7.7 Separation of Source and Channel Coding

A Separation Theorem
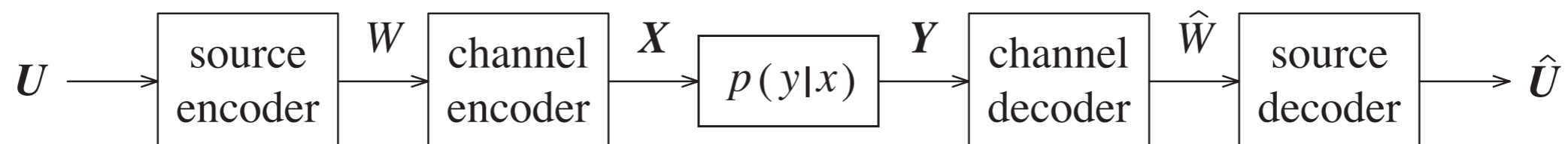
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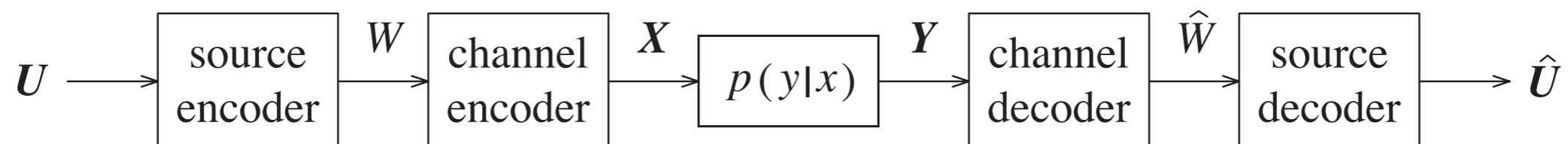
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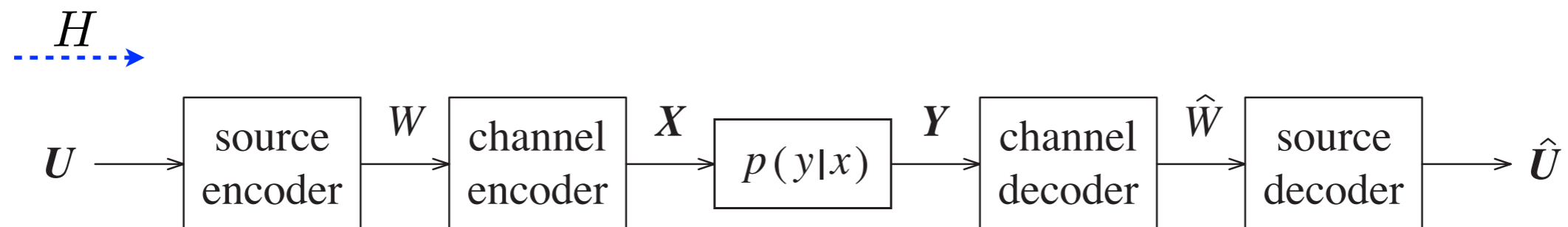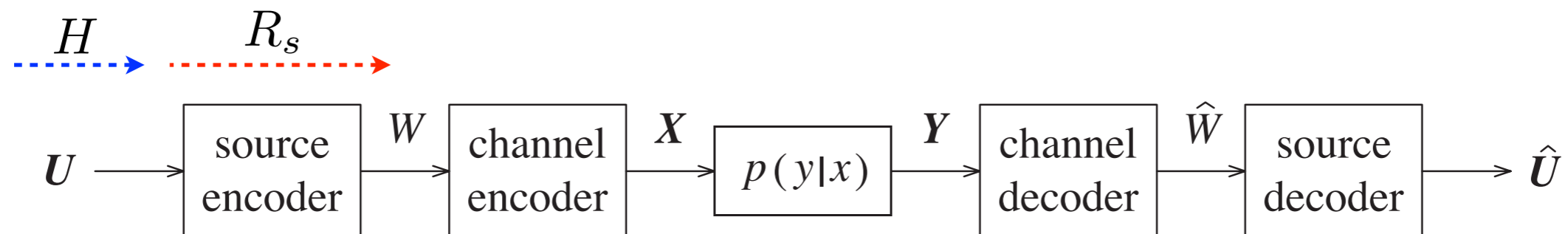
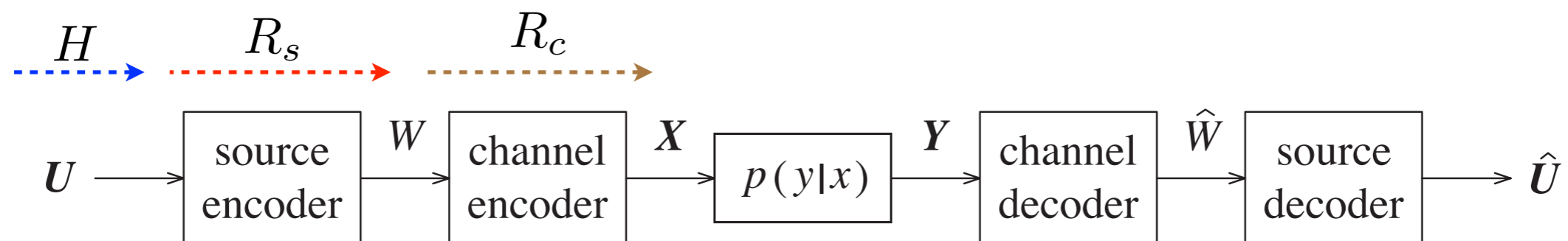- Specifically, choose R_s and R_c such that

