香港中文大學
The Chinese University of Hong Kong

7.4 Achievability

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- For every input distribution $p(x)$, prove that the rate $I(X;Y)$ is achievable by showing for large n the existence of a channel code such that
  1. the rate of the code is arbitrarily close to $I(X;Y)$;
  2. the maximal probability of error λ_{max} is arbitrarily small.
- Choose the input distribution $p(x)$ to be one that achieves the channel capacity, i.e., $I(X;Y) = C$.

Lemma 7.17 Let $(\mathbf{X}', \mathbf{Y}')$ be n i.i.d. copies of a pair of generic random variables (X', Y'), where $X' \sim X$, $Y' \sim Y$, and X' and Y' are independent. Then

$$\Pr\{(\mathbf{X}', \mathbf{Y}') \in T^n_{[XY]\delta}\} \leq 2^{-n(I(X;Y)-\tau)},$$

where $\tau \to 0$ as $\delta \to 0$.

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where $\tau \to 0$ as $\delta \to 0$.

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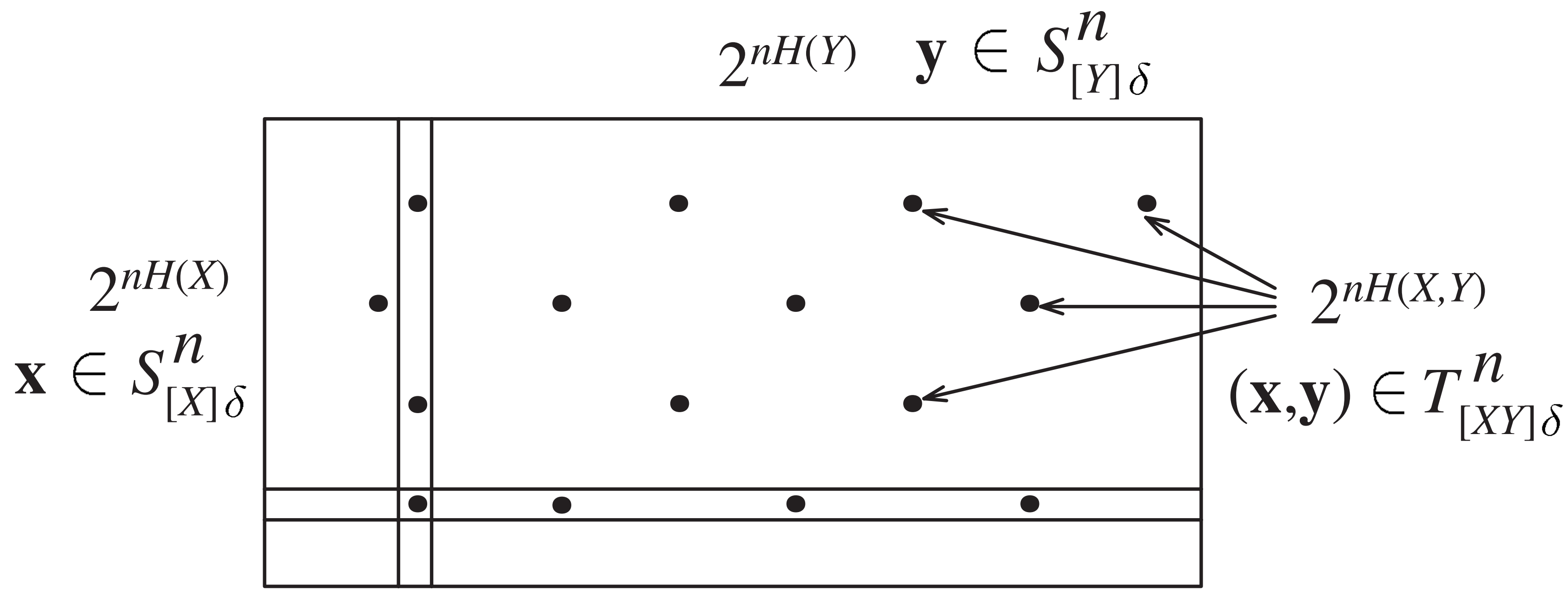
$$\tau = \xi + \eta + \zeta \to 0$$

as $\delta \to 0$. The lemma is proved.

An Interpretation of Lemma 7.17



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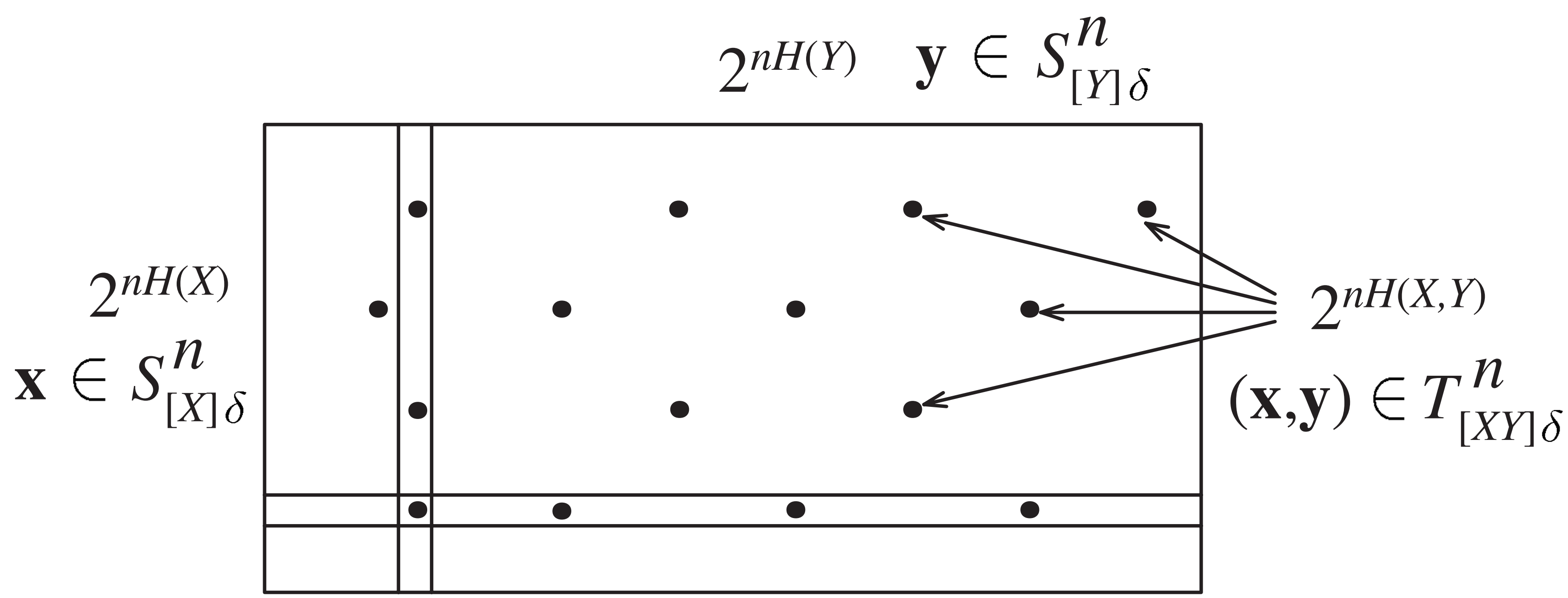

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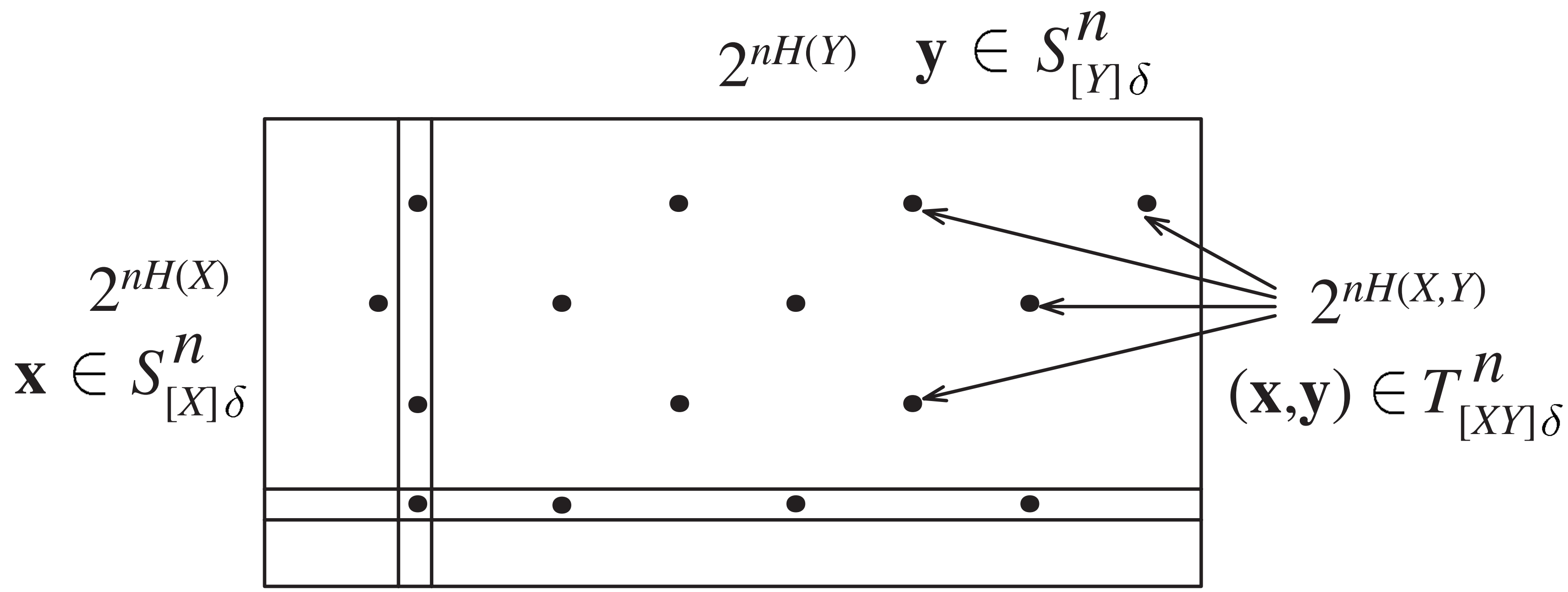

- Randomly choose a row with uniform distribution and randomly choose a column with uniform distribution.
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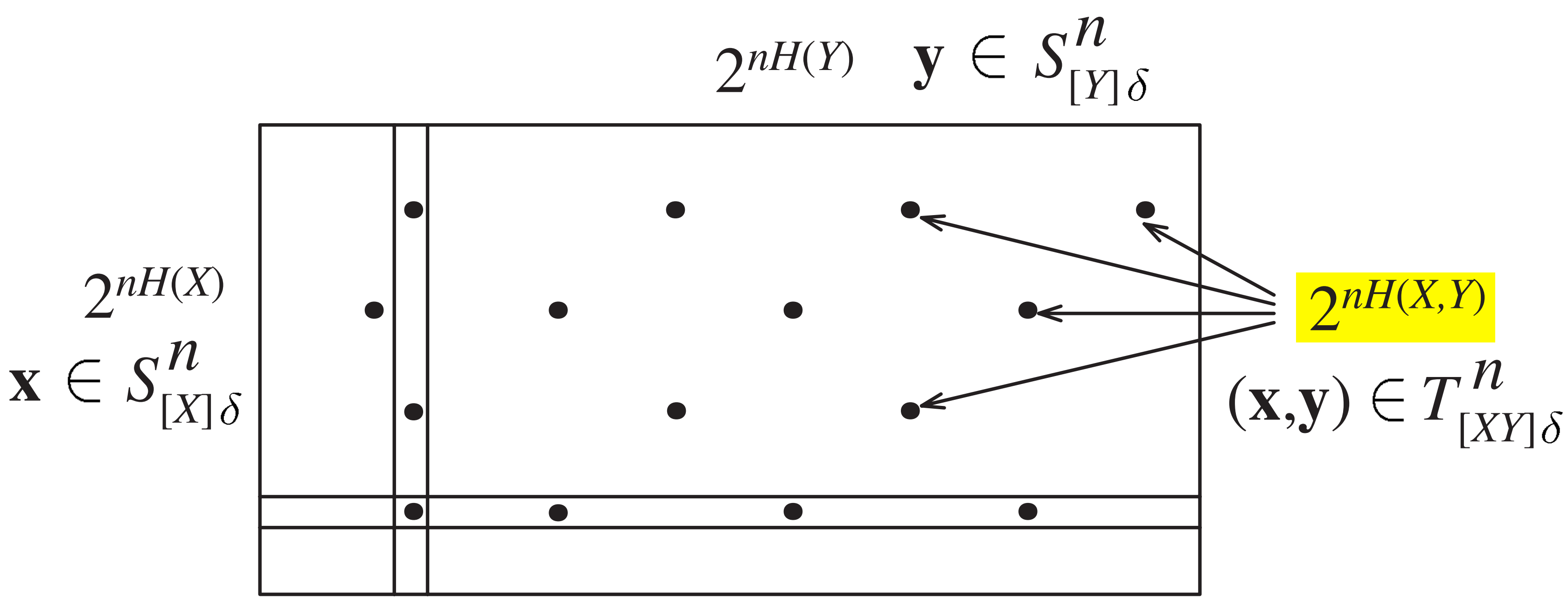

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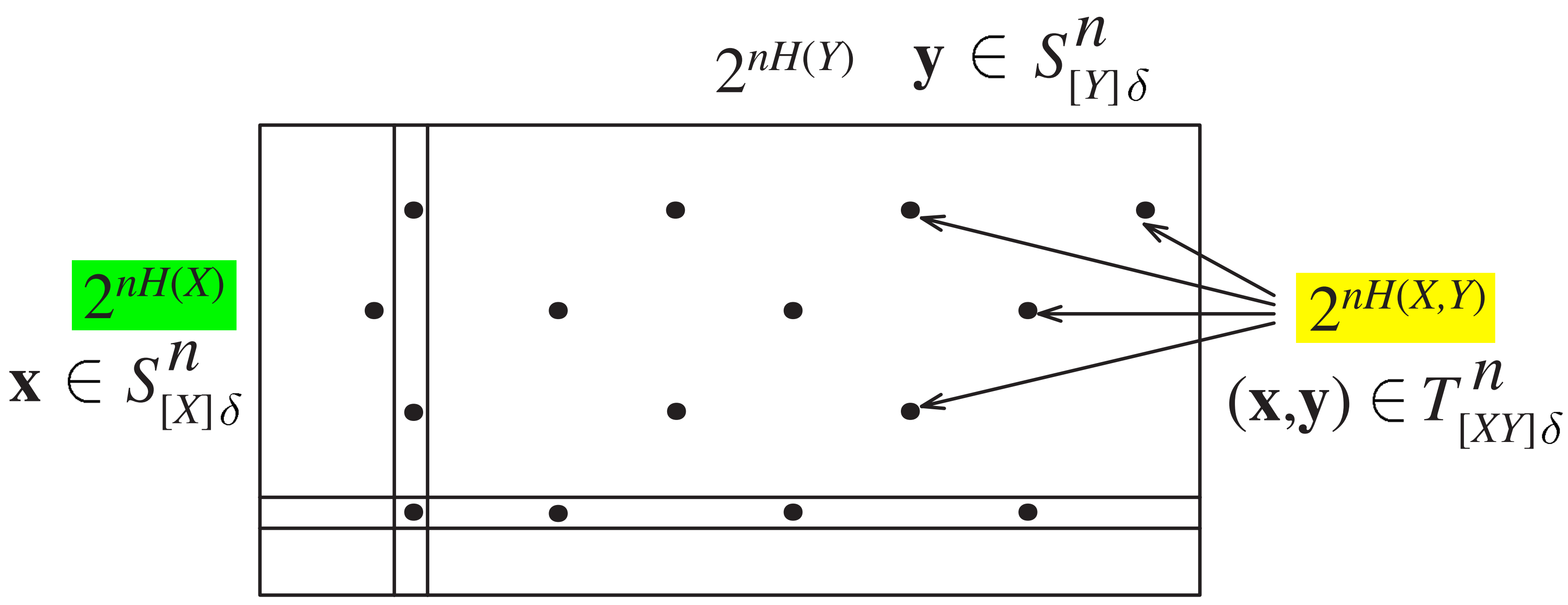

