# BATS: Achieving the Capacity of Networks with Packet Loss

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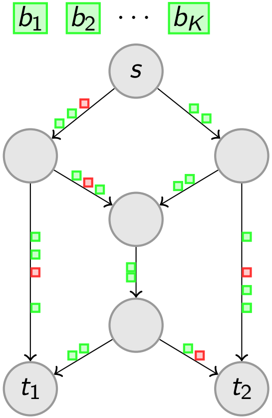
# Outline

1. File Transmission in Packet Networks
2. Two Classes of Solutions
   - Fountain Codes in Networks
   - Chunked Codes
3. BATS Codes

# Transmission through Packet Networks (Erasure Networks)

One 20MB file ≈ 20,000 packets

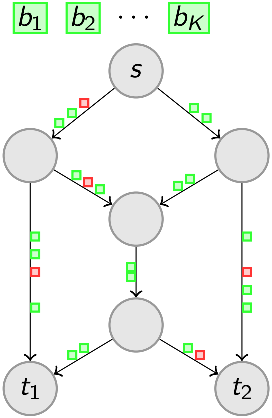


# Transmission through Packet Networks (Erasure Networks)

One 20MB file ≈ 20,000 packets

**A practical solution**

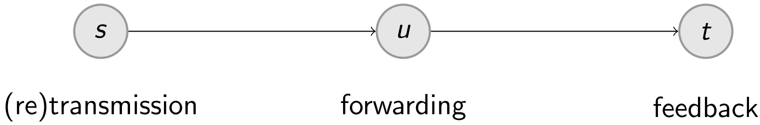
- low computational and storage costs
- high transmission rate
- small protocol overhead



# Routing Networks

**Retransmission**

- Example: TCP
- Not scalable for multicast
- Cost of feedback



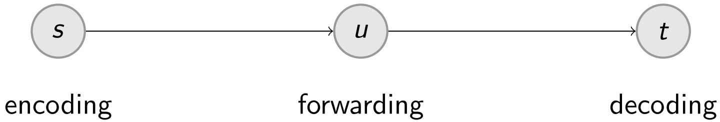
# Routing Networks

**Retransmission**

- Example: TCP
- Not scalable for multicast
- Cost of feedback

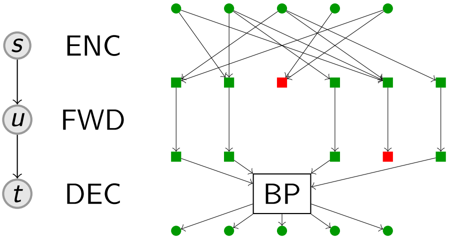
**Forward error correction**

- Example: fountain codes
- Scalable for multicast
- Neglectable feedback cost



# Complexity of Fountain Codes with Routing

- $K$ packets, $T$ symbols in a packet.
- Encoding: $\mathcal{O}(T)$ per packet.
- Decoding: $\mathcal{O}(T)$ per packet.
- Routing: $\mathcal{O}(1)$ per packet and fixed buffer size.

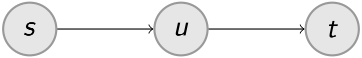




[Luby02] M. Luby, "LT codes," in Proc. 43rd Ann. IEEE Symp. on Foundations of Computer Science, Nov. 2002.

[Shokr06] A. Shokrollahi, "Raptor codes," IEEE Trans. Inform. Theory, vol. 52, no. 6, pp. 2551-2567, Jun 2006.

# Achievable Rates




Both links have a packet loss rate 0.2.

The capacity of this network is 0.8.

| Intermediate | End-to-End | Maximum Rate |
|---|---|---|
| forwarding | retransmission | 0.64 |
| forwarding | fountain codes | 0.64 |

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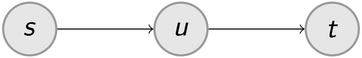


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| Intermediate | End-to-End | Maximum Rate |
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| forwarding | retransmission | 0.64 |
| forwarding | fountain codes | 0.64 |
| network coding | random linear codes | 0.8 |

# Multicast capacity of erasure networks

**Theorem**

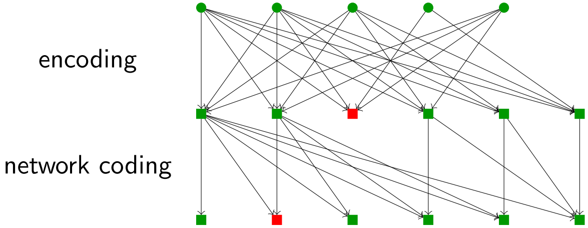
*Random linear network codes achieve the capacity of a large range of multicast erasure networks.*

[Wu06] Y. Wu, "A trellis connectivity analysis of random linear network coding with buffering," in Proc. IEEE ISIT 06, Seattle, USA, Jul. 2006.

