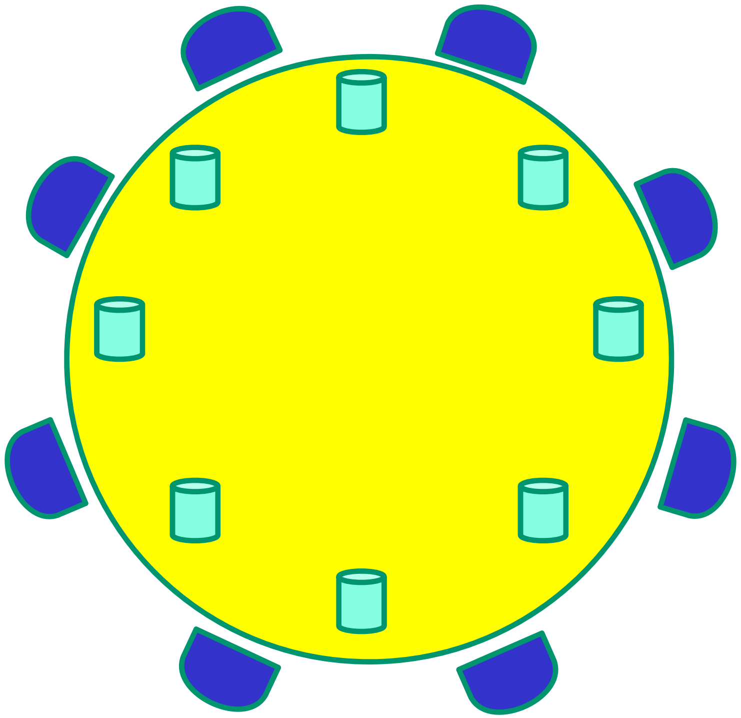
The symmetry is not broken for the dynamics, just hidden for the ground state.

No external field is needed for this to occur (spontaneous).

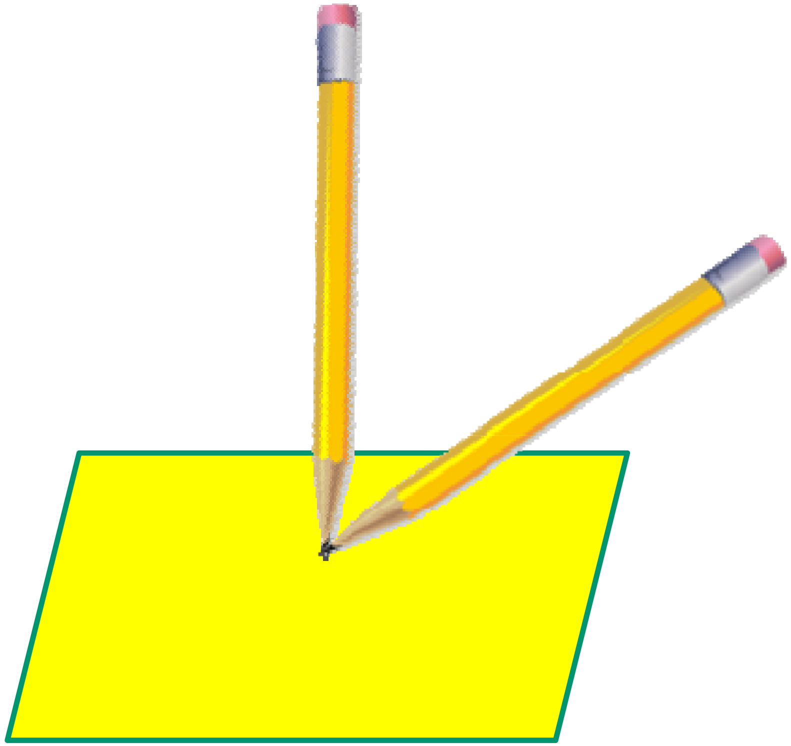
Usually happen for systems with many possible degenerate ground states, each hides the symmetry, but all together reveals it.

Examples of SSB

SSB: ground state of a system does not exhibit a symmetry of the Lagrangian



The system is symmetric w.r.t. left/right for each seat. The first person picking his/her glass induces SSB.



Axial symmetric pencil in vertical position is not the ground state.

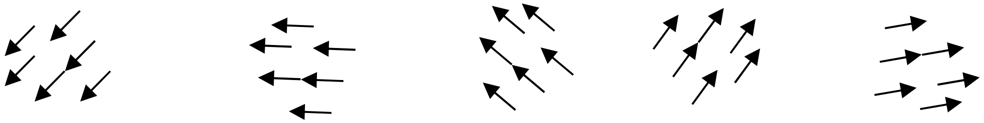
The g. s. chooses a direction randomly: SSB.

Note that SSB ≠ no symmetry. The Lagrangian (dynamics) has the symmetry. It's not exhibited by one g.s., but restored if all degenerate g.s. are taken together.