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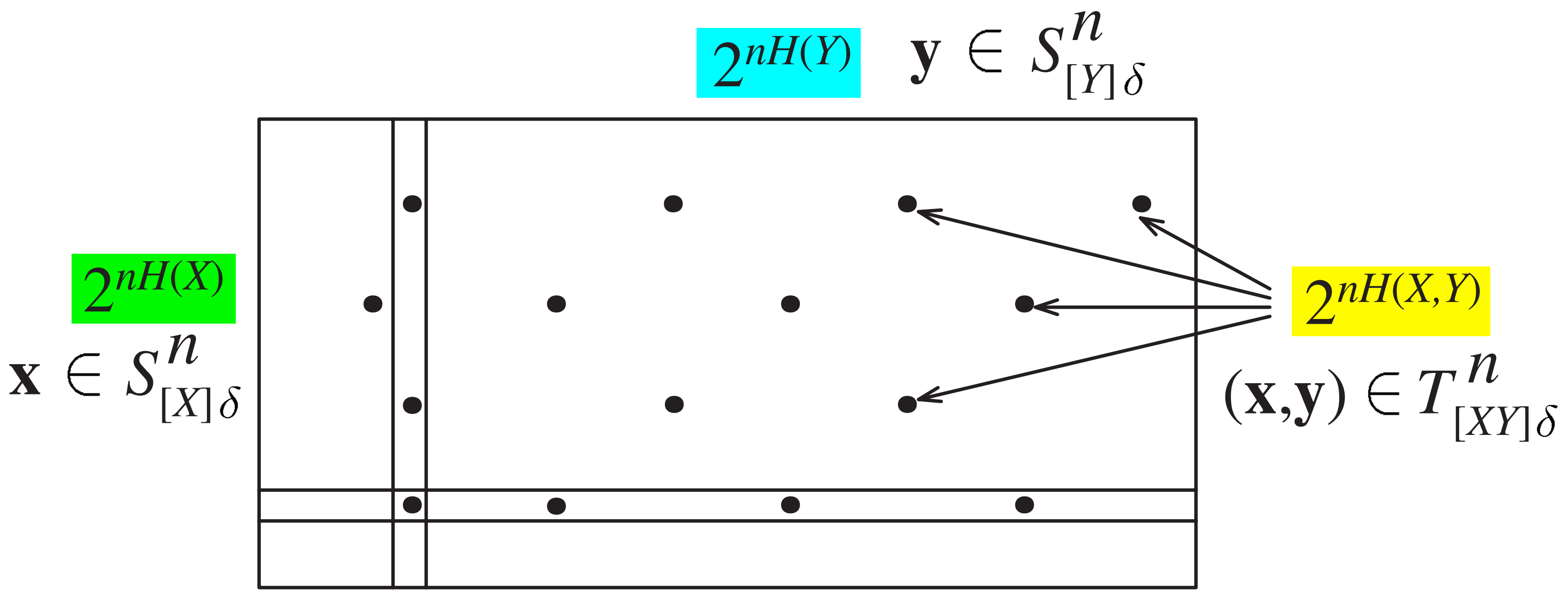

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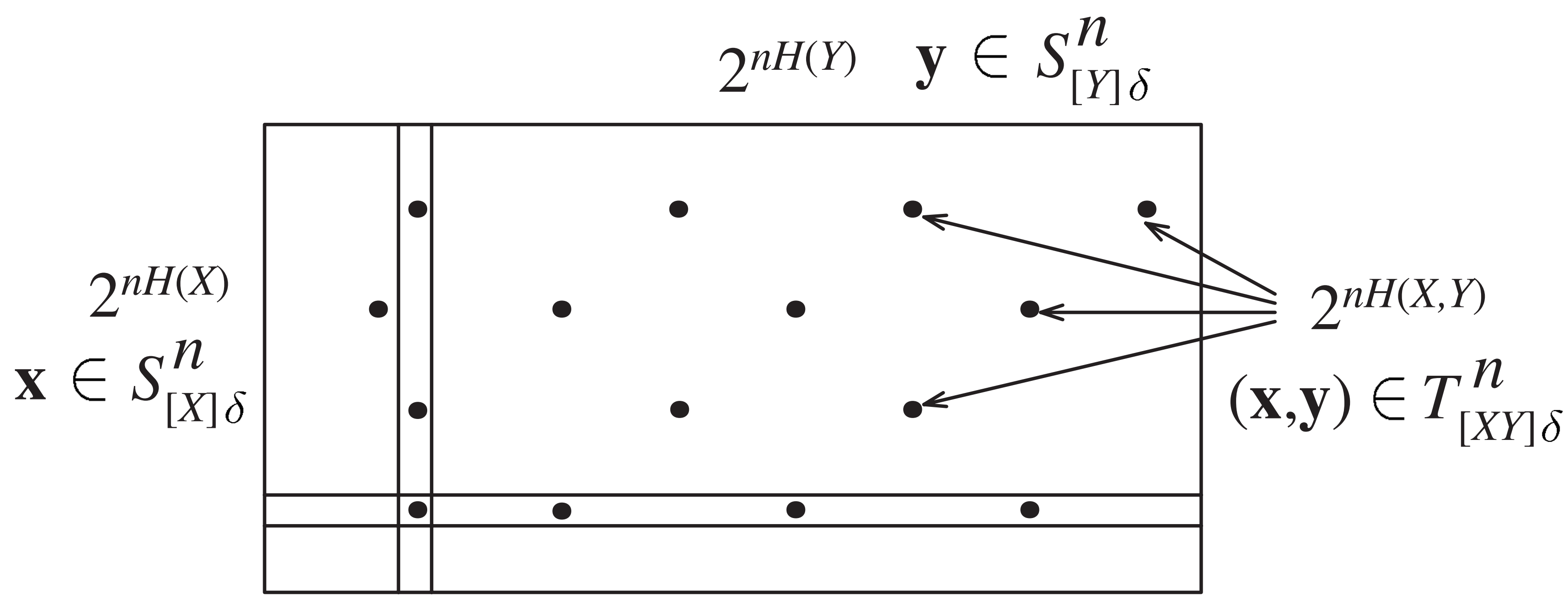

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1. Fix $\epsilon > 0$ and input distribution $p(x)$. Let δ to be specified later.

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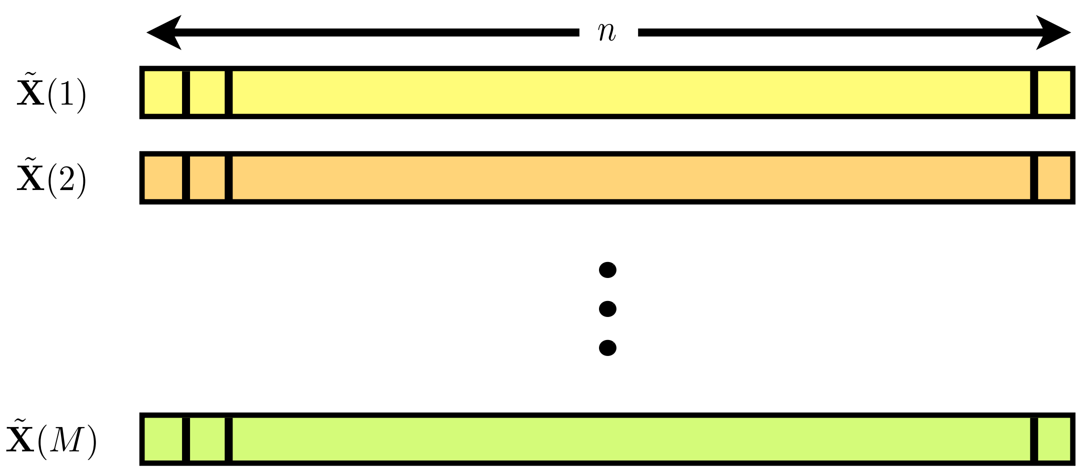

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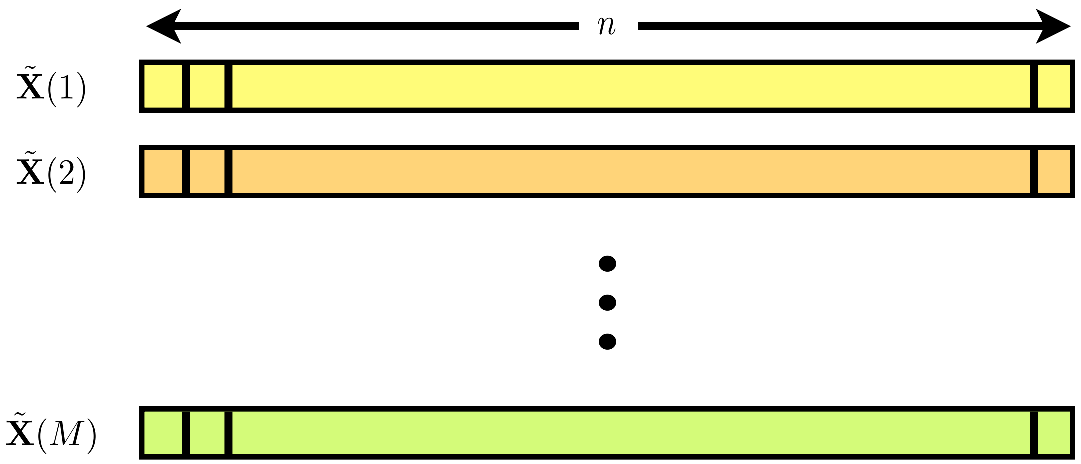

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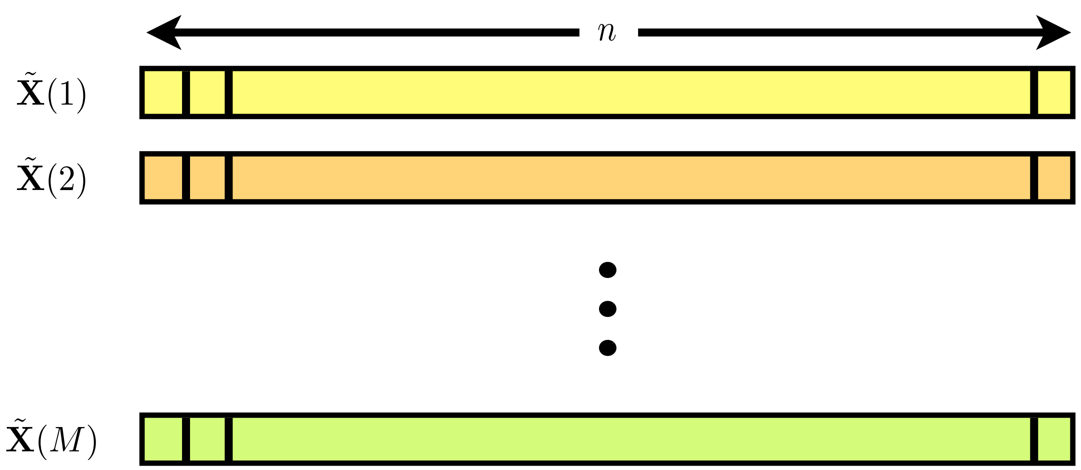

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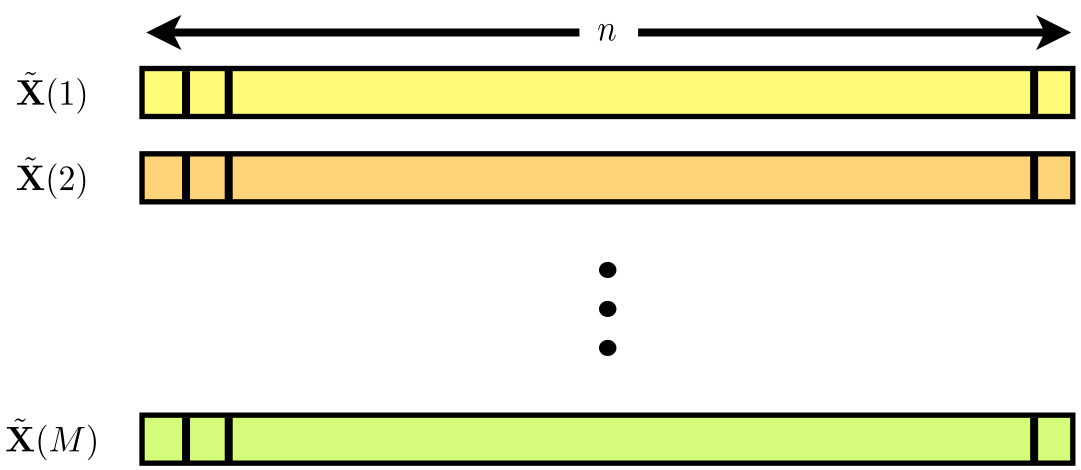

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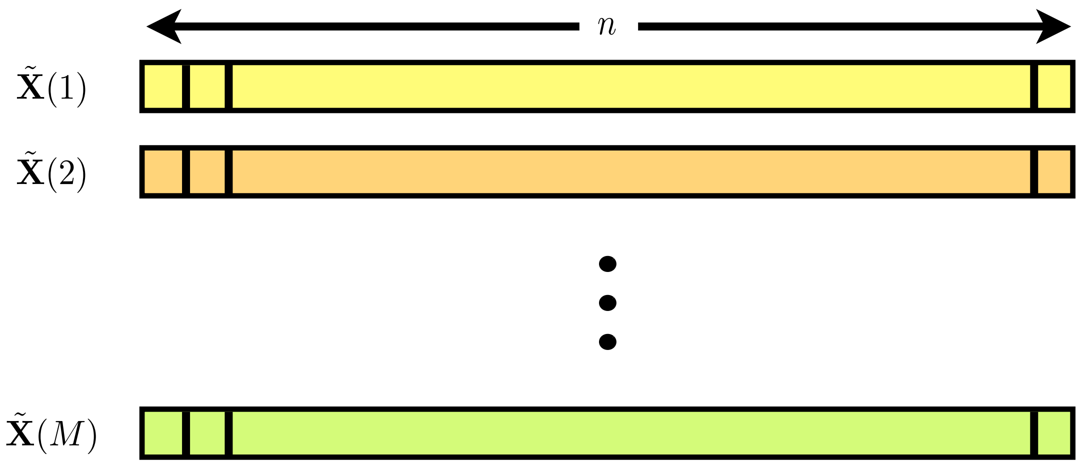

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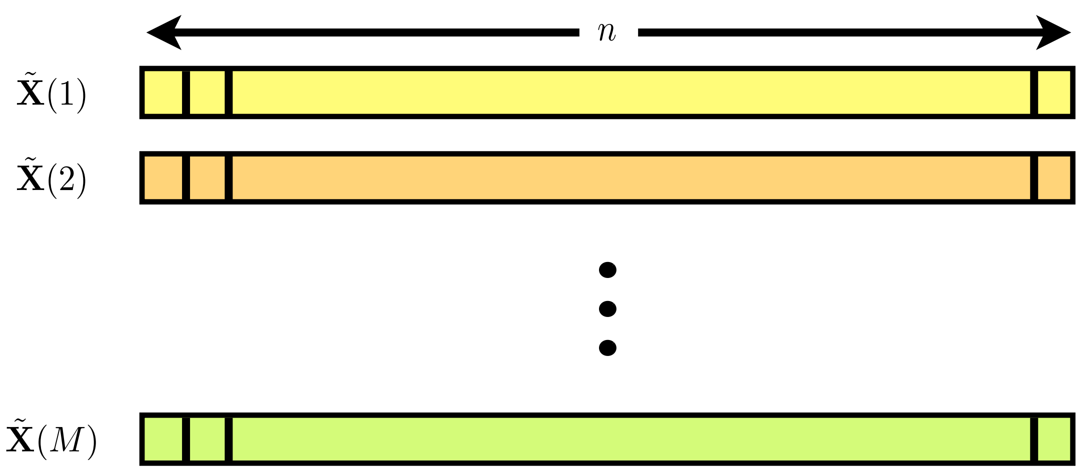

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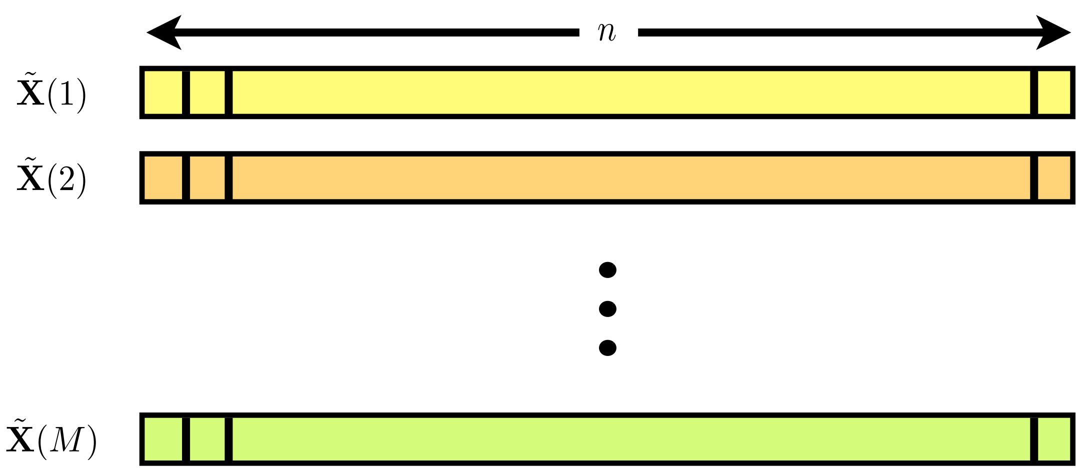