LMKE08] D. S. Lun, M. Médard, R. Koetter, and M. Effros, "On coding for reliable communication over packet networks," Physical Communication, vol. 1, no. 1, pp. 320, 2008.

# Complexity of Linear Network Coding

- Encoding: $\mathcal{O}(TK)$ per packet.
- Decoding: $\mathcal{O}(K^2 + TK)$ per packet.
- Network coding: $\mathcal{O}(TK)$ per packet. Buffer $K$ packets.



# Quick Summary

**Routing + fountain**

- low complexity
- low rate

**Network coding**

- high complexity
- high rate

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# Fountain Codes with Coding in Intermediate Nodes

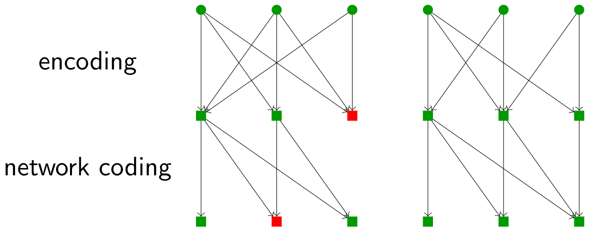
- Network coding changes the degree distribution of the received packets such that the low decoding complexity cannot be guaranteed.
- Works for special cases: P2P file sharing [CHKS09] and line networks [GS08].
  - Difficult to extend.
  - In the intermediate nodes, computational cost is $\mathcal{O}(TK)$ per packet and storage cost is $K$ packets.


[CHKS09] M.-L. Champel, K. Huguenin, A.-M. Kermarrec, and N. L. Scouarnec. LT network codes. Research Report RR-7035, INRIA, 2009.

[GS08] R. Gummadi and R. Sreenivas. Relaying a fountain code across multiple nodes. In Proc. IEEE ITW 08, pages 149–153, May 2008.

# Chunk Based Network Coding

- Using chunks to reduce complexity [CWJ03]
  - Encoding complexity: $\mathcal{O}(TKL)$
  - Decoding complexity: $\mathcal{O}(KL^2 + TKL)$
- Buffer requirement in the intermediate nodes?





[CWJ03] P. A. Chou, Y. Wu, and K. Jain. Practical network coding. In Proc. Allerton Conf. Comm., Control, and Computing, Oct. 2003.

# Scheduling of Chunks

- Sequential scheduling of chunks
  - Protocol overhead
  - Not scalable for multicast

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- Random scheduling of chunks [MHL06]
  - Intermediate network nodes cache $K$ packets.
  - Less efficient when a major fraction of all the chunks have been decoded.

[MHL06] P. Maymounkov, N. J. A. Harvey, and D. S. Lun. Methods for efficient network coding. In Proc. Allerton Conf. Comm., Control, and Computing, Sept. 2006.

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- Sequential scheduling of chunks
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- Random scheduling of chunks [MHL06]
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  - Overlapped Chunks [SZK09] [HB10]
    - Improve the throughput of random scheduling
    - Cannot reduce the buffer size

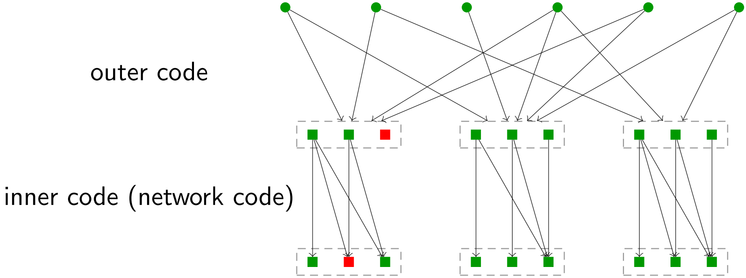

[MHL06] P. Maymounkov, N. J. A. Harvey, and D. S. Lun. Methods for efficient network coding. In Proc. Allerton Conf. Comm., Control, and Computing, Sept. 2006.

[SZK09] D. Silva, W. Zeng, and F. R. Kschischang. Sparse network coding with overlapping classes. In Proc. NetCod 09, pages 74–79, 2009.