More examples of SSB

Eg. ferromagnet: $U = -\mathbf{s}_i \cdot \mathbf{s}_{i\pm1}$ *lowest energy state: all spins aligned*

symmetric w.r.t. rotation: no preferred direction



Zero external field

$T = 0$

These are all degenerate ground states, but each 'breaks' the symmetry of the Hamiltonian. All possible g.s. together 'restores' the symmetry.

$<\mathbf{s}> \neq 0$ *for any one possible g.s.*

$<\mathbf{s}> = 0$ *if averaged over all possible g.s.*

$<\mathbf{s}> = 0 \rightarrow \neq 0$ *indication of SSB*

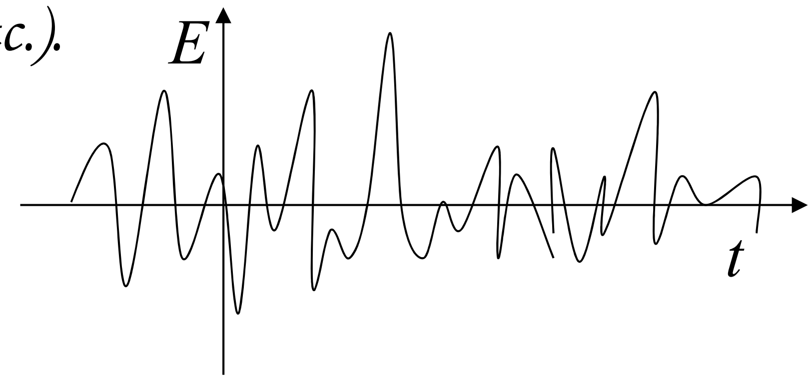
order parameter *condensate*

External fields can also break the symmetry, but not SSB: Explicit Symmetry Breaking. $U = -\mathbf{s}_i \cdot \mathbf{s}_{i\pm1} - \mathbf{B} \cdot \mathbf{s}_i$

Vacuum

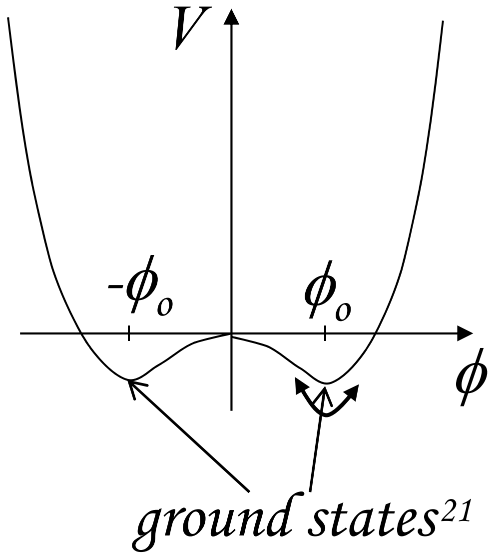
-In quantum physics, the vacuum is not empty. It's full of quantum fluctuations (energy E, particle no. n, etc.).

- vacuum = lowest energy state



-usually, the average of fluctuations is zero: eg. $<n> = 0$, $<s> = 0$, … if the vacuum is symmetric

- SSB: possible to have symmetry broken spontaneously in the ground state eg. $<\phi> = \phi_o \neq 0$ (cf. magnetized medium) because of self interaction energy V.



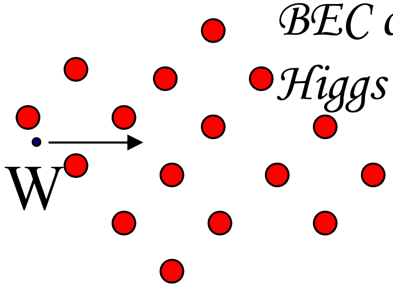
Higgs Mechanism

-*there exists a field ϕ (bosonic), which condenses in the vacuum (BEC), s.t. $<\phi> = \phi_o \neq 0$, because of self interaction V (P.E.)*

- *Higgs field interacts with* W, Z: *interaction energy $\rightarrow$ effective mass m_A*