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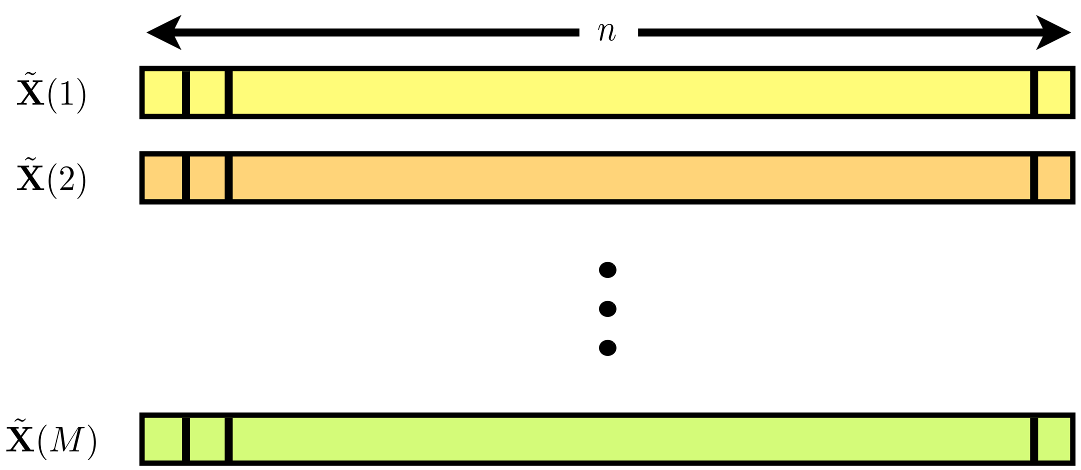

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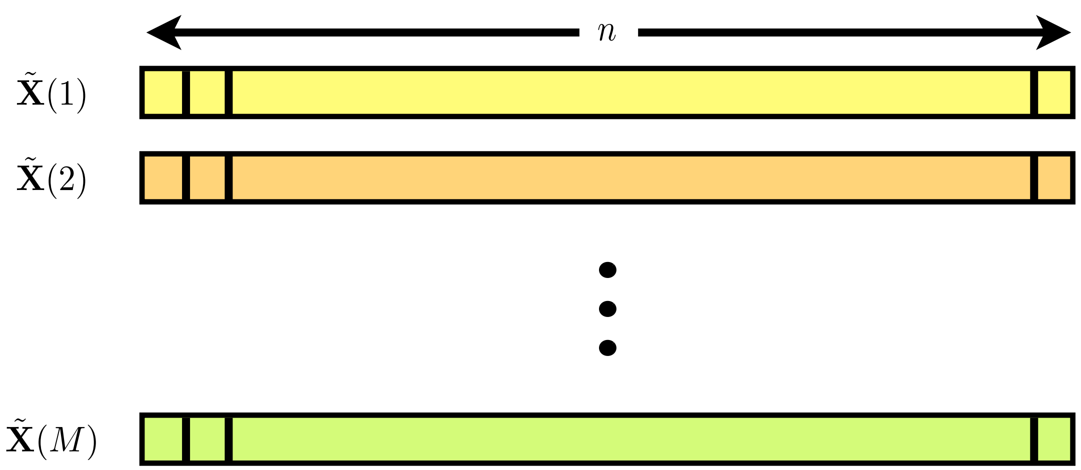

- Generate each component according to $p(x)$.
- There are a total of $|\mathcal{X}|^{Mn}$ possible codebooks that can be constructed.
- Regard two codebooks whose sets of codewords are permutations of each other as two different codebooks.

2. Reveal the codebook $\mathcal{C}$ to both the encoder and the decoder.

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1. Fix $\epsilon > 0$ and input distribution $p(x)$. Let δ to be specified later.

2. Let M be an even integer satisfying

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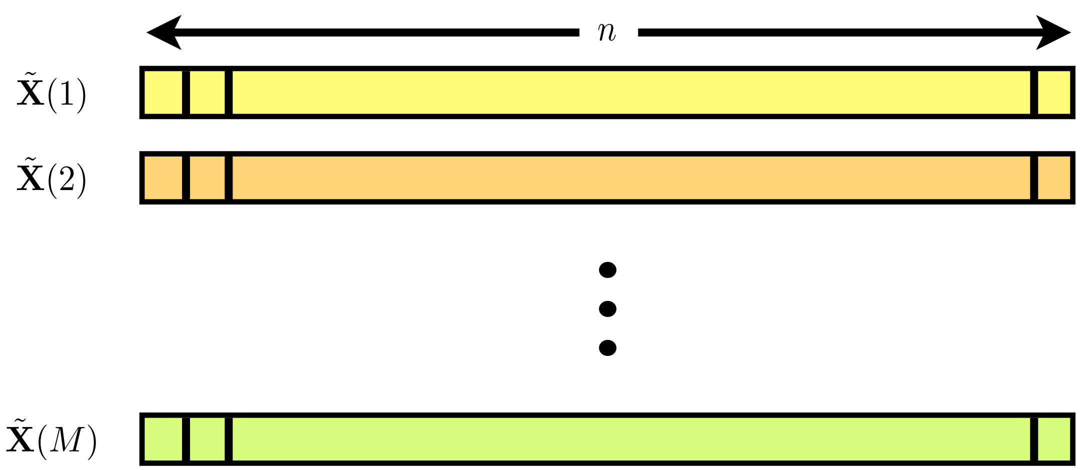

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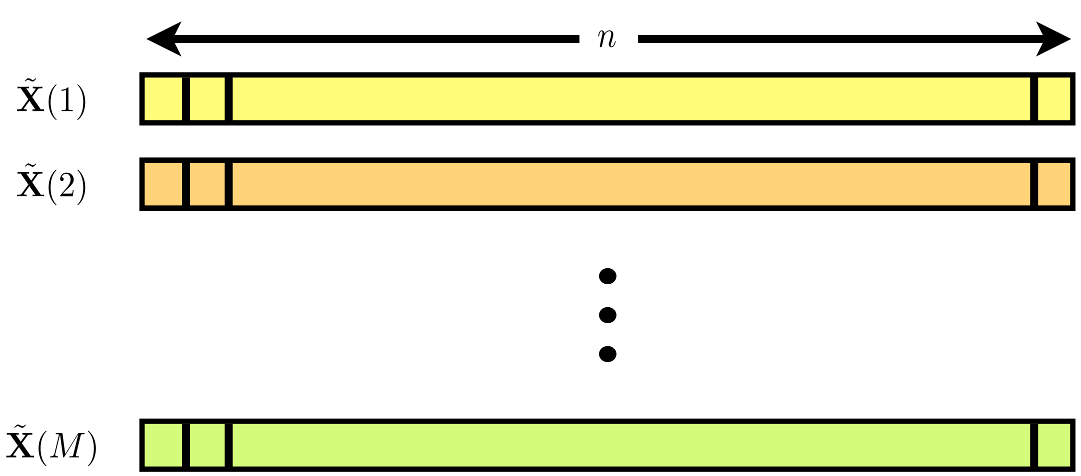

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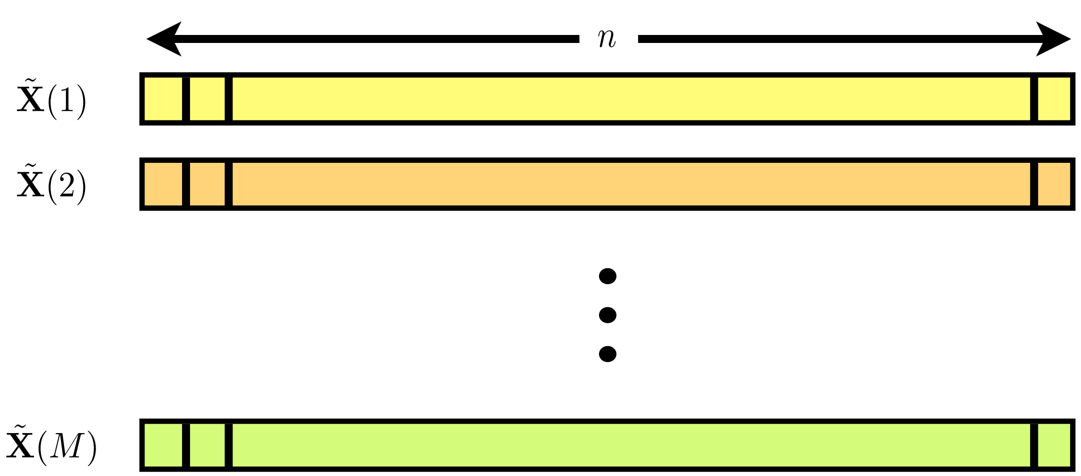

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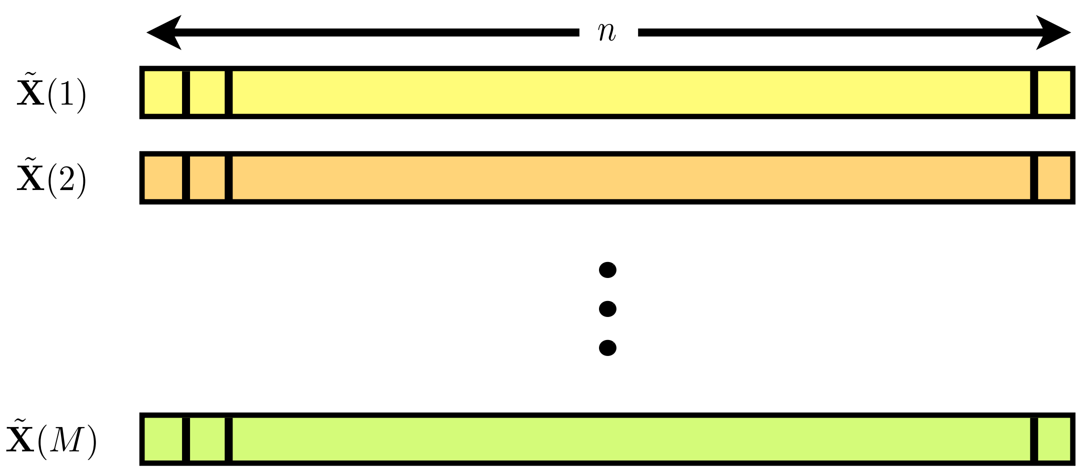

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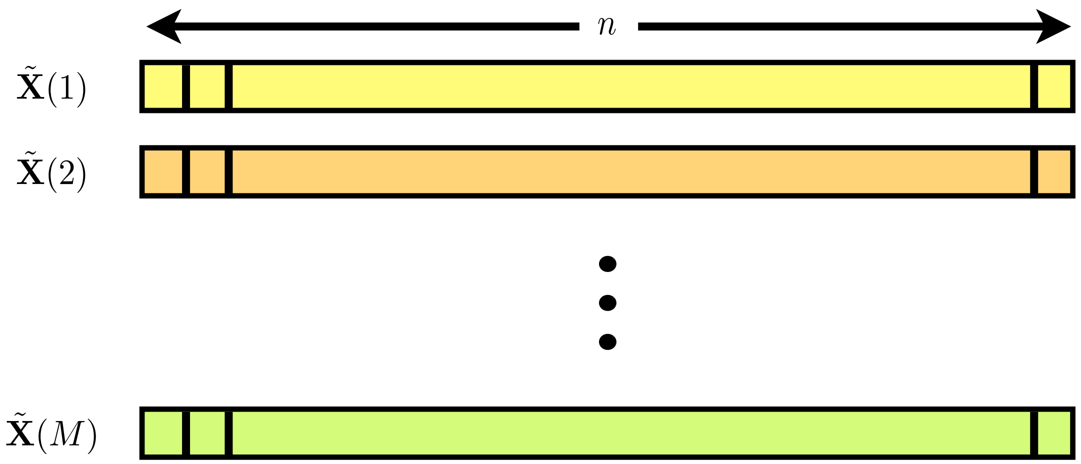

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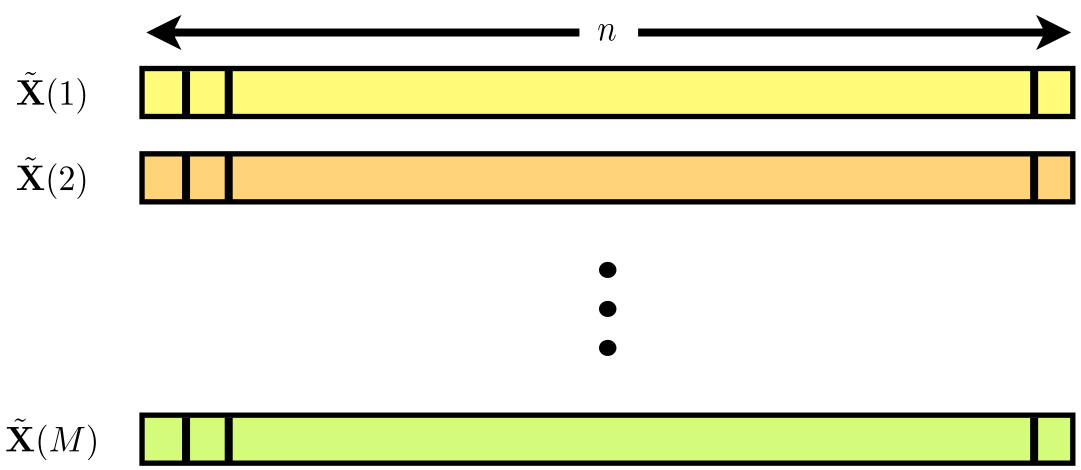

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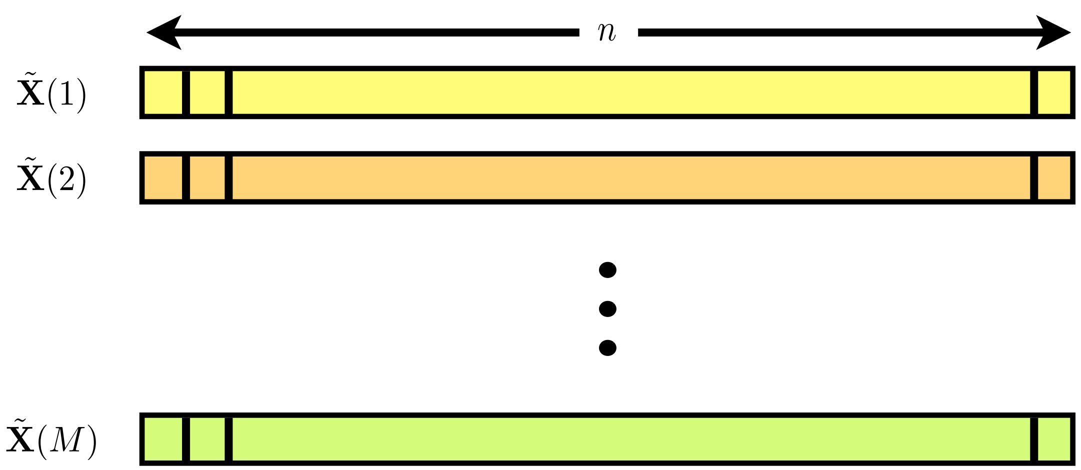

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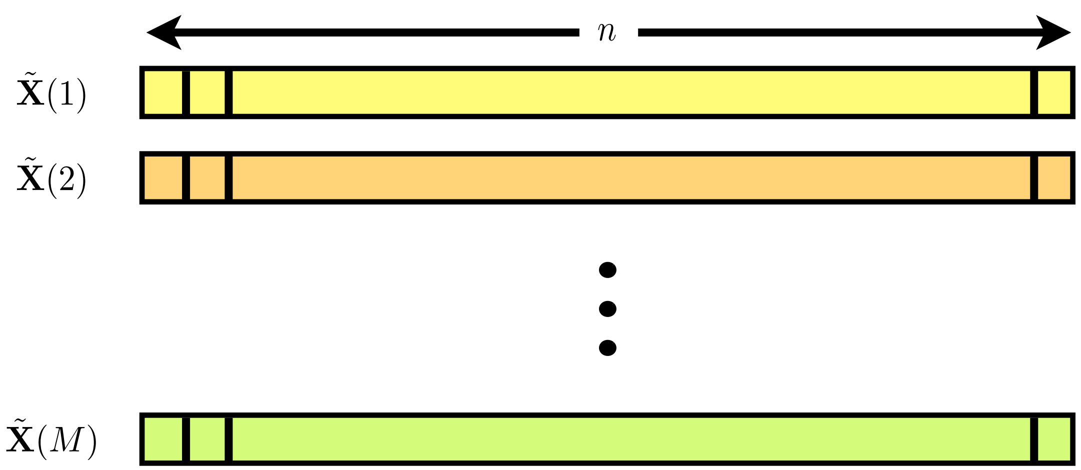

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6. The sequence $\mathbf{Y}$ is decoded to the message w if

- $(\tilde{\mathbf{X}}(w), \mathbf{Y}) \in T^n_{[XY]\delta}$, and
- there does not exists $w' \neq w$ such that $(\tilde{\mathbf{X}}(w'), \mathbf{Y}) \in T^n_{[XY]\delta}$.