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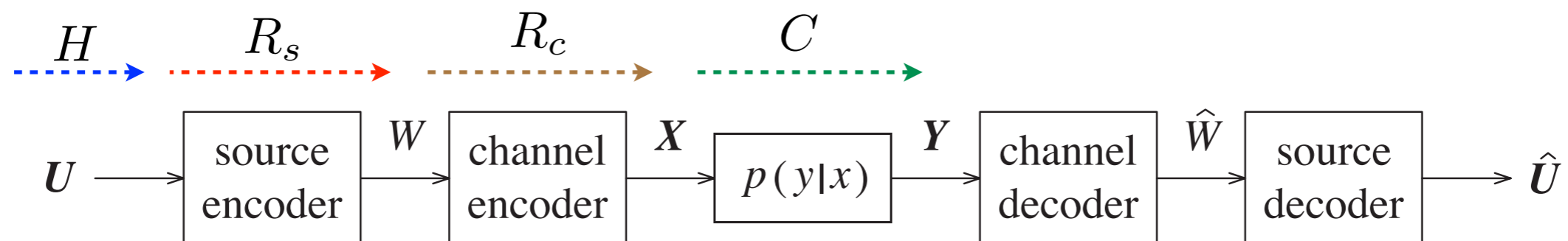
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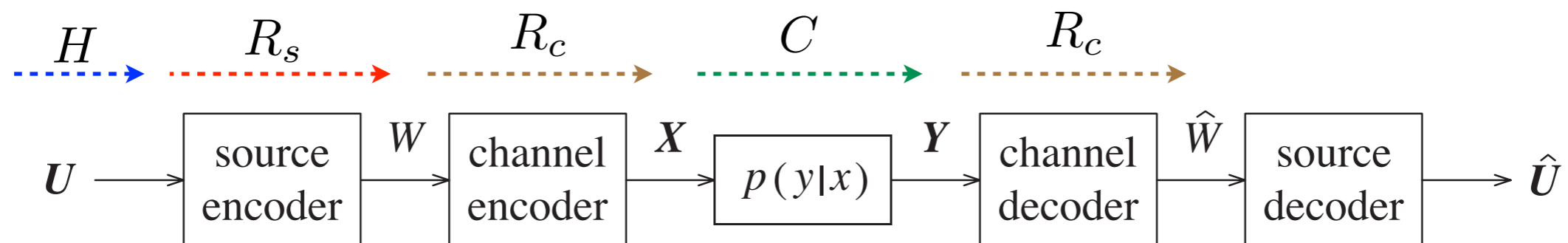- Specifically, choose R_s and R_c such that

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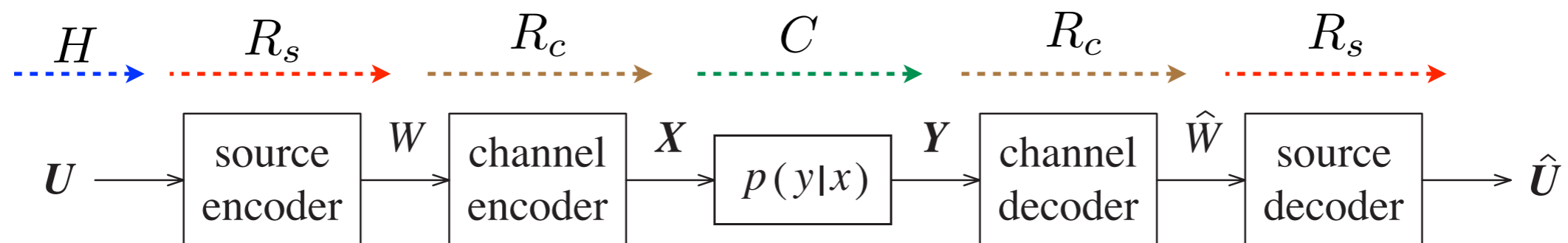
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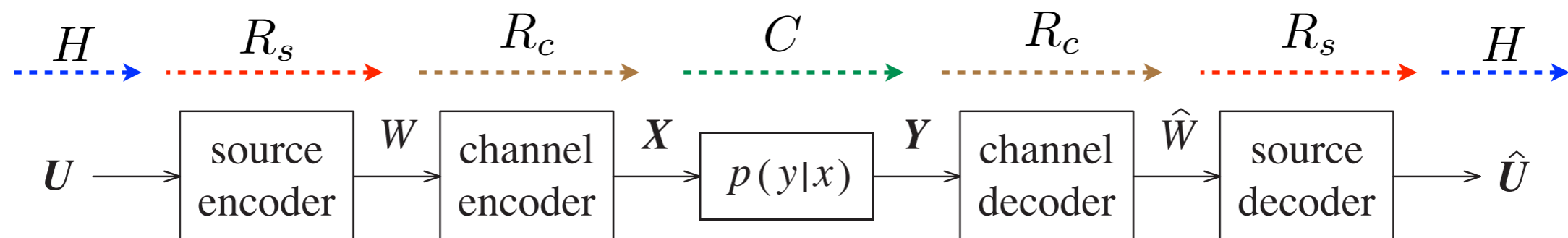