$$\phi_o^* \phi_o A_\mu A^\mu \rightarrow \tfrac{1}{2}\, m_A^2 A_\mu A^\mu$$

- *But* W, Z *are intrinsically massless $\rightarrow$ gauge symmetry ok*



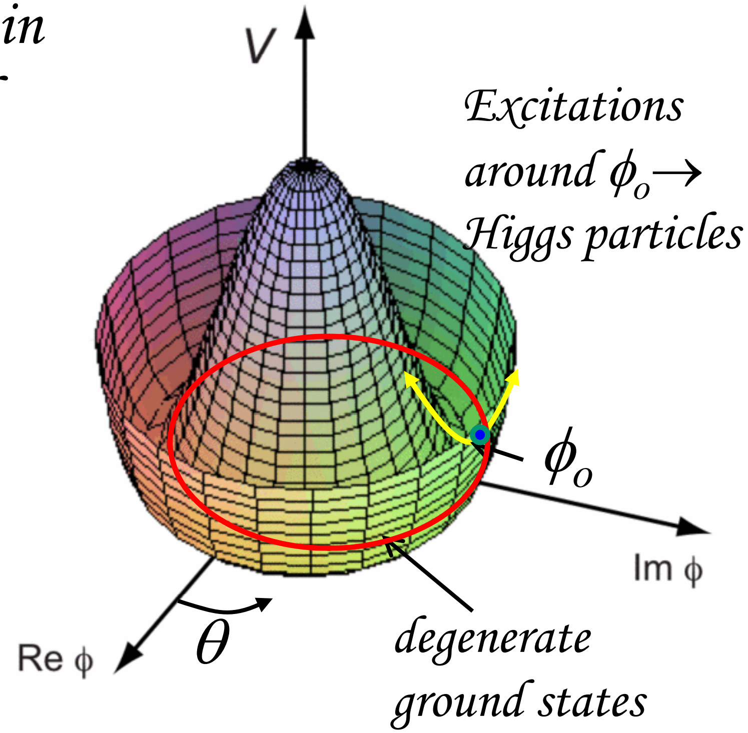


Illustration taken from: http://www.scholarpedia.org/article/Englert-Brout-Higgs-Guralnik-Hagen-Kibble_mechanism_(history)

Higgs and fermion mass

Fermion mass can also be generated by interacting with Higgs field ϕ:

$$\mathcal{L} = \mathcal{L}_o + g\bar{\psi}\phi\psi$$

Yukawa coupling

SSB: $<\phi> \neq 0 \rightarrow$ mass term of ψ

$$m_i \sim g_i <\phi>$$

Fermion masses generated from interacting with Higgs field!

Neutrinos, photons, gluons are assumed to be massless in the Standard Model $\rightarrow$ they do not interact with Higgs field; remain massless.

Significance of the recent discovery

- Found a new heavy scalar particle ($m \sim 125$ GeV), Higgs-like

- New elementary particle!

- If Higgs boson:

Confirm Higgs Mechanism, Standard Model (SM)

Confirm gauge theory approach to interaction

Confirm electroweak unification, boost confidence on GUT

Discover a new force different from EM, Weak, Strong, gravity

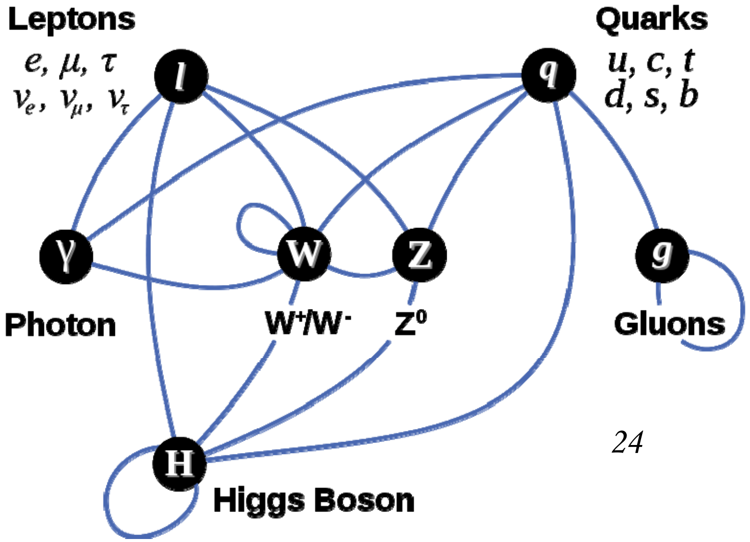
Constrain details of Higgs mechanism and physics beyond SM

- If not Higgs: even more exciting!