[HB10] A. Heidarzadeh and A. H. Banihashemi. Overlapped chunked network coding. In Proc. ITW 10, pages 1–5,2010.

# Outline

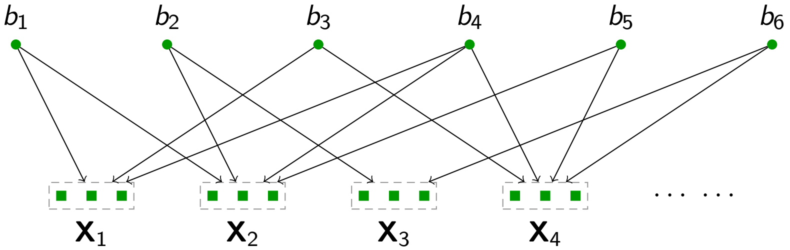
# Batched Sparse (BATS) Codes




[YY11] S. Yang and R. W. Yeung. Coding for a network coded fountain. ISIT 2011, Saint Petersburg, Russia, 2011.

# Encoding of BATS Code: Outer Code

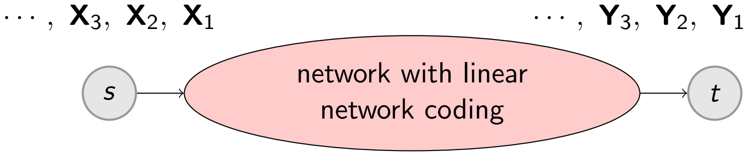
- Apply a "matrix fountain code" at the source node:
  1. Obtain a degree $d$ by sampling a degree distribution $\Psi$.
  2. Pick $d$ distinct input packets randomly.
  3. Generate a batch of $M$ coded packets using the $d$ packets.
- Transmit the batches sequentially.




$$\mathbf{X}_i = \begin{bmatrix} b_{i1} & b_{i2} & \cdots & b_{id_i} \end{bmatrix} \mathbf{G}_i = \mathbf{B}_i \mathbf{G}_i.$$

# Encoding of BATS Code: Inner Code

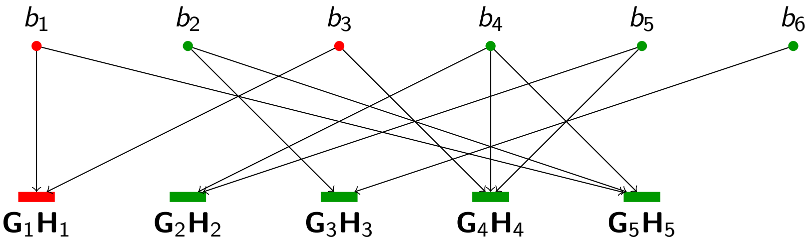
- The batches traverse the network.
- Encoding at the intermediate nodes forms the inner code.
- Linear network coding is applied in a causal manner within a batch.




$$\mathbf{Y}_i = \mathbf{X}_i \mathbf{H}_i, \; i = 1, 2, \ldots.$$

# Belief Propagation Decoding

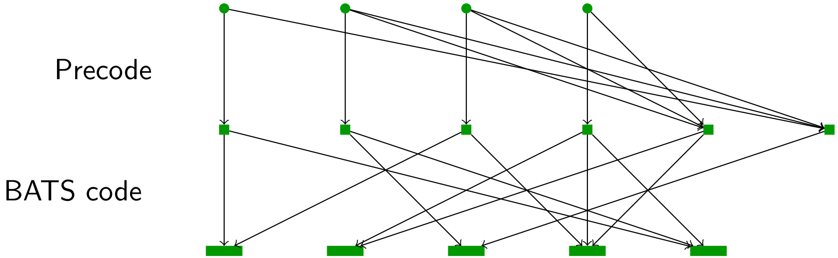
1. Find a check node $i$ with $\text{degree}_i = \text{rank}(\mathbf{G}_i\mathbf{H}_i)$.
2. Decode the $i$th batch.
3. Update the decoding graph. Repeat 1).




The linear equation associated with a check node: $\mathbf{Y}_i = \mathbf{B}_i\mathbf{G}_i\mathbf{H}_i$.

# Precoding

- Precoding by a fixed-rate erasure correction code.
- The BATS code recovers $(1-\eta)$ of its input packets.




[Shokr06] A. Shokrollahi, Raptor codes, IEEE Trans. Inform. Theory, vol. 52, no. 6, pp. 25512567, Jun. 2006.