Otherwise, $\mathbf{Y}$ is decoded to a constant message in $\mathcal{W}$. Denote by $\hat{W}$ the message to which $\mathbf{Y}$ is decoded.

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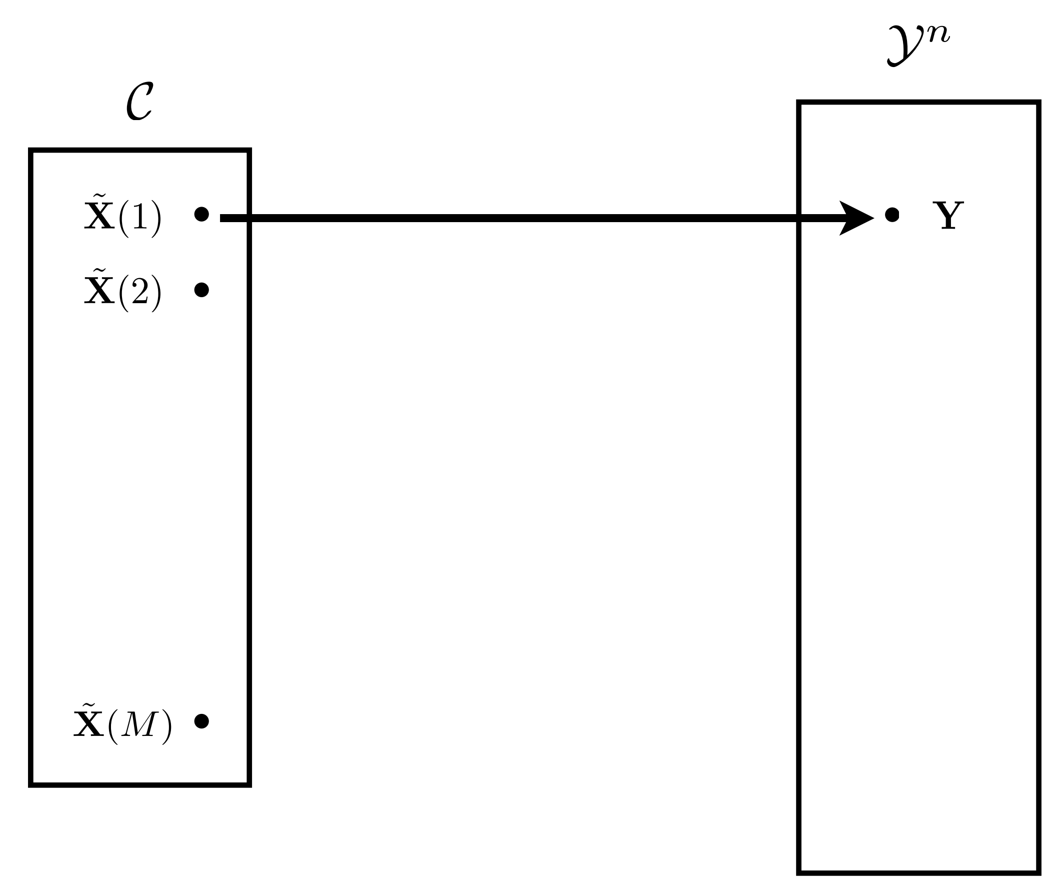

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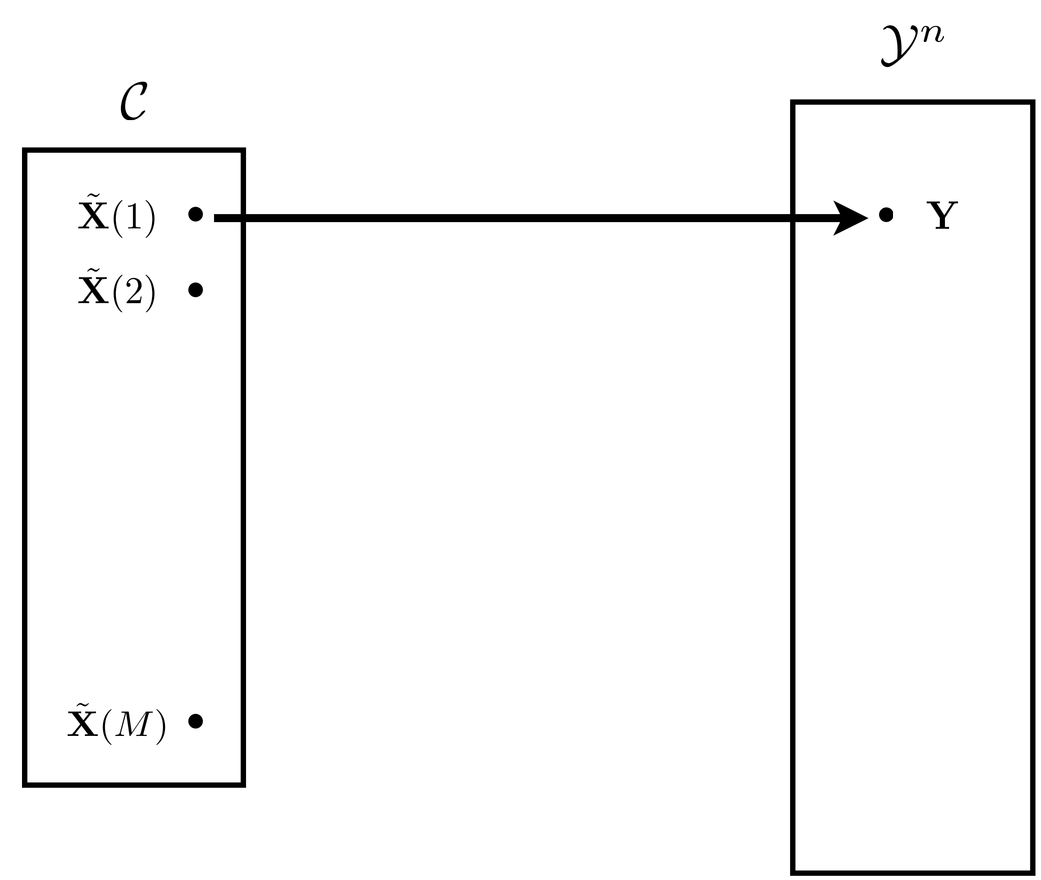

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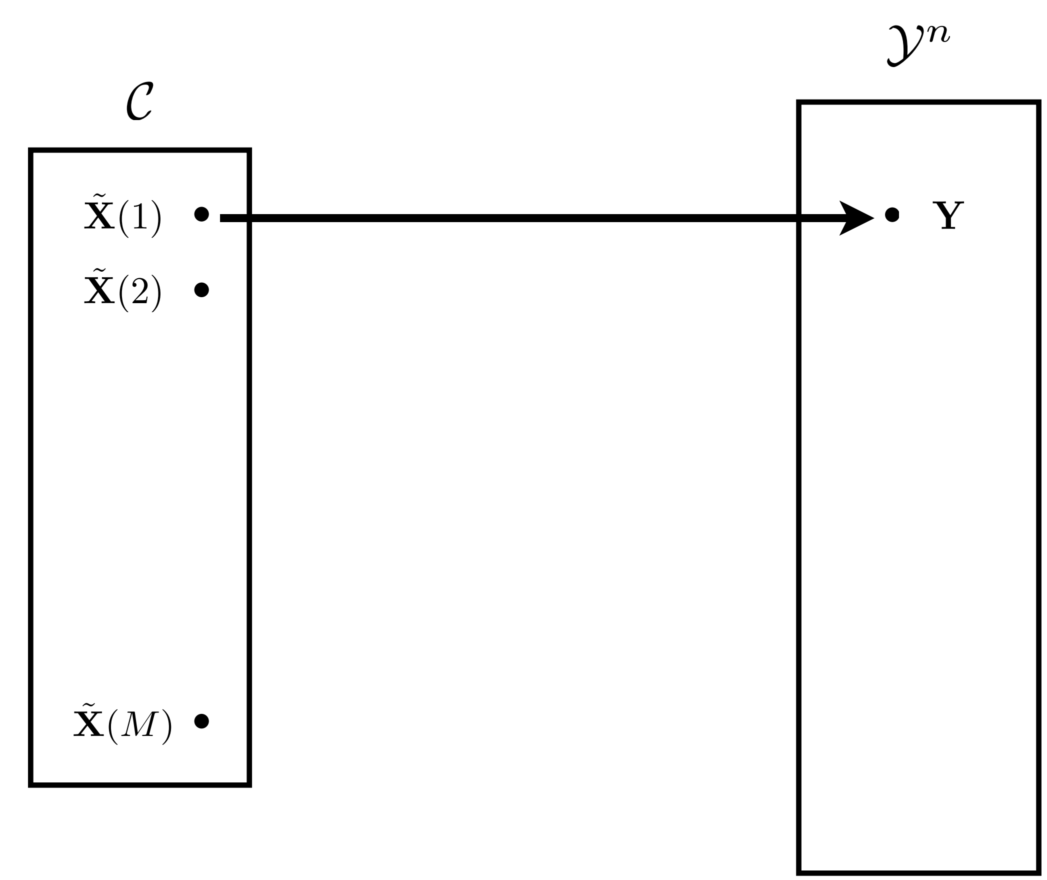

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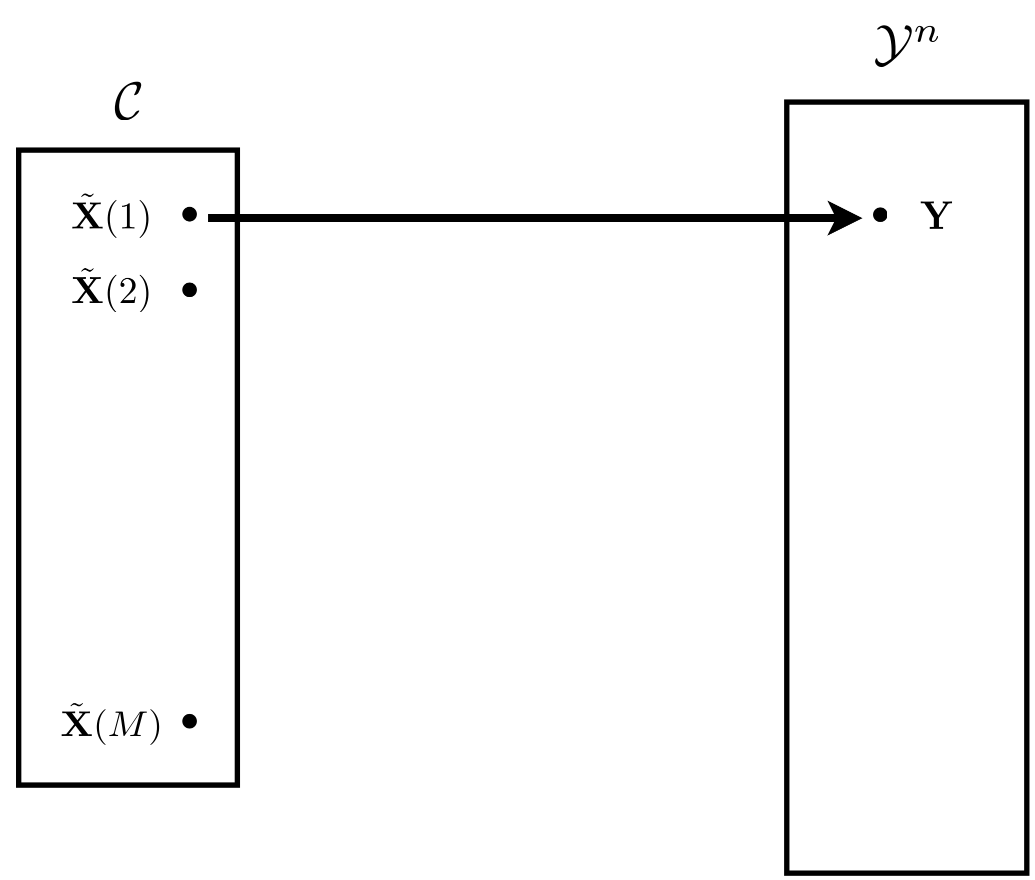

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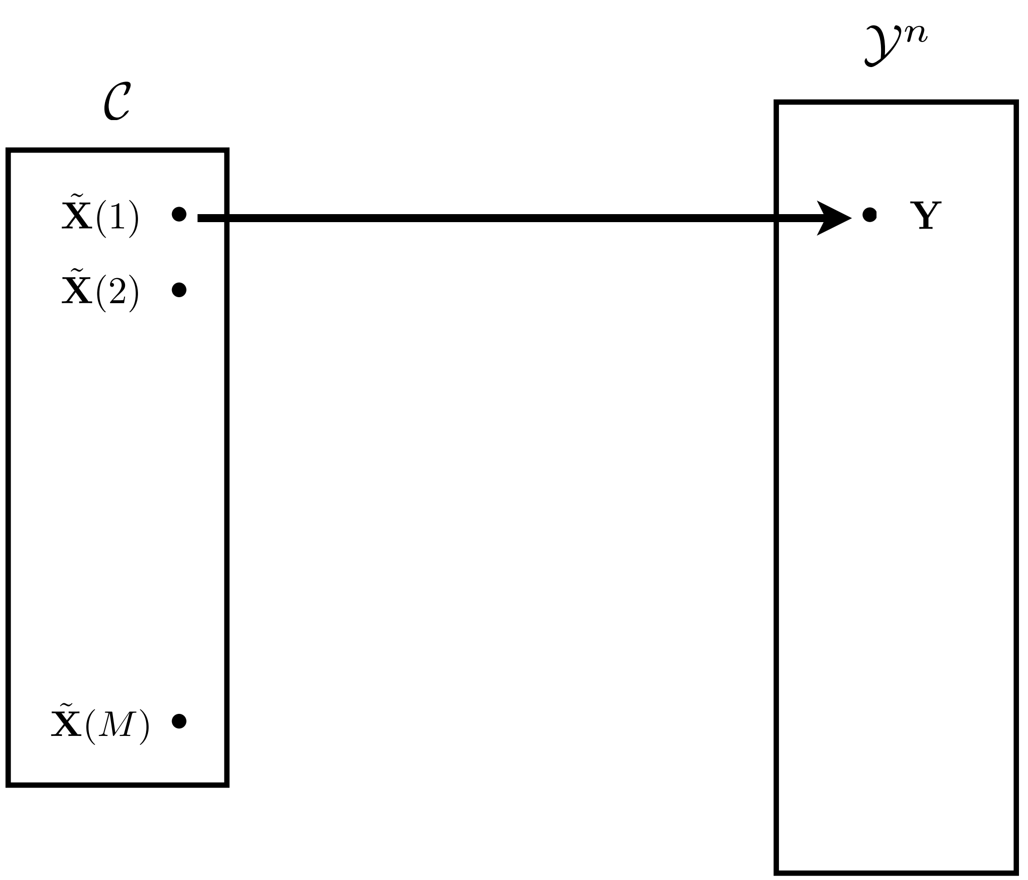