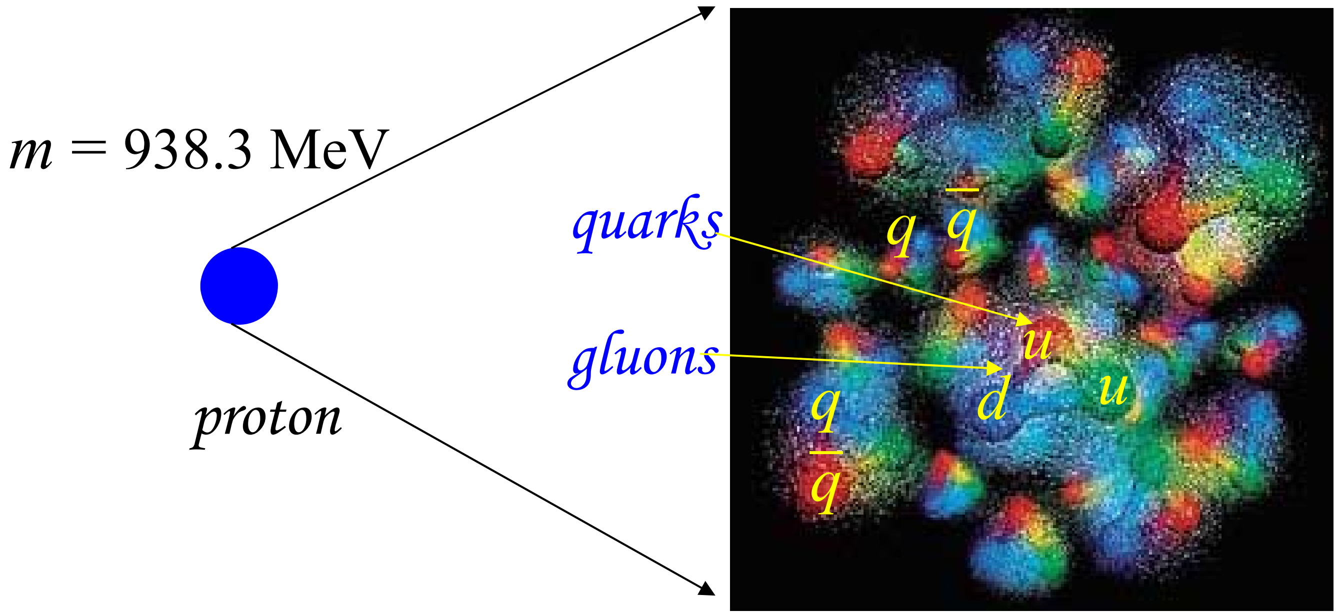
- gives mass of e → sizes of atoms

But not origin of all masses!

Not even masses of elementary particles!



Mass of a proton/neutron



Most of your mass is in protons/neutrons

proton = u, u, d + gluons + $q\bar{q}$ pairs

u, d quark rest mass (few MeVs) negligible, gluons have zero rest mass

Where is the mass of the proton?

Major contributions: KE of quarks and gluons ~ 300 MeV

PE stored in the gluons, particularly instantons

proven by mc^2!

The "God particle"

蘋果日報:「它是所有物質的質量之源，是促成宇宙形成的重要粒子, …」http://www.youtube.com/watch?v=av_hWBQ7C_8

都市日報：「有一種說法認為，找到「上帝粒子」，就找到萬物之源。」
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LA Times: "…the so-called God particle that theorists believe gives all other particles mass."

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Chu Ming-chung, Department of Physics, CUHK
mcchu@phy.cuhk.edu.hk

Classical field theory

Generalized coordinates: q_i, $\dot{q}_i$; $i = 1, \ldots, N$

$L(x_i, \dot{x}_i) \rightarrow L(q_i, \dot{q}_i)$

q_i could even be a function of (x, t).
Eg. normal modes of a string.

- Field theory: use fields as generalized coordinates
- Lagrangian $\rightarrow$ Lagrangian density $\quad L(q_i, q_i) \rightarrow \mathcal{L}(\phi_i, \partial_\mu \phi_i)$

$\phi_i(x)$ is a field (i labels different fields)

space-time coordinates

$\partial_\mu \phi_i$ is treated as an independent field

Conjugate momentum density: $\pi_i \equiv \partial \mathcal{L} / \partial \dot{\phi}_i$

Hamiltonian density: $\quad \mathcal{H}(\phi_i, \pi_i) \equiv \pi_i \dot{\phi}_i - \mathcal{L}(\phi_i, \partial_\mu \phi_i)$

Ref.: Goldstein, 'Classical Mechanics'. Landau, 'Classical Field Theory'.

Lagrangian Mechanics

Noether's Theorem: symmetry $\rightarrow$ conservation

||

invariance under some operations

Eg. translation invariance $(\mathbf{x} \rightarrow \mathbf{x} + \mathbf{a}) \Rightarrow$ *momentum conservation*

Global $U(1)$ *symmetry:* $\psi \rightarrow e^{i\theta}\psi \Rightarrow \partial_\mu j^\mu = 0;\ j^\mu =$ *E.M. current*

Gauge symmetry $\rightarrow$ Charge conservation!

Local $U(1)$ *symmetry:* $\psi \rightarrow e^{i\theta(x)}\psi \Rightarrow$ *Maxwell equations!*

Gauge fields $\rightarrow$ dynamics (eg. electrodynamics)

Spontaneous Symmetry Breaking

1. The mass term

Massless, free Klein-Gordon field: $\mathcal{L} = ½\, \partial_\mu\phi\, \partial^\mu\phi$ *Vacuum:* $\phi = 0$

$\mathcal{L} = ½\, [\partial_\mu\phi\, \partial^\mu\phi - m^2\phi^2]$ (1)

mass $= (-a)^{1/2}$ *, a = coefficient of the quadratic term in* ϕ *(mass term)*

Mass is the energy needed for an excited state above the vacuum.