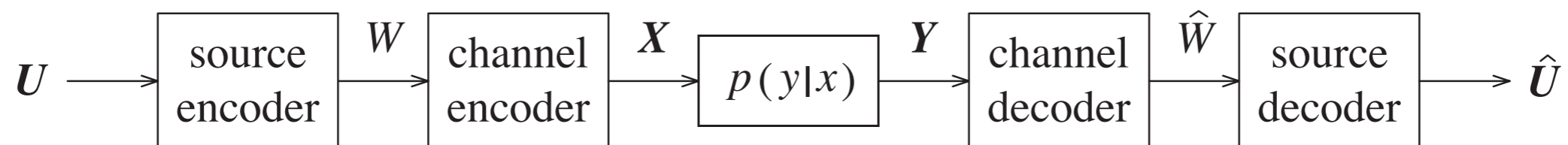- Specifically, choose R_s and R_c such that

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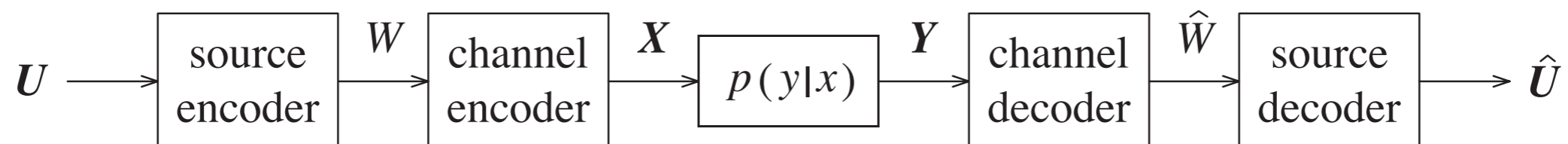
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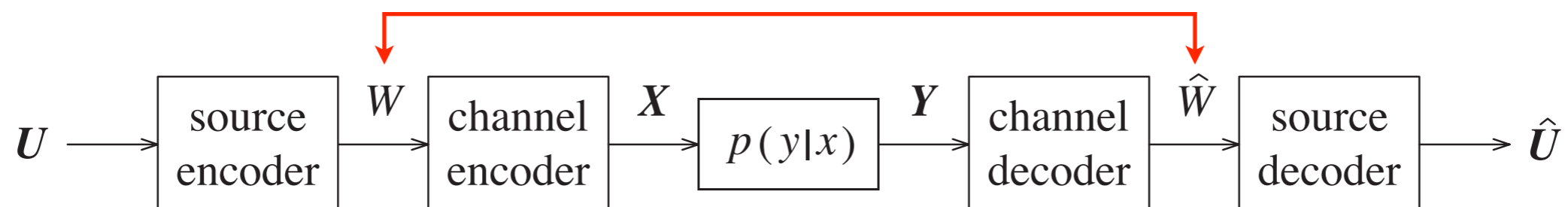
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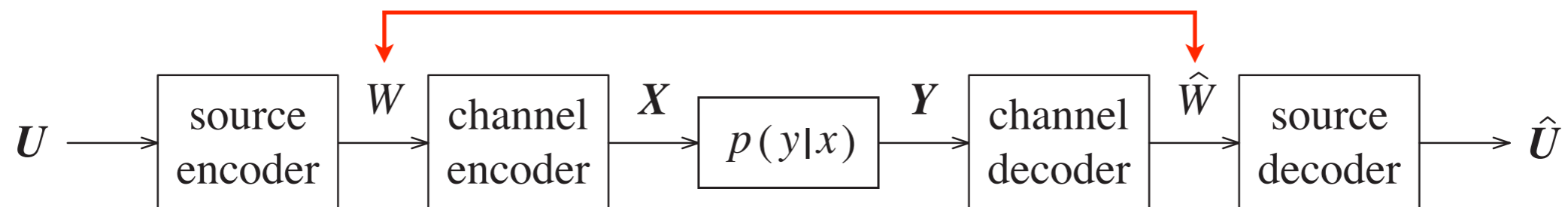
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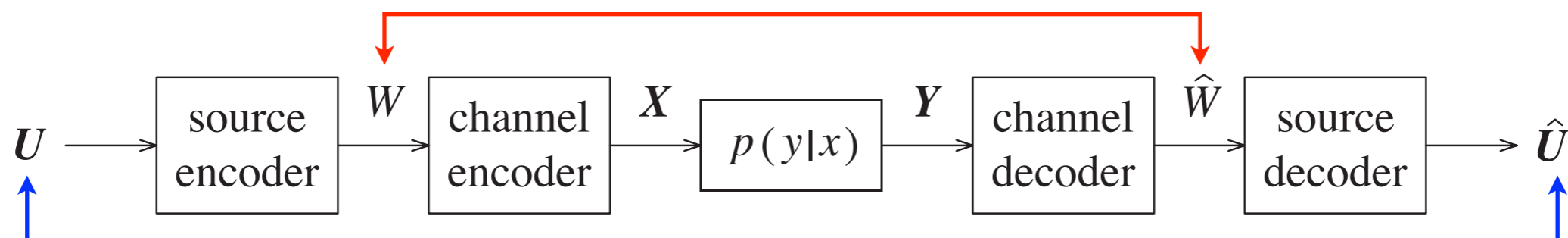
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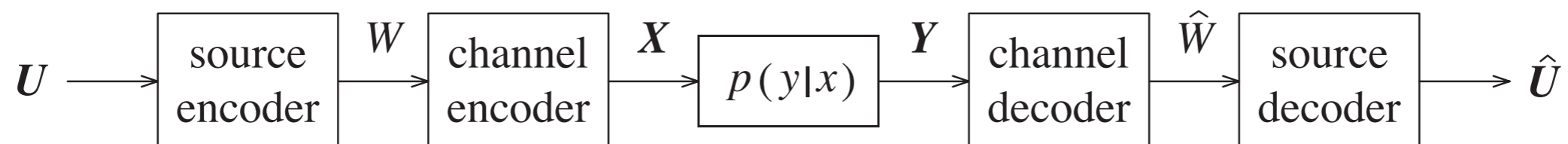
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- It can be shown that even with complete feedback, reliable communication is impossible if $H > C$. See textbook for details.

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Remark For finite block length, the probability of error generally can be reduced by using joint source-channel coding.