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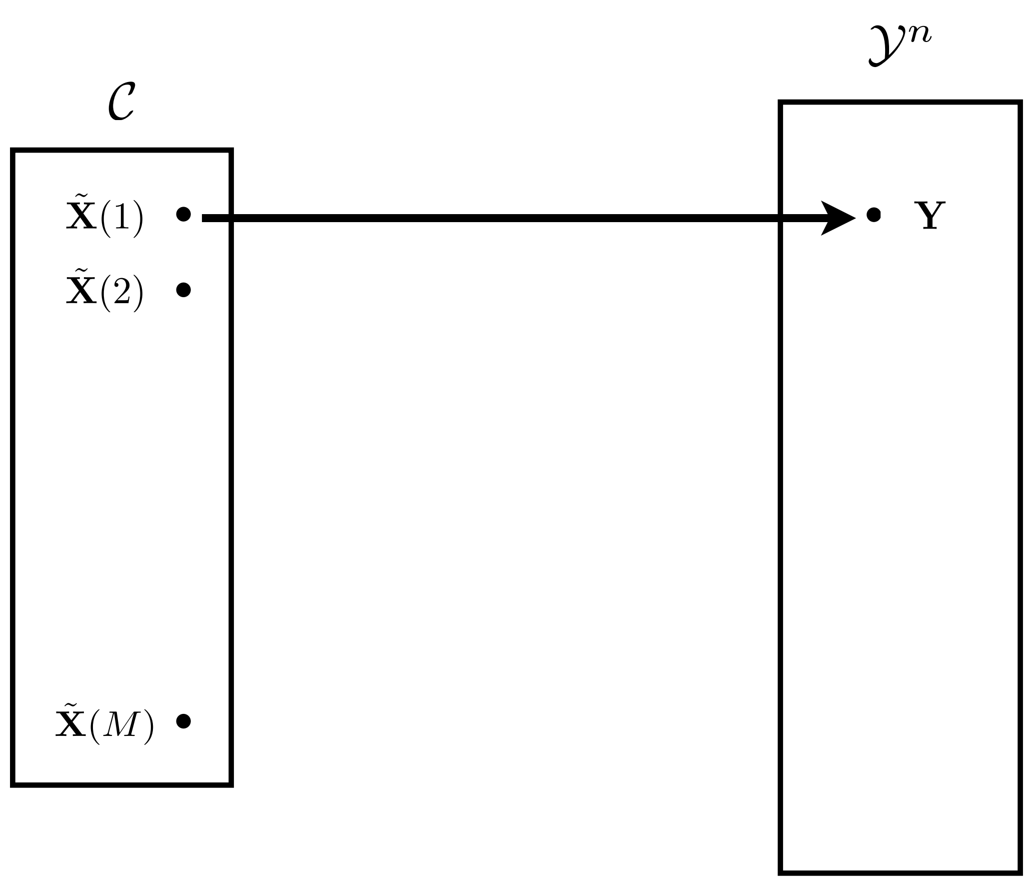

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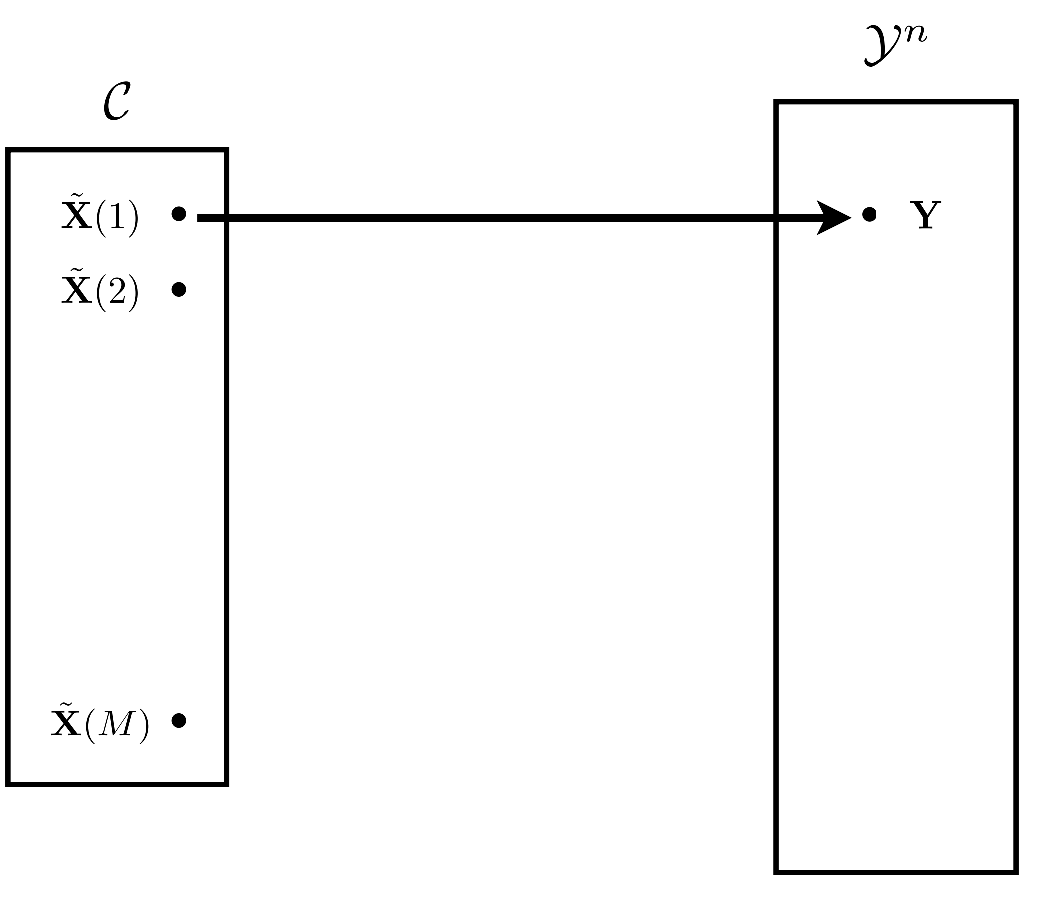

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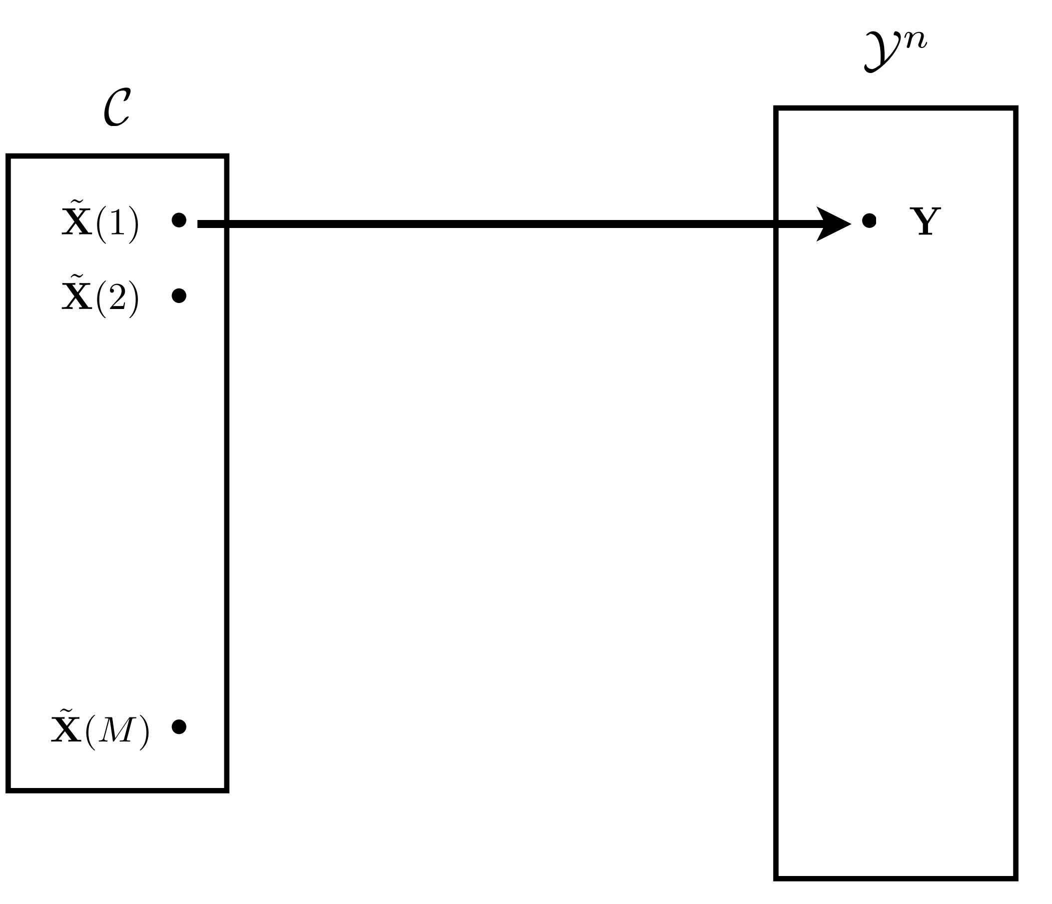

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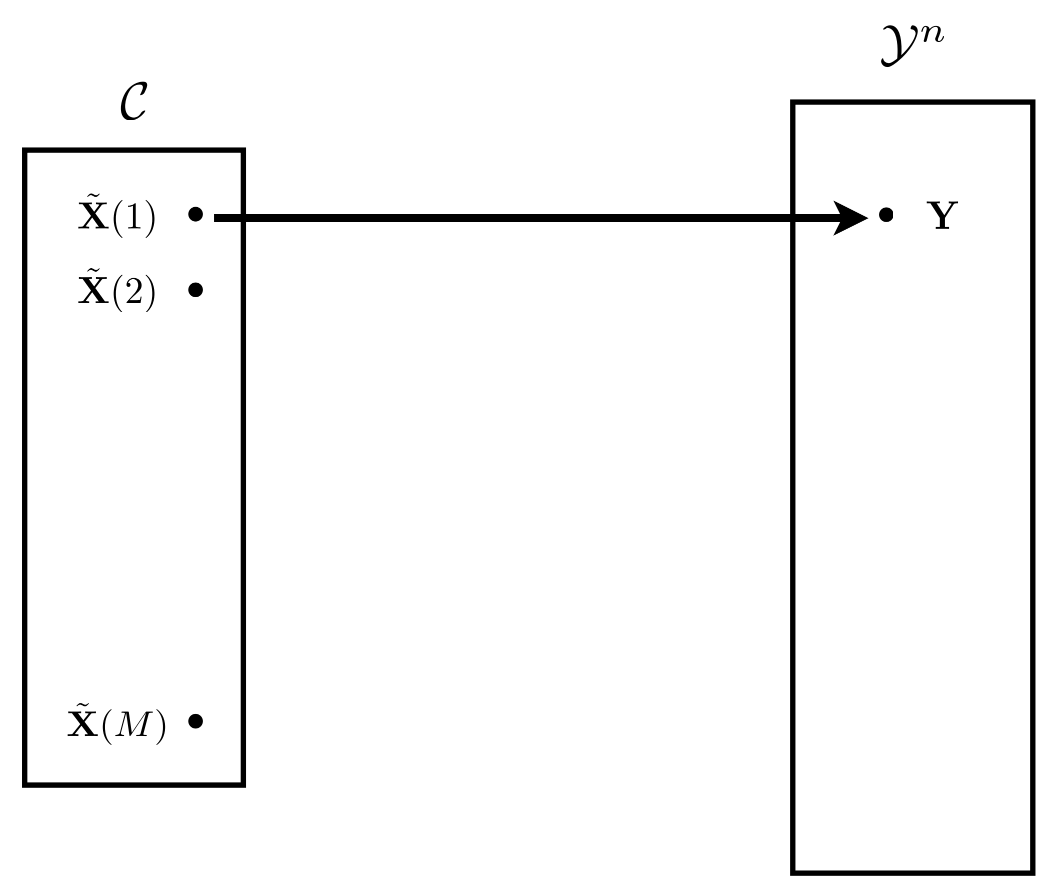

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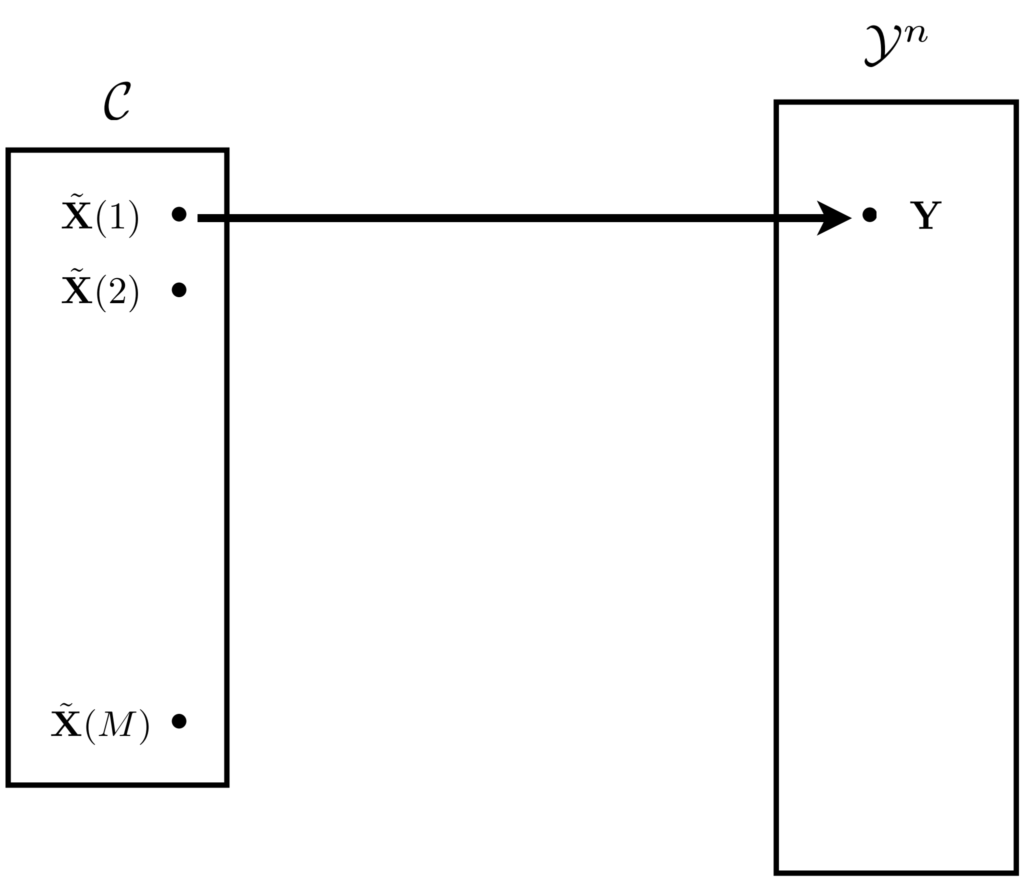

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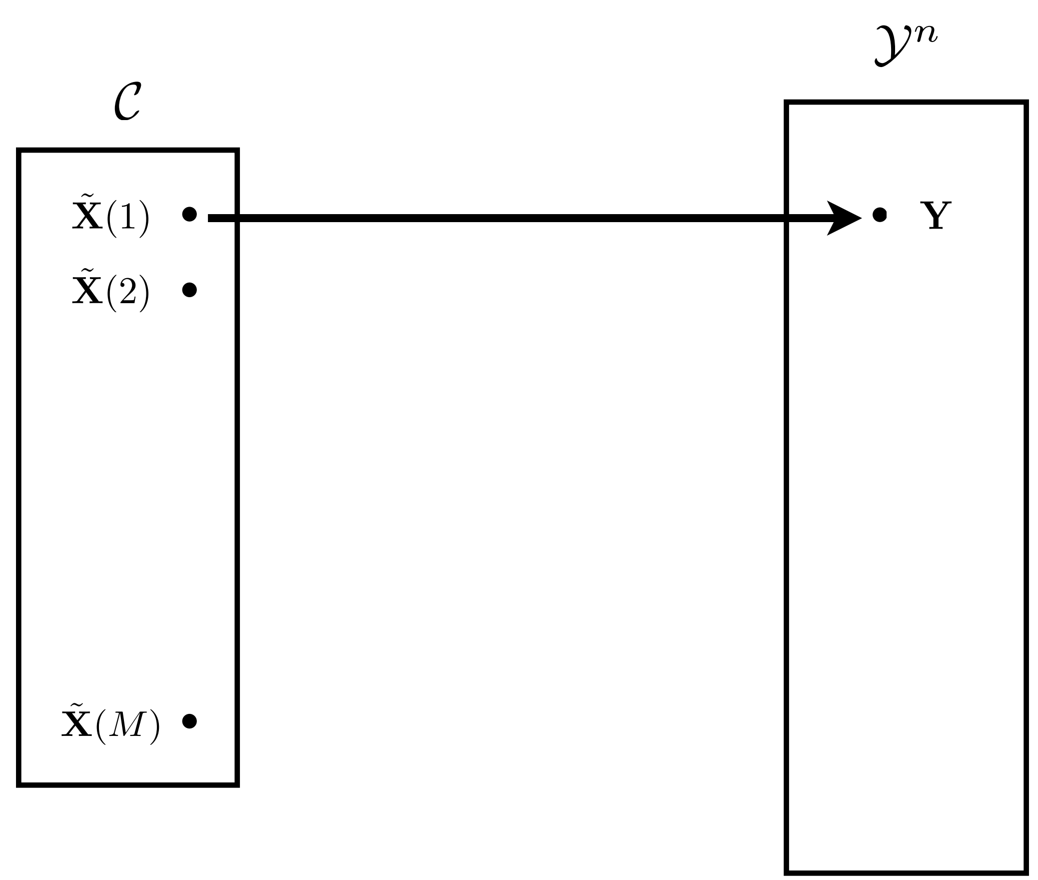