With interaction: addition of a potential can hide the mass term. Eg.

$\mathcal{L} = ½\, \partial_\mu\phi\, \partial^\mu\phi + ½\, \mu^2\phi^2 - ¼\, \lambda^2\phi^4$ (2)

The mass is not $-i\,\mu$*, (would be tachyon, but not physical)*

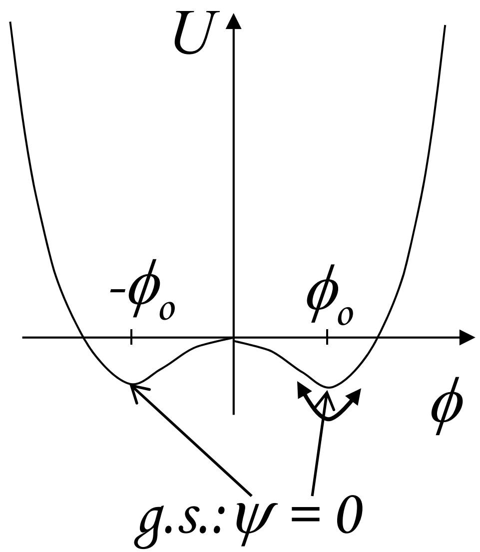
because the vacuum (ground state) is not $\phi = 0$*.*

$U = -\tfrac{1}{2}\,\mu^2\phi^2 + \tfrac{1}{4}\,\lambda^2\phi^4 \qquad (3)$

Minima at $\phi = \pm\phi_o;\ \phi_o \equiv \mu/\lambda$

2 degenerate vacuua: $\phi = \pm\phi_o$

Unstable (false) vacuum: $\phi = 0$



Expand around $\phi = \pm\phi_o$: $\quad \psi \equiv \phi - (\pm\phi_o)$

$$U = -\tfrac{1}{2}\,\mu^2(\psi \pm \phi_o)^2 + \tfrac{1}{4}\,\lambda^2(\psi \pm \phi_o)^4$$

$$= -\tfrac{1}{2}\,\mu^2\psi^2 - \pm\,\mu^2\phi_o\psi - \tfrac{1}{2}\,\mu^2\phi_o^2 + \tfrac{1}{4}\lambda^2\psi^4 + (3/2)\lambda^2\psi^2\phi_o^2$$
$$\pm\,\lambda^2\psi^3\phi_o \pm \lambda^2\psi\phi_o^3 + \tfrac{1}{4}\,\lambda^2\phi_o^4$$

$$= \mu^2\psi^2 - \tfrac{1}{4}\,\mu^4/\lambda^2 \pm \lambda\,\mu\,\psi^3 + \tfrac{1}{4}\lambda^2\psi^4 \quad (4)$$

$$\boxed{\mathcal{L} = \tfrac{1}{2}\,\partial_\mu\psi\,\partial^\mu\psi - \mu^2\psi^2 + \tfrac{1}{4}\,\mu^4/\lambda^2 \pm \lambda\,\mu\,\psi^3 - \tfrac{1}{4}\lambda^2\psi^4 \qquad (5)}$$

KE for ψ $\qquad$ *mass for* $\psi = 2^{1/2}\,\mu$ $\qquad$ *potential for* ψ

Zero mass around $\phi = 0$, *but unphysical.*

Physical excitations around $\psi = 0$ *have mass* $= 2^{1/2}\,\mu$.

Note that (2) *and* (5) *are identical, except for a change of variable.*

Symmetry: $\mathcal{L}(\phi) = \mathcal{L}(-\phi)$

Spontaneous symmetry breaking (SSB): $\mathcal{L}(\psi) \neq \mathcal{L}(-\psi)$ (ψ^3 *term in* (5))

Interaction with the medium (described by the potential) $\rightarrow$ *mass*

2. SSB of continuous symmetry: Goldstone Theorem

Generalize (2) *to two (real) fields:*

$$\mathcal{L} = \tfrac{1}{2}\,\partial_\mu\phi_1\partial^\mu\phi_1 + \tfrac{1}{2}\,\partial_\mu\phi_2\partial^\mu\phi_2 + \tfrac{1}{2}\,\mu^2(\phi_1^{\,2} + \phi_2^{\,2}) - \tfrac{1}{4}\lambda^2(\phi_1^{\,2} + \phi_2^{\,2})^2 \quad (6)$$