# Degree Distribution

We need a degree distribution $\Psi$ such that

1. The BP decoding succeeds with high probability.
2. The encoding/decoding complexity is low.
3. The coding rate is high.

# A Sufficient Condition

Define

$$\Omega(x) = \sum_{r=1}^{M} h^*_{r,r} \sum_{d=r+1}^{D} d\Psi_d I_{d-r,r}(x) + \sum_{r=1}^{M} h_{r,r} r \Psi_r,$$

where $h^*_{r,r}$ is related to the rank distribution of $H$ and $\mathrm{I}_{a,b}(x)$ is the *regularized incomplete beta function*.

## Theorem

*Consider a sequence of decoding graph BATS$(K, n, \{\Psi_{d,r}\})$ with constant $\theta = K/n$. The BP decoder is asymptotically error free if the degree distribution satisfies*

$$\Omega(x) + \theta \ln(1 - x) > 0 \quad \text{for } x \in (0, 1 - \eta),$$

# An Optimization Problem

$$
\begin{aligned}
\max \quad & \theta \\
\text{s.t.} \quad & \Omega(x) + \theta \ln(1-x) \geq 0, \quad 0 < x < 1-\eta \\
& \Psi_d \geq 0, \quad d = 1, \cdots, D \\
& \sum_d \Psi_d = 1.
\end{aligned}
$$

- $D = \lceil M/\eta \rceil$
- Solver: Linear programming by sampling some $x$.

# Complexity of Sequential Scheduling

| | | |
|---|---|---|
| Source node encoding | | $\mathcal{O}(TM)$ per packet |
| Destination node decoding | | $\mathcal{O}(M^2 + TM)$ per packet |
| Intermediate Node | buffer | $\mathcal{O}(TM)$ |
| | network coding | $\mathcal{O}(TM)$ per packet |

- $T$: length of a packet
- $K$: number of packets
- $M$: batch size

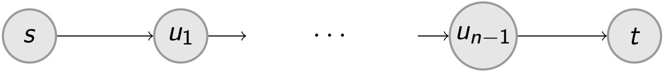
# Achievable Rates

## Optimization

$$
\begin{aligned}
\max \quad & \theta \\
\text{s.t.} \quad & \Omega(x_k) + \theta \ln(1 - x_k) \geq 0, \quad x_k \in (0, 1 - \eta) \\
& \Psi_d \geq 0, \quad d = 1, \cdots, \lceil M/\eta \rceil \\
& \sum_d \Psi_d = 1.
\end{aligned}
$$

- The optimal values of $\theta$ is very close to $\mathrm{E}[\mathrm{rank}(H)]$.
- It can be proved when $\mathrm{E}[\mathrm{rank}(H)] = M \Pr\{\mathrm{rank}(H) = M\}$.

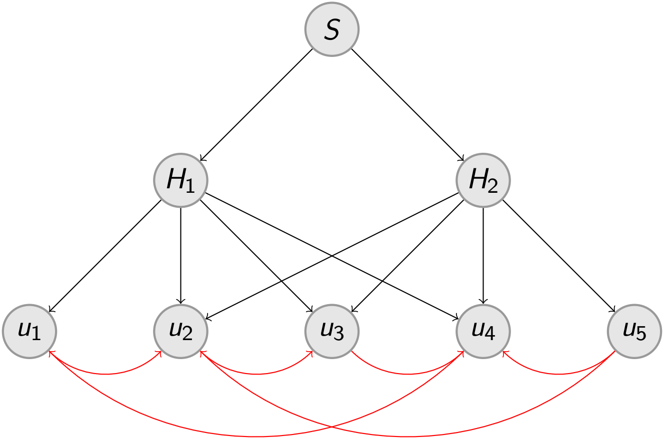
# Multi-hop Wireless Transmisson on IEEE802.11




All links have a packet loss rate 0.2.

| Intermediate Operation | Maximum Rate |
|---|---|
| forwarding | $0.8^n \to 0,\ n \to \infty$ |
| network coding | 0.8 |

# BATS in Content Distribution: NeP2P



# Summary

- BATS codes provide a digital fountain solution with linear network coding:
  - Outer code at the source node is a matrix fountain code.
  - Linear network coding at the intermediate nodes forms the inner code.
  - Prevents BOTH packet loss and delay from accumulating along the way.
- The more hops between the source node and the sink node, the larger the benefit.
- Future work:
  - Proof of (nearly) capacity achieving
  - Design of intermediate operations to maximize the throughput and minimize the buffer size

# References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