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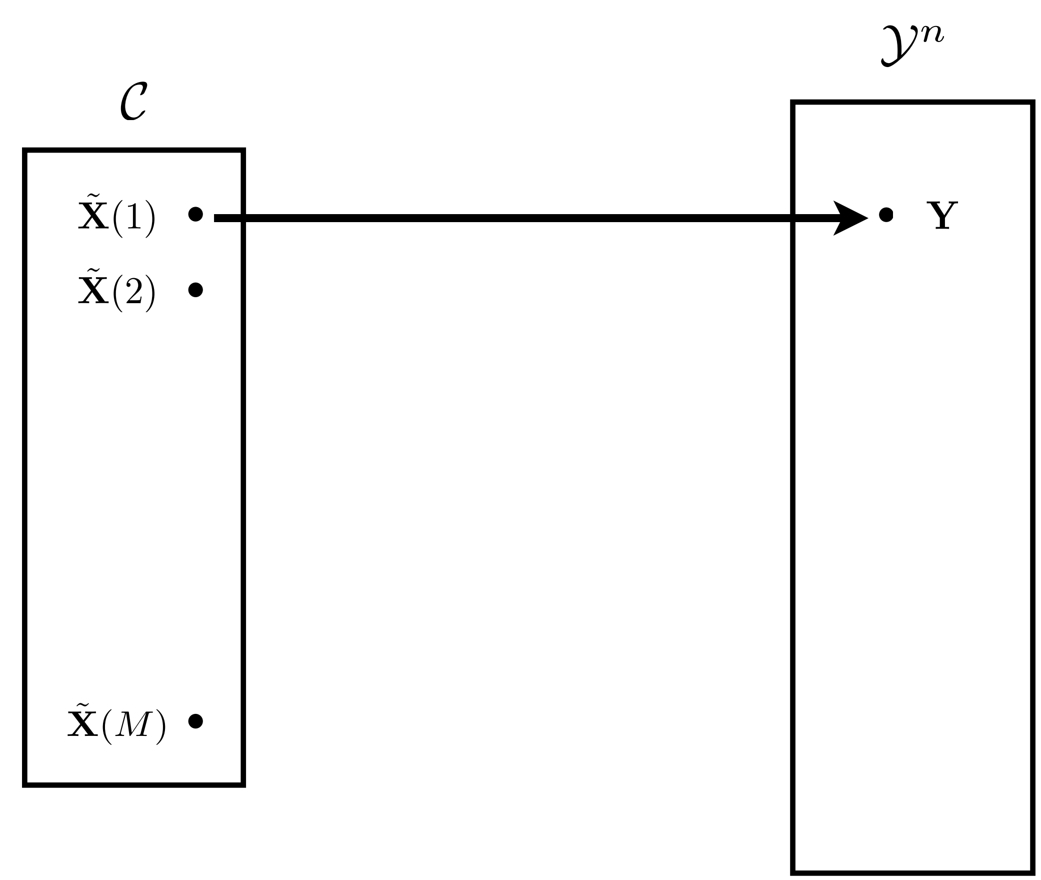

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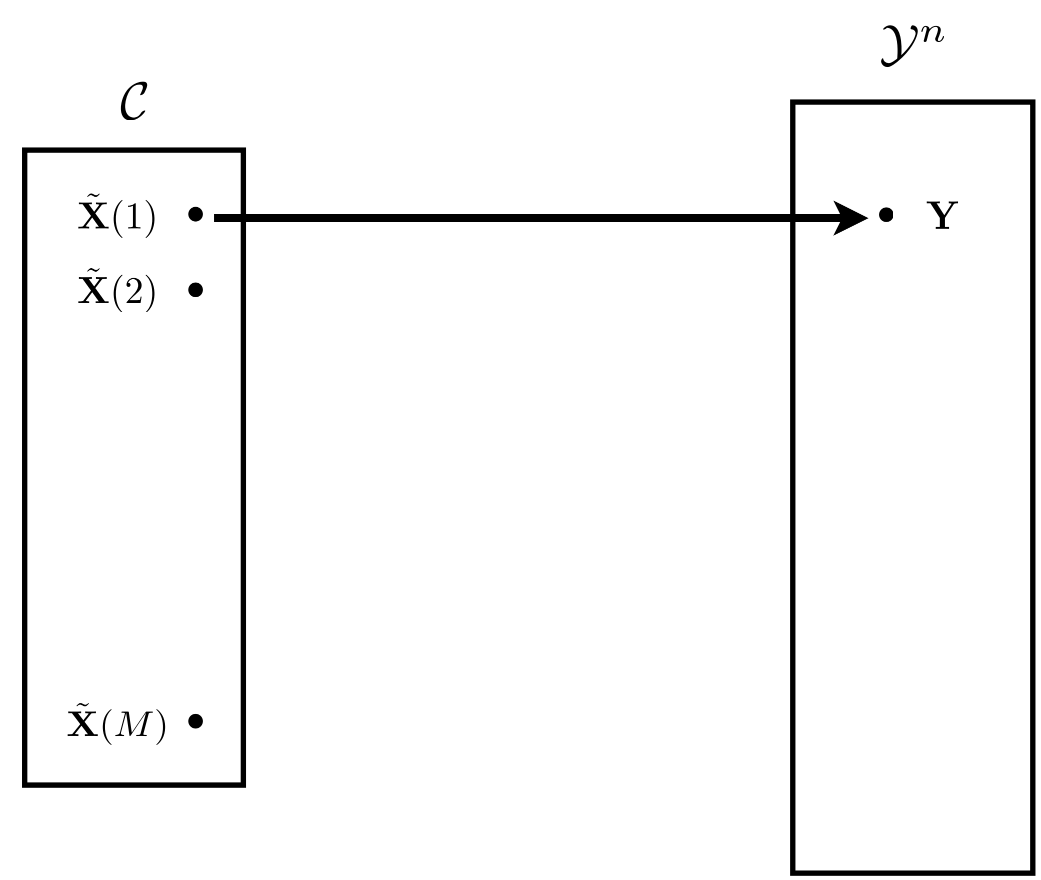

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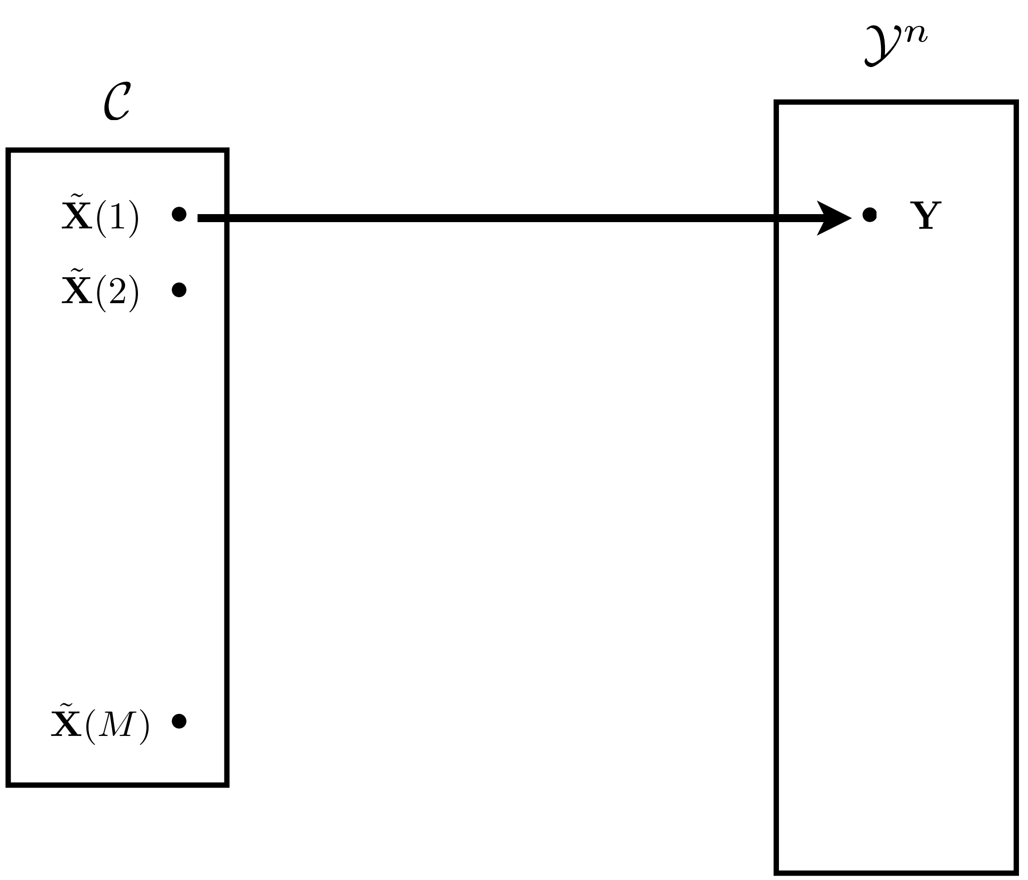

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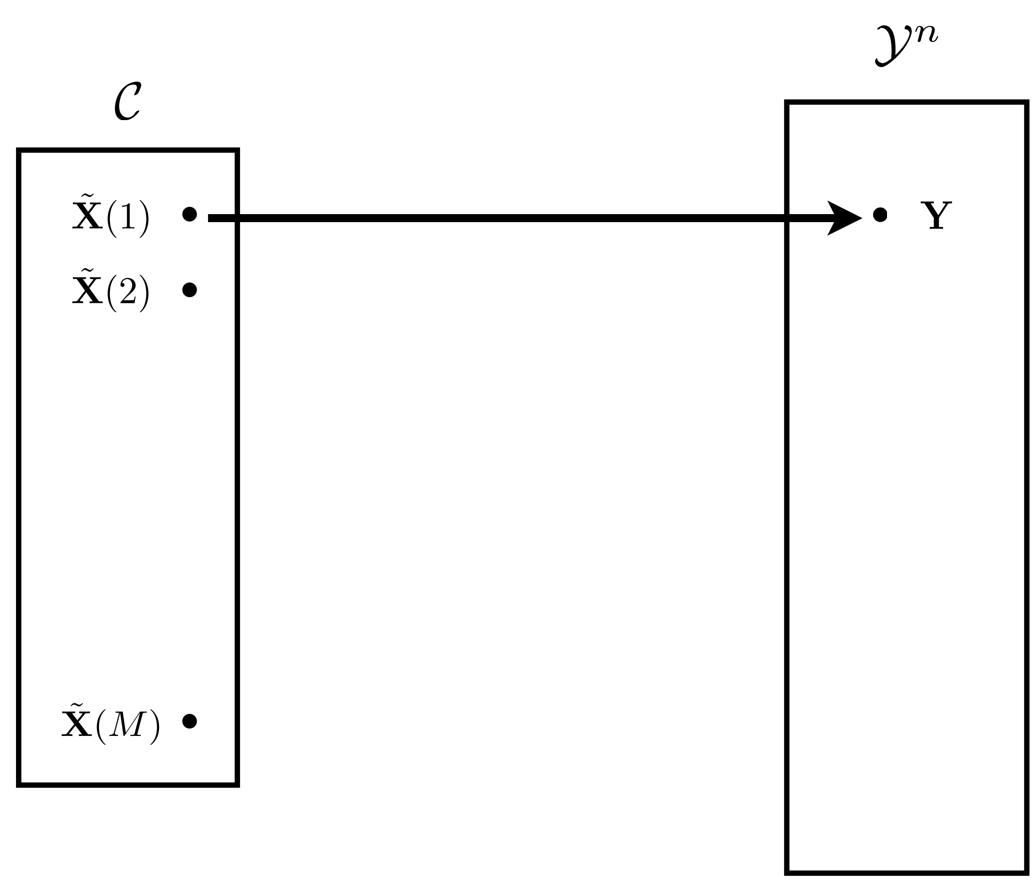

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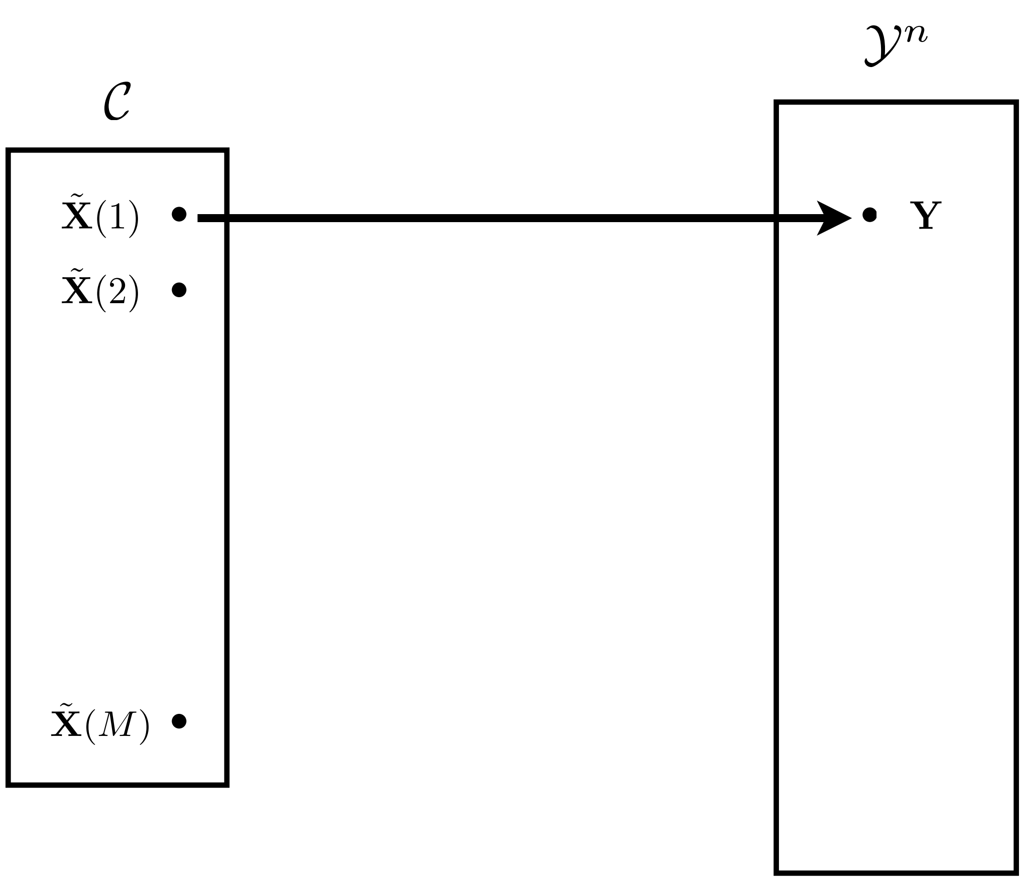