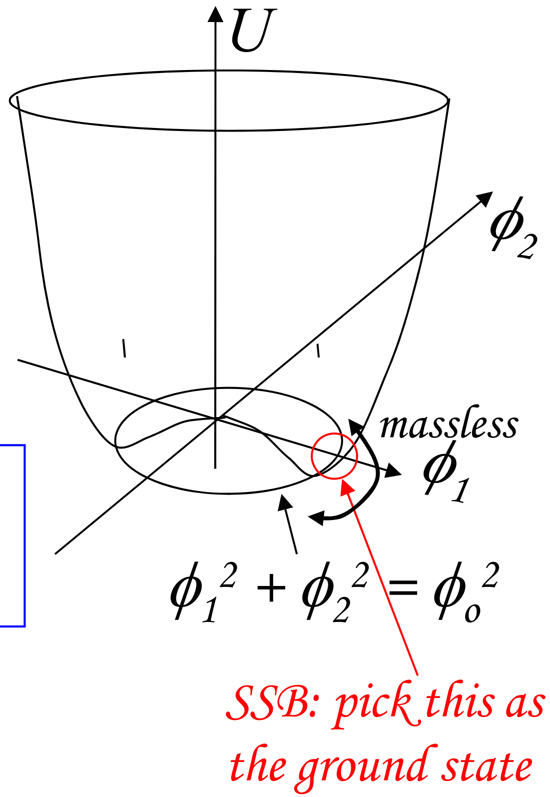
Minima of U: $\phi_1^{\,2} + \phi_2^{\,2} = \phi_o^{\,2} \equiv \mu^2/\lambda^2$

SSB: choose $\phi_1 = \phi_o$, $\phi_2 = 0$ *as the vacuum*

Expand around this vacuum: $\psi \equiv \phi_1 - \phi_o$

Mass term for ϕ_2: $\tfrac{1}{2}\,\mu^2 - \tfrac{1}{2}\,\lambda^2\phi_o^{\,2} = 0$

$$\boxed{\begin{aligned}\mathcal{L} &= \tfrac{1}{2}\,\partial_\mu\psi\partial^\mu\psi + \tfrac{1}{2}\,\partial_\mu\phi_2\partial^\mu\phi_2 - \mu^2\psi^2 \\ &\quad + \mu\lambda(\psi^3 + \psi\phi_2^{\,2}) - \tfrac{1}{4}\lambda^2(\psi^2 + \phi_2^{\,2})^2 + \mu^4/(4\lambda^2) \quad (7)\end{aligned}}$$



ψ acquires a mass $2^{1/2}\mu$, but ϕ_2 becomes massless!

> *Goldstone Theorem: Spontaneous breaking of a continuous symmetry gives rise to a massless scalar (Nambu-Goldstone Boson).*

But there's no massless scalar particle observed!

4. What is Higgs Mechanism?

Higgs mechanism = SSB of gauge symmetry

Pictorially, it's clear that the symmetry in (6) *is rotation in* (ϕ_1, ϕ_2) *plane,*
$\rightarrow$*can be represented in 'polar form' as symmetry w.r.t.* θ.

$\phi \equiv \phi_1 + i\phi_2$

$$\mathcal{L} = \tfrac{1}{2}\, (\partial_\mu \phi)^* \partial^\mu \phi + \tfrac{1}{2}\, \mu^2 (\phi^* \phi) - \tfrac{1}{4} \lambda^2 (\phi^* \phi)^2 \qquad (8)$$

$\mathcal{L}$ *is clearly invariant w.r.t.* $\phi \rightarrow \exp(i\theta)\phi$ ($U(1)$ *transformation*).

Make local $\theta(x) \rightarrow U(1)$ *gauge theory:*

$$\boxed{\mathcal{L} = \tfrac{1}{2}\, (\mathcal{D}_\mu \phi)^* \mathcal{D}^\mu \phi + \tfrac{1}{2}\, \mu^2 (\phi^* \phi) - \tfrac{1}{4}\, \lambda^2 (\phi^* \phi)^2 - (1/16\pi) F^{\mu\nu} F_{\mu\nu} \quad (9)}$$

$\mathcal{D}_\mu \equiv \partial_\mu + iqA_\mu$ *Must couple to* $\mathbf{A}$ *to make* $\mathcal{L}$ *gauge invariant!*

Now SSB!

SSB: Expand around the vacuum $\phi = (\phi_o, 0) \rightarrow$ $\boxed{\phi = (\phi_o + \psi)\exp(i\phi_2/\phi_o)}$

$<\phi> = \phi_o$

$\psi \equiv \phi_1 - \phi_o$ *(real);* $\phi_o \equiv \mu/\lambda$

excitations

massive scalar *Goldstone boson (massless)*

$$\boxed{\begin{aligned}\mathcal{L} = {} & \tfrac{1}{2}\,\partial_\mu\psi\partial^\mu\psi - \mu^2\psi^2 + \tfrac{1}{2}\,\partial_\mu\phi_2\partial^\mu\phi_2 \\ & + \tfrac{1}{2}(q\,\mu/\lambda)^2 A_\mu A^\mu - 2i(q\,\mu/\lambda)(\partial_\mu\phi_2)A^\mu - (1/16\pi)F_{\mu\nu}F^{\mu\nu} \\ & + \textit{interaction terms} \qquad (10)\end{aligned}}$$

mass term for gauge field! $\boxed{m_A = 2\pi^{1/2} q\mu/\lambda}$ *gauge field energy*

The mass of the gauge field comes from gauging $(\partial_\mu \rightarrow \mathcal{D}_\mu)$ *and SSB around* $\phi \sim \phi_o$ $\Big\}$ $\phi^*\phi A_\mu A^\mu \rightarrow \tfrac{1}{2}\, m_A^{\,2} A_\mu A^\mu$

Note that Higgs mass $(\sim \mu)$ *is related to massive boson mass* m_A, *but also depends on the potential* (λ).

But there is still the unseen massless scalar (Goldstone boson) ϕ_2 …

Higgs Mechanism

Make use of the remaining gauge freedom in A_μ *to cancel* ϕ_2*!*

$A_\mu \rightarrow A_\mu + (i/q\phi_o)\, \partial_\mu \phi_2$

$$\mathcal{L} = \tfrac{1}{2}\,(\partial_\mu \psi)^* \partial^\mu \psi \;-\; \mu^2 \psi^2 + \tfrac{1}{2}(q\,\mu/\lambda)^2 A_\mu A^\mu - (1/16\pi) F^{\mu\nu} F_{\mu\nu}$$
$$+ \text{ interaction terms} \qquad (11)$$

Higgs mechanism: SSB of gauge symmetry $(SU(2)) \longrightarrow$ *massive vector boson* A_μ*. Goldstone boson got 'eaten' by the third polarization of* A_μ*.*

Electroweak theory: Higgs mechanism to make $\underbrace{W^\pm, Z}_{A_\mu}$ *heavy*

Higgs particle: ψ *(scalar particle(s), composite, …), mass related to those of* W*,* Z*, but depends on details of potential.*

Higgs condensate fills the vacuum: $<\phi> = \phi_o$

Higgs mechanism for superconductivity: Ginzburg-Landau Model

$\phi(x)$ = *macroscopic complex wavefunction of Cooper pairs (Landau: superfluid of electrons)*

$= |\phi(x)| \exp[i\theta(x)]$

Density of pairs: $|\phi(x)|^2$

Cooper pairs are charged $\rightarrow$ *coupled to external EM fields*

Hamiltonian density:

$H = (1/4m)[\mathcal{D}\phi^* \cdot \mathcal{D}\phi] + V(\phi)$

$\mathcal{D} \equiv \nabla + i2e\mathbf{A}$ *minimal coupling;* $q = 2e$ *for a Cooper pair*

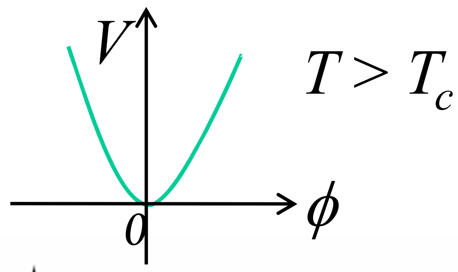
$V(\phi) = a|\phi(x)|^2 + b|\phi(x)|^4$

Taylor expansion around $\phi = 0$; a, b *depend on* T

Local $U(1)$ *symmetry* $\theta(x)$

Ginzburg-Landau Model

Normal state, $a > 0$ for $T > T_c$: $<\phi> = 0$
*massless **A**, massive ϕ*



$a < 0$ for $T < T_c$: minima of V shifted to
$|\phi_o|^2 = -a/b$, arbitrary phase

SSB: ground state of system picks a particular phase and breaks $U(1)$:
$<\phi> = \phi_o$ (Cooper pair condensate)

$$(1/4m)[\mathcal{D}\phi^* \cdot \mathcal{D}\phi] = (1/4m)[4e^2\phi_o{}^2\mathbf{A}^2]$$

***A** acquires an effective mass $e\phi_o/m^{1/2}$*
$\rightarrow$ Meissner effect
*massive **A** (short range),*
massive ϕ (gap)