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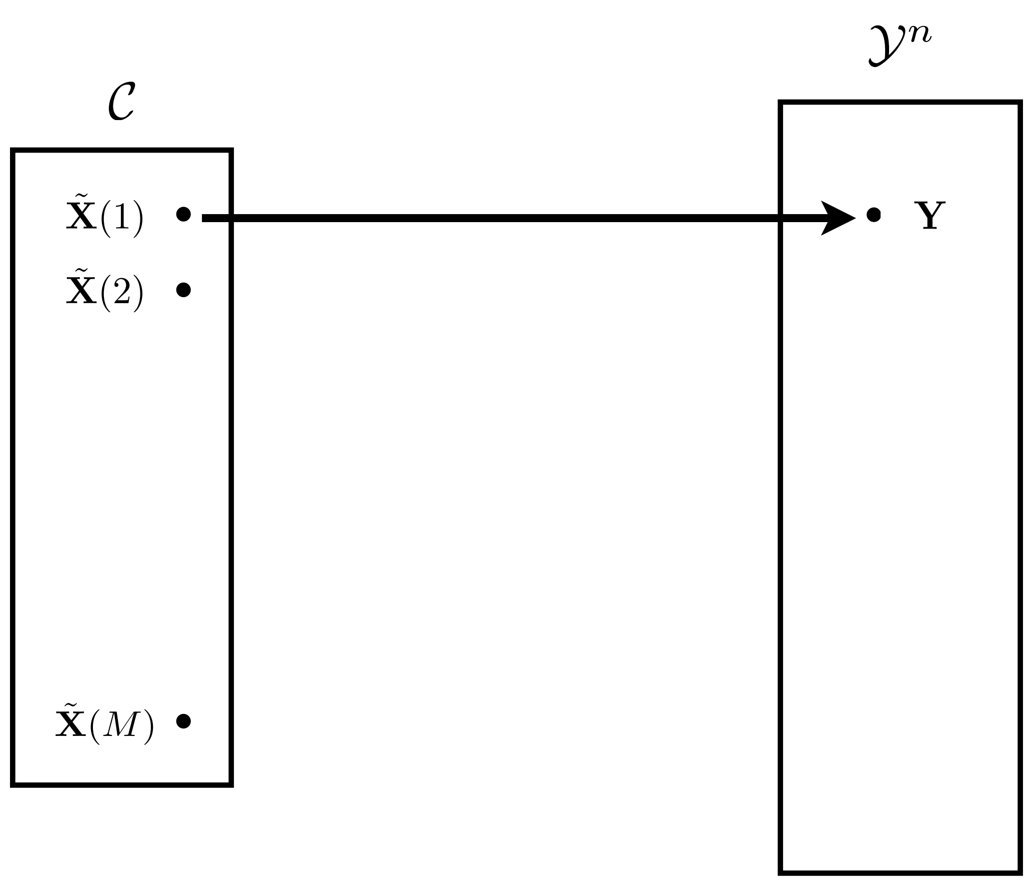

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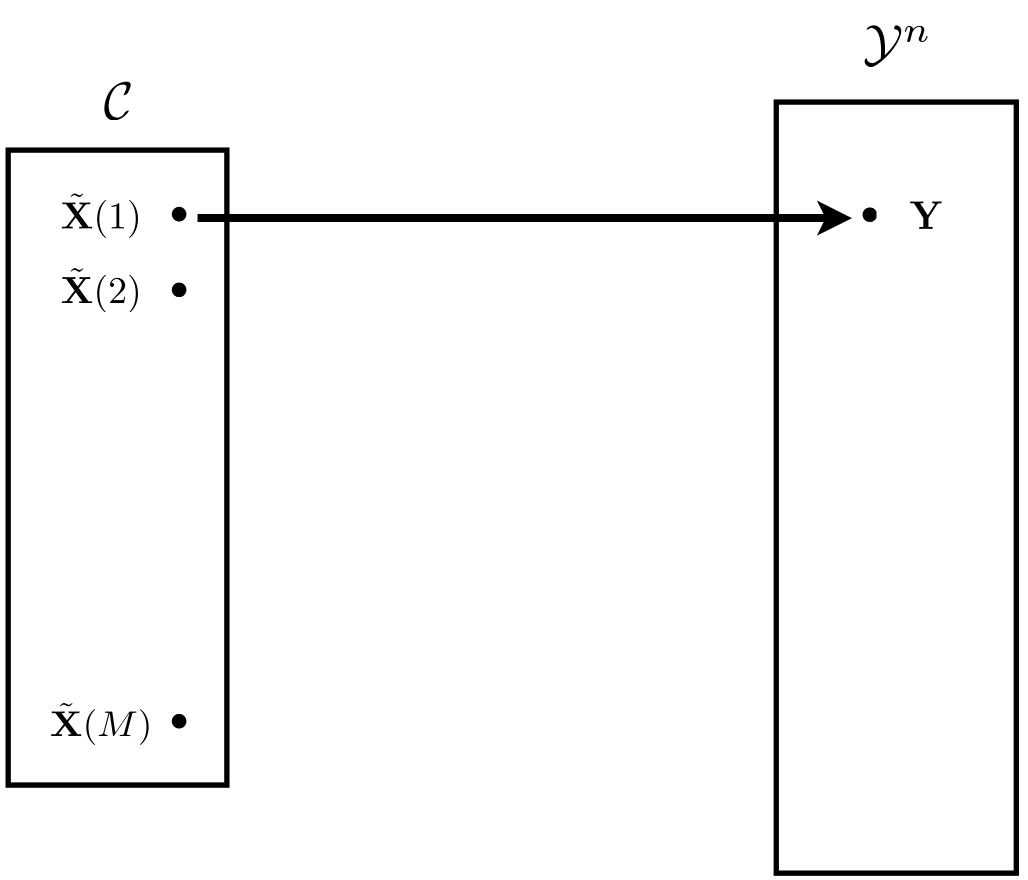

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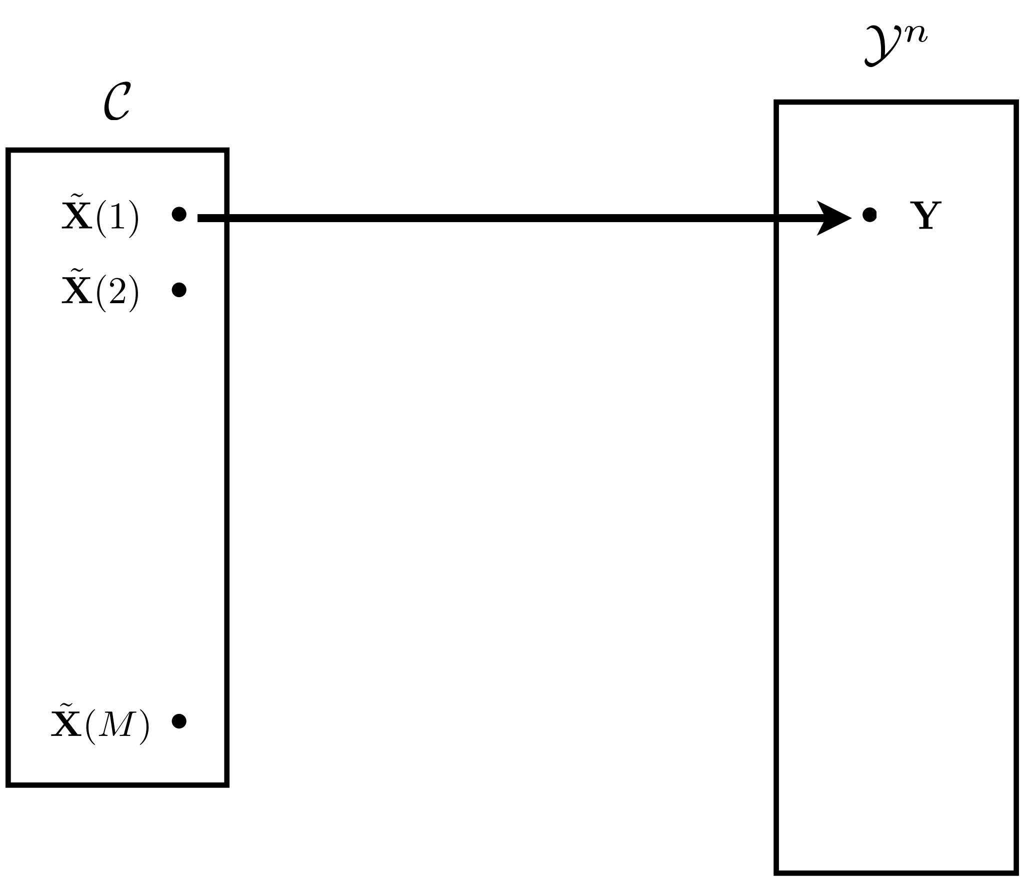

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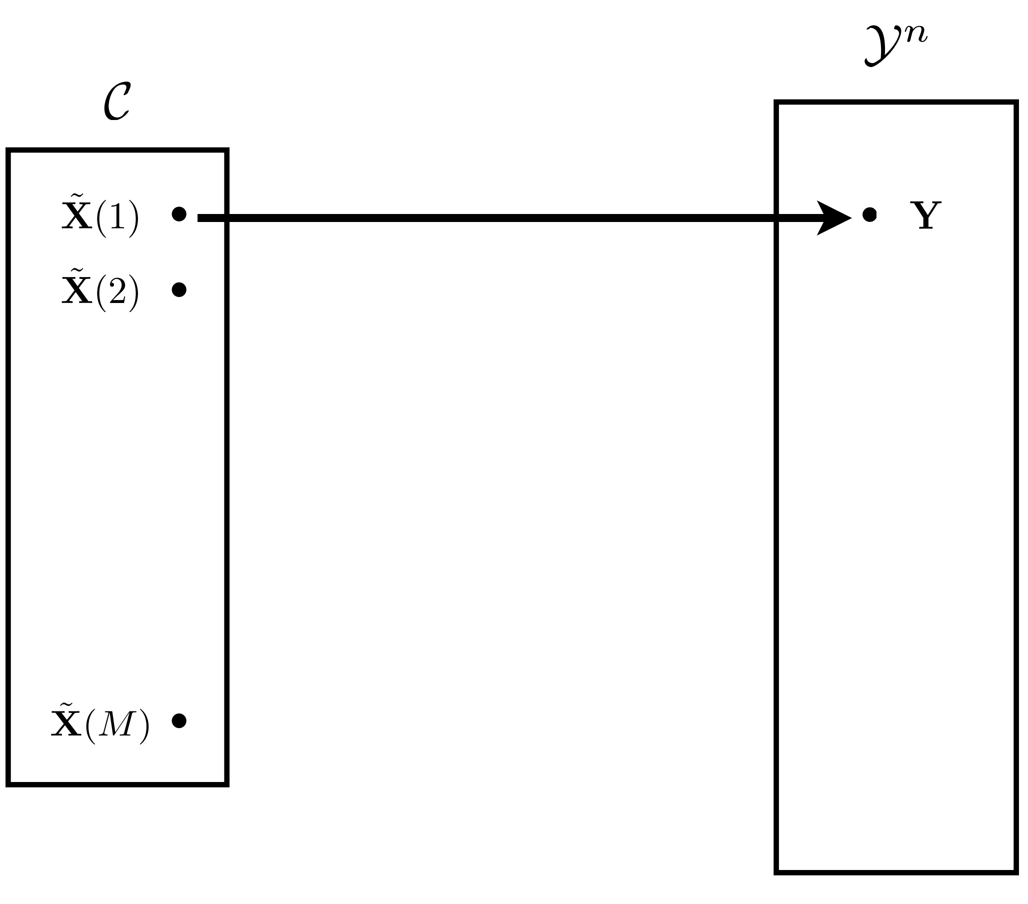

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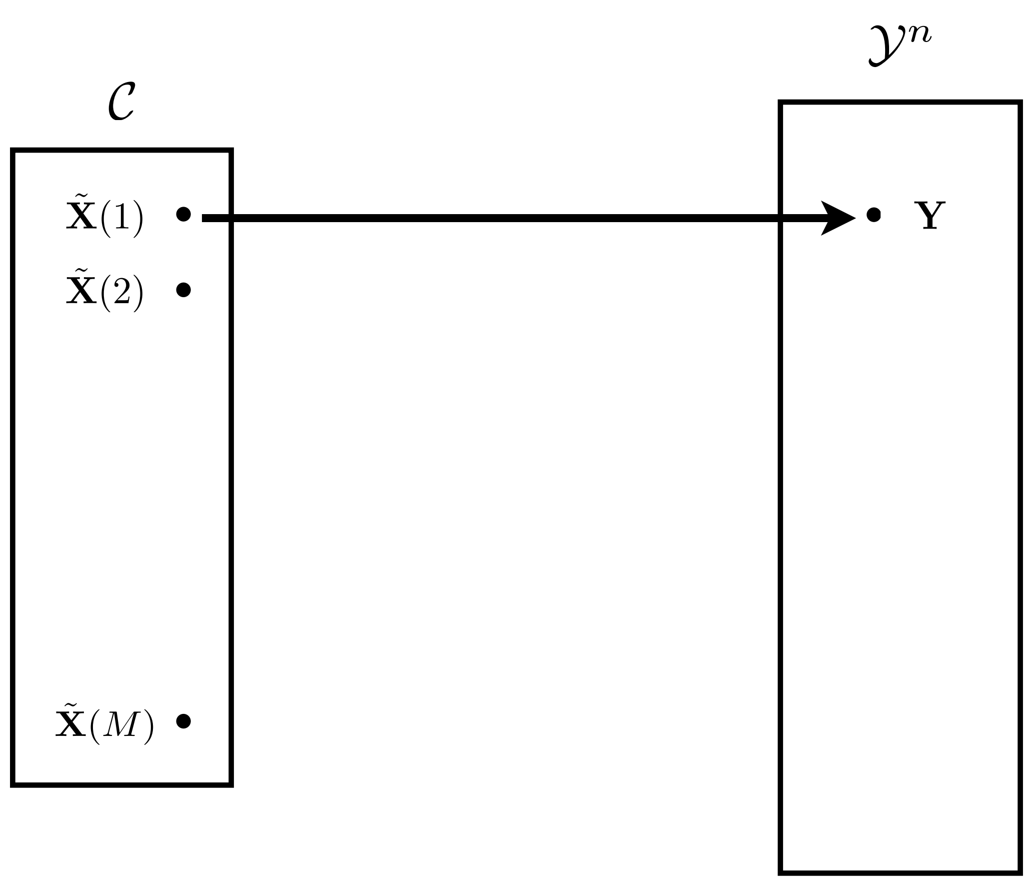

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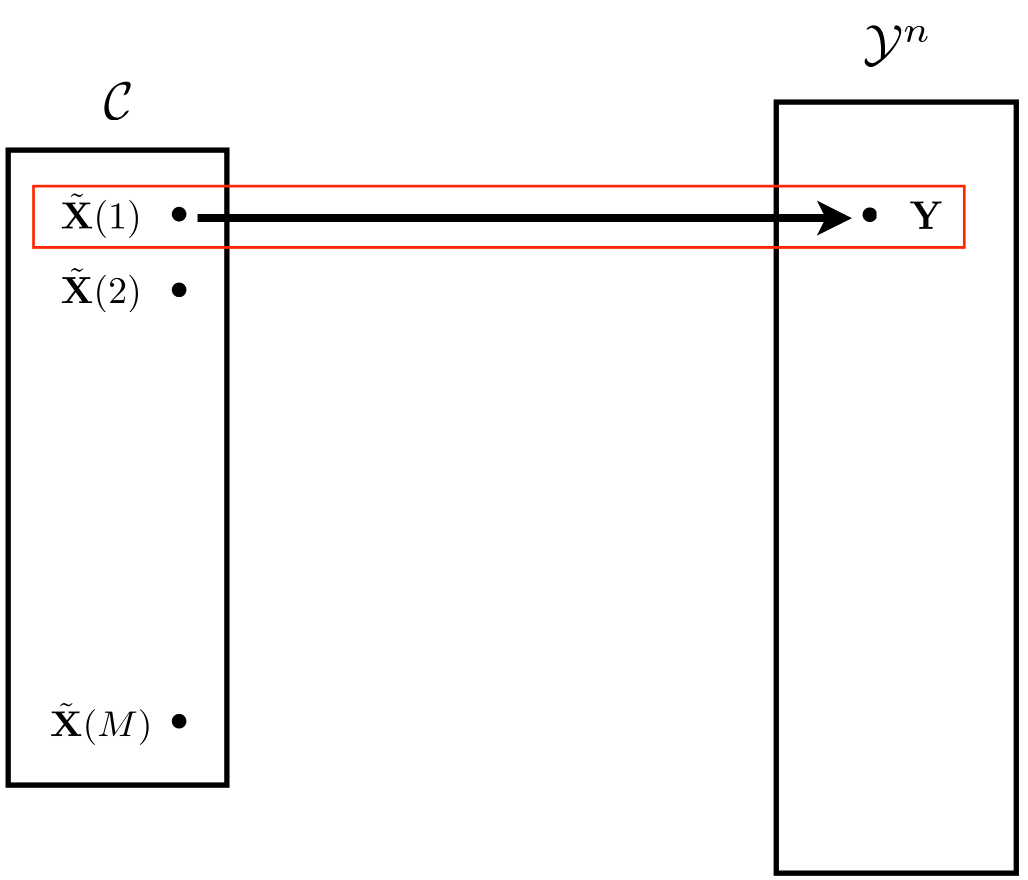

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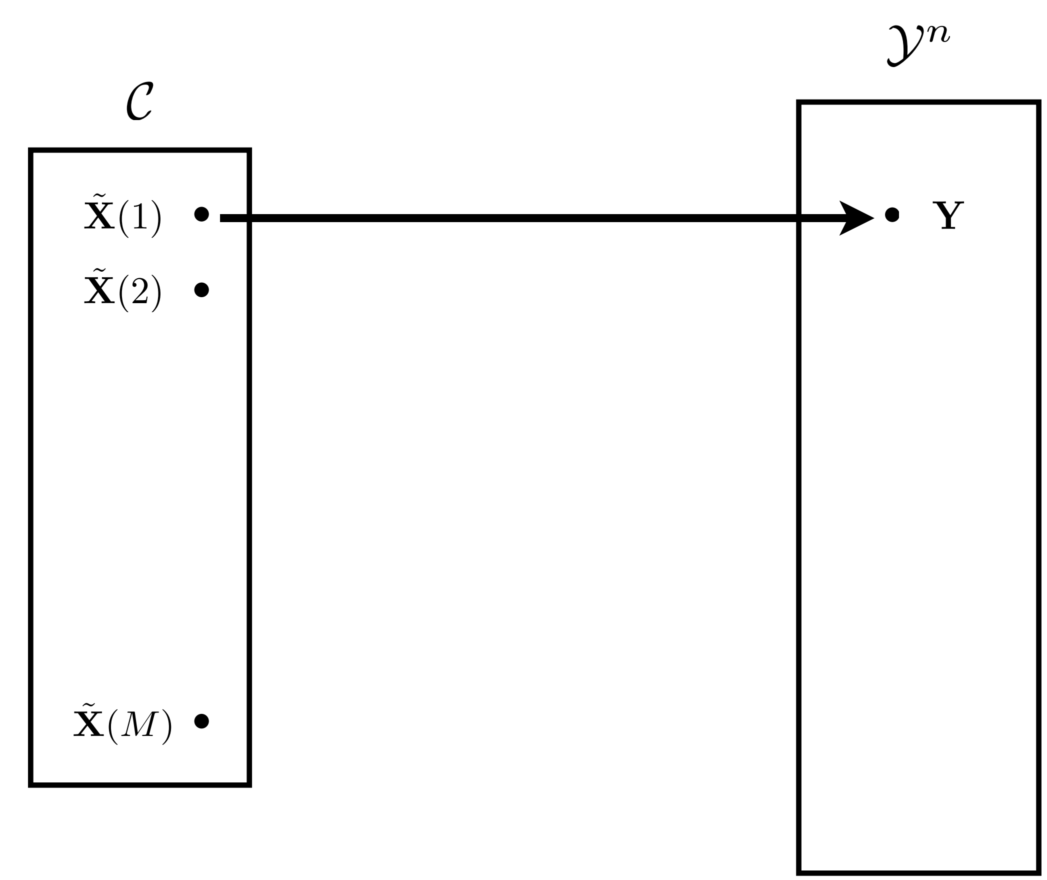

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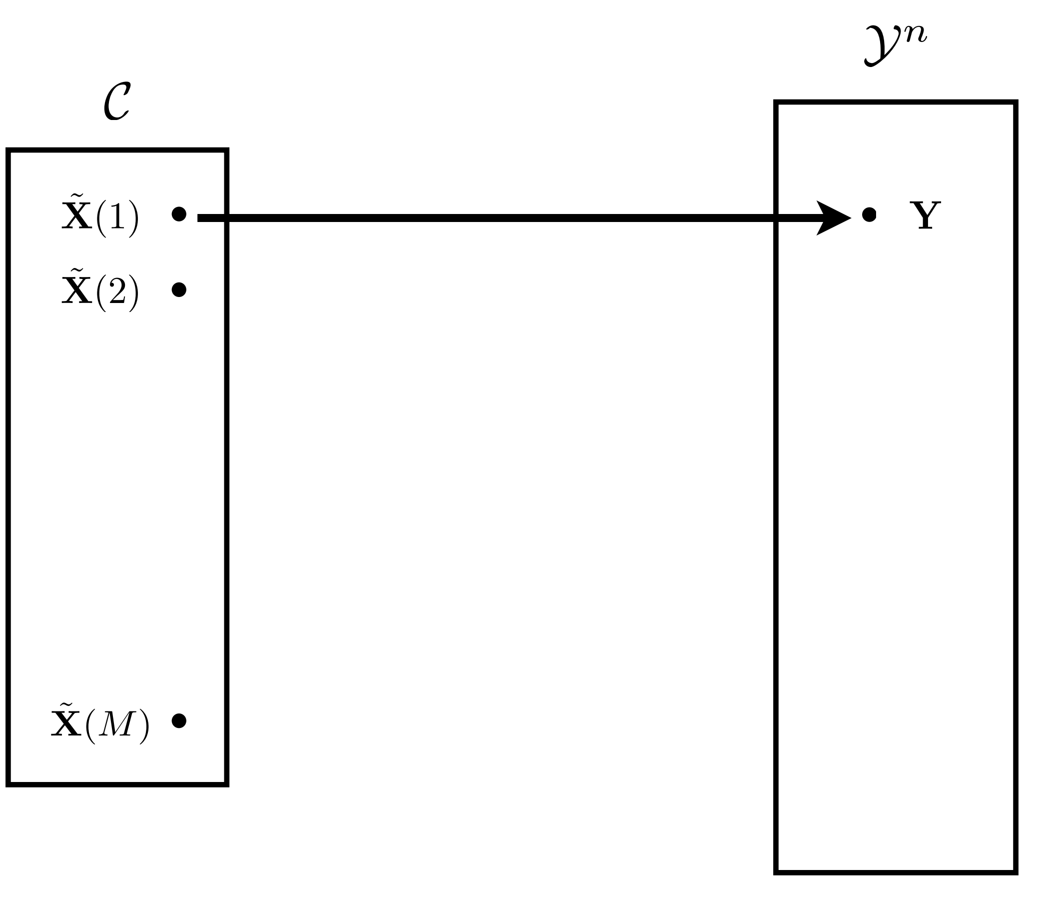

Performance Analysis

1. We need to show that $\Pr\{Err\} = \Pr\{\hat{W} \neq W\}$ can be made arbitrarily small.

2. Consider

$$\begin{aligned} \Pr\{Err\} &= \sum_{w=1}^{M} \Pr\{Err|W = w\}\Pr\{W = w\} \\ &= \Pr\{Err|W = 1\} \sum_{w=1}^{M} \Pr\{W = w\} \\ &= \Pr\{Err|W = 1\}. \end{aligned}$$

Assume without loss of generality that the message 1 is chosen.

3. For $1 \leq w \leq M$, define the event

$$E_w = \{(\tilde{\mathbf{X}}(w), \mathbf{Y}) \in T^n_{[XY]\delta}\}.$$

4. If E_1 occurs but E_w does not occur for all $2 \leq w \leq M$, then no decoding error. Therefore,

$$\Pr\{Err^c|W = 1\} \geq \Pr\{E_1 \cap E_2^c \cap E_3^c \cap \cdots \cap E_M^c|W = 1\}.$$

5. Consider

$$\begin{aligned} &\Pr\{Err|W = 1\} \\ &= 1 - \Pr\{Err^c|W = 1\} \\ &\leq 1 - \Pr\{E_1 \cap E_2^c \cap E_3^c \cap \cdots \cap E_M^c|W = 1\} \\ &= \Pr\{(E_1 \cap E_2^c \cap E_3^c \cap \cdots \cap E_M^c)^c|W = 1\} \\ &= \Pr\{E_1^c \cup E_2 \cup E_3 \cup \cdots \cup E_M|W = 1\}. \end{aligned}$$

6. By the union bound,

$$\Pr\{Err|W = 1\} \leq \Pr\{E_1^c|W = 1\} + \sum_{w=2}^{M} \Pr\{E_w|W = 1\}.$$

7. By strong JAEP,

$$\Pr\{E_1^c|W = 1\} = \Pr\{(\tilde{\mathbf{X}}(1), \mathbf{Y}) \notin T^n_{[XY]\delta}|W = 1\} < \nu.$$

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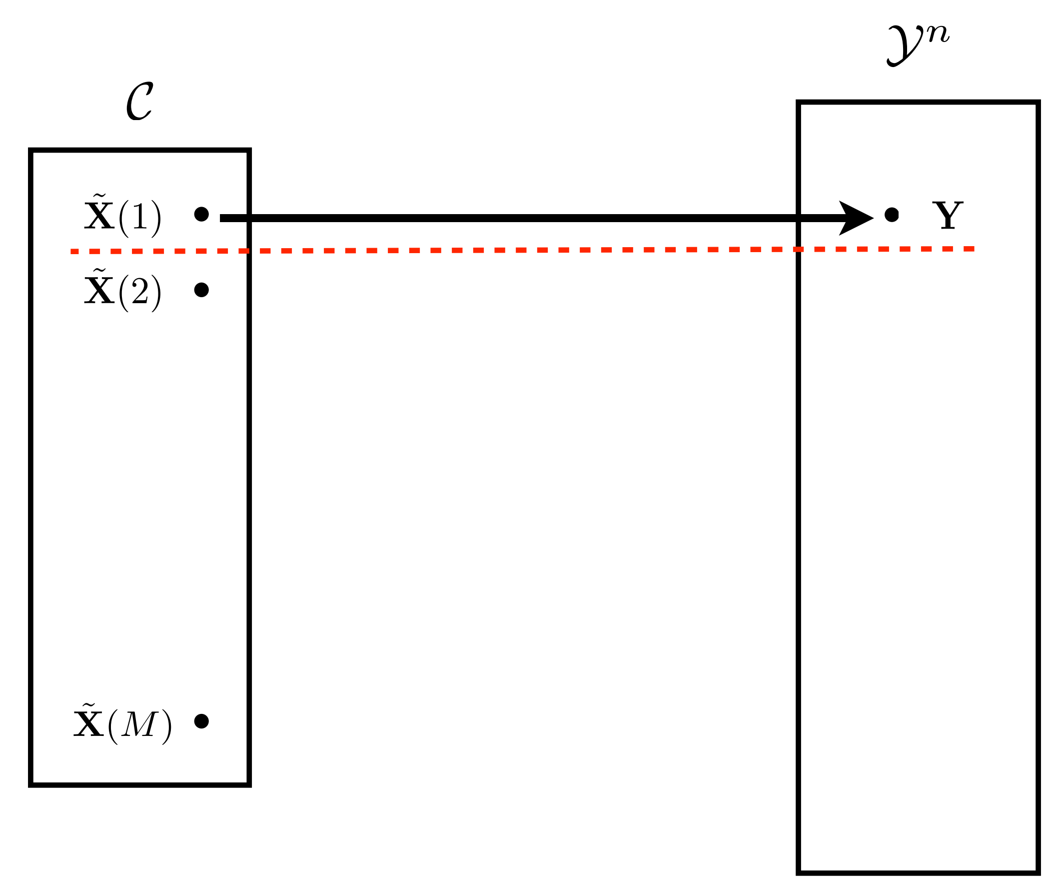

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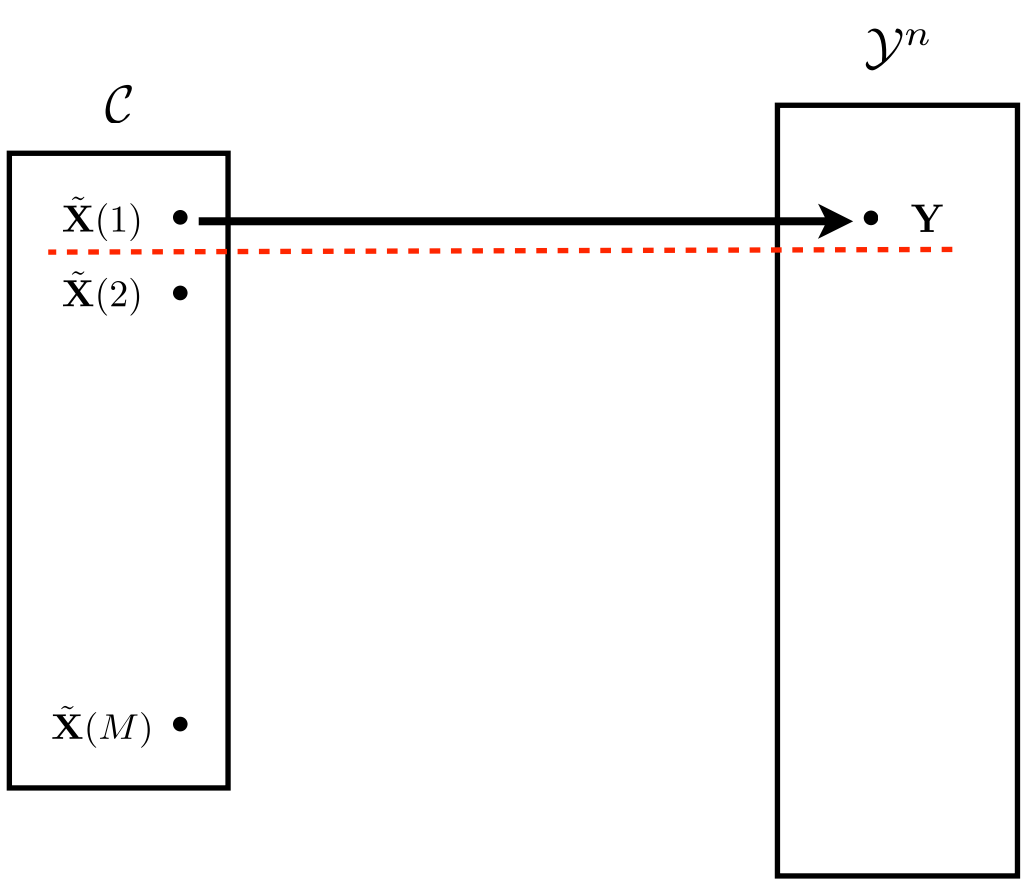

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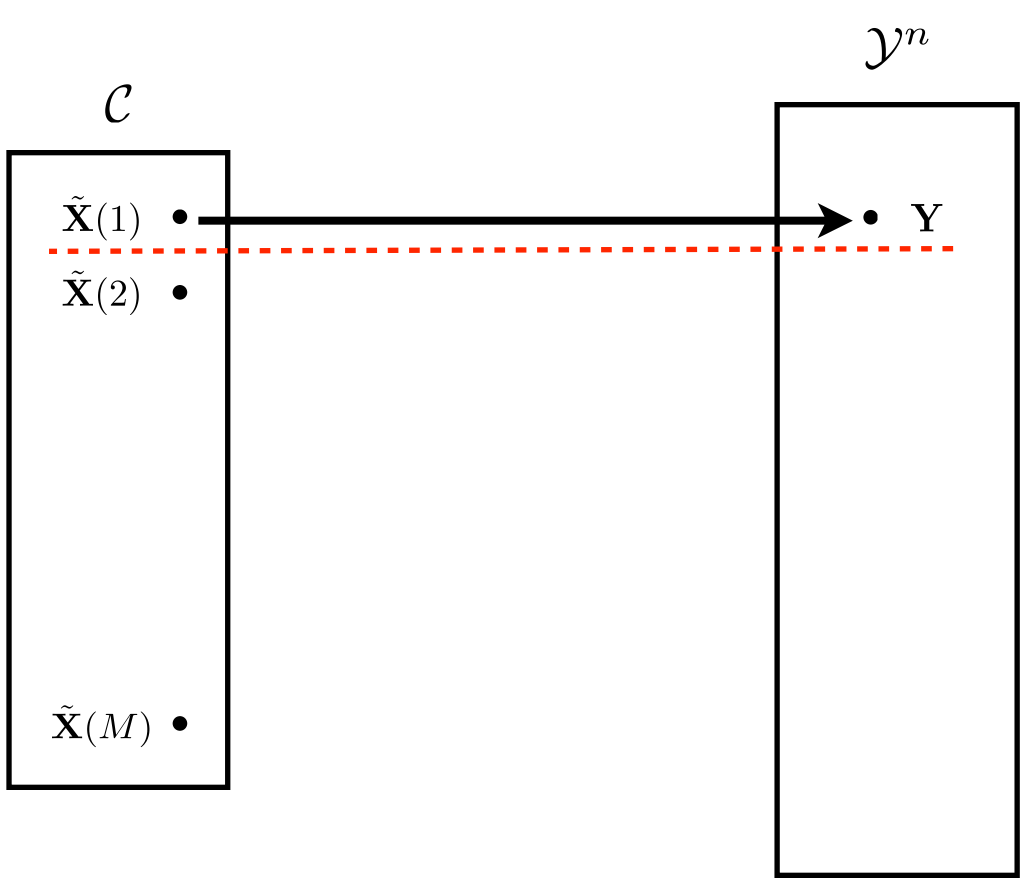

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