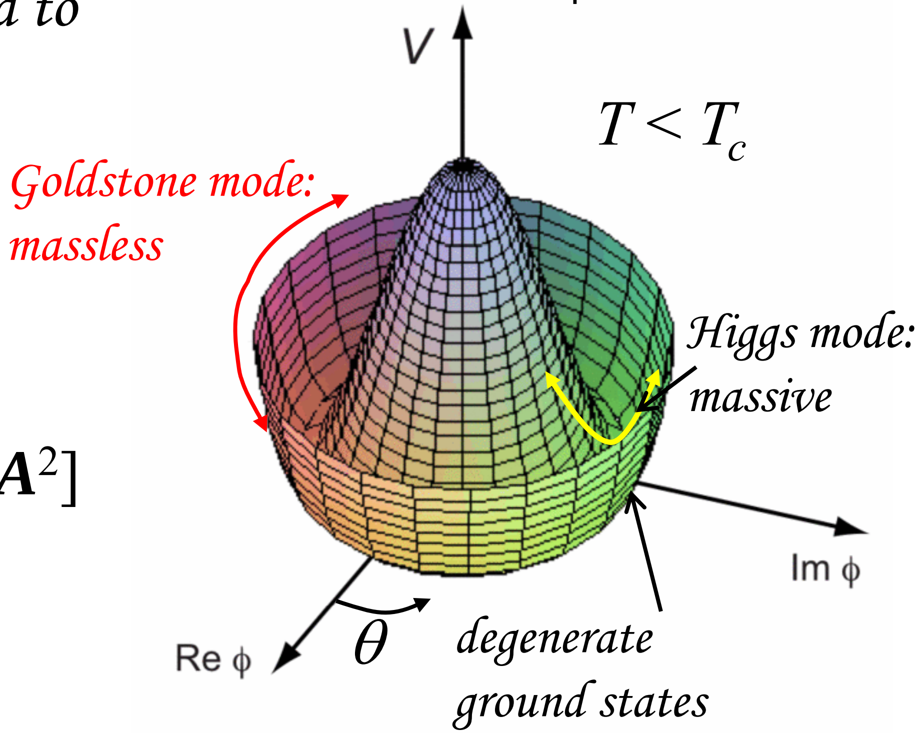
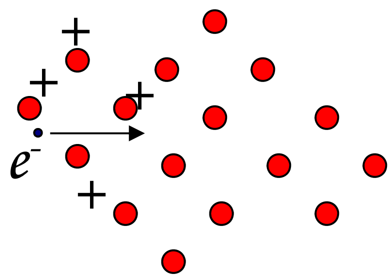


Illustration taken from:
http://www.scholarpedia.org/article/Englert-Brout-Higgs-Guralnik-Hagen-Kibble_mechanism_(history)

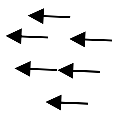
SSB and mass

- *Ground state is the vacuum of a system*
- *must identify the correct vacuum before finding the excitations and their masses!*



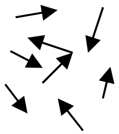
Eg. effective mass of e^- depends on the lattice arrangement. May even have directional dependence!

Eg. this is g.s. (vacuum)



$<\mathbf{s}> \neq 0$ $<E_0> = -A$

not this

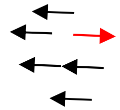


$<\mathbf{s}> = 0$ $<E_0> = 0$

Zero external field

$T = 0$

Excitations above the vacuum will have different energies above the two states. Eg.: energy (mass) for



$<E_1> = -A(1-2/N)$

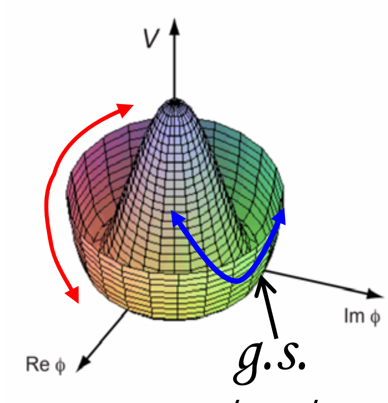
$m = 2A/N$ *for true vacuum*

$m = -A(1-2/N)$ *for false vacuum (negative mass indicates wrong vacuum)*

Higgs mechanism summary

Gauge theory: generates interactions, renormalizable, but gauge invariance $\to$ *massless gauge fields*

SSB: ground state may 'break' symmetry (but the Lagangian does not). Excitations around the g.s. $\to$ *massive particles (Higgs) + massless Goldstone bosons*



g.s.
$\phi \sim \phi_o$

SSB of gauge fields: $\to$ *effective mass for gauge fields*

$$\partial_\mu \to \mathcal{D}_\mu \quad , \phi \sim \phi_o \quad \to \quad (1/8\pi) m_A^2 A_\mu A^\mu$$

Make use of gauge freedom of A^μ *to get rid of Goldstone bosons. Only massive boson (Higgs) and massive gauge fields remain.*

Summary

- Spontaneous Symmetry Breaking

  - Ground state of a system may not show the symmetries of the Lagrangian (eg. $<\phi> \neq 0$)

  - Need to expand around the true vacuum to identify the mass term

  - SSB of continuous symmetry $\rightarrow$ massless scalar (Goldstone boson)

- Higgs mechanism

  - Assume existence of a scalar field with some continuous gauge symmetry $\rightarrow$ coupling to gauge field

  - SSB of gauge symmetry $\rightarrow$ massive gauge field

  - Use gauge freedom to eliminate Goldstone boson

  - Electroweak: massive $W^{\pm}$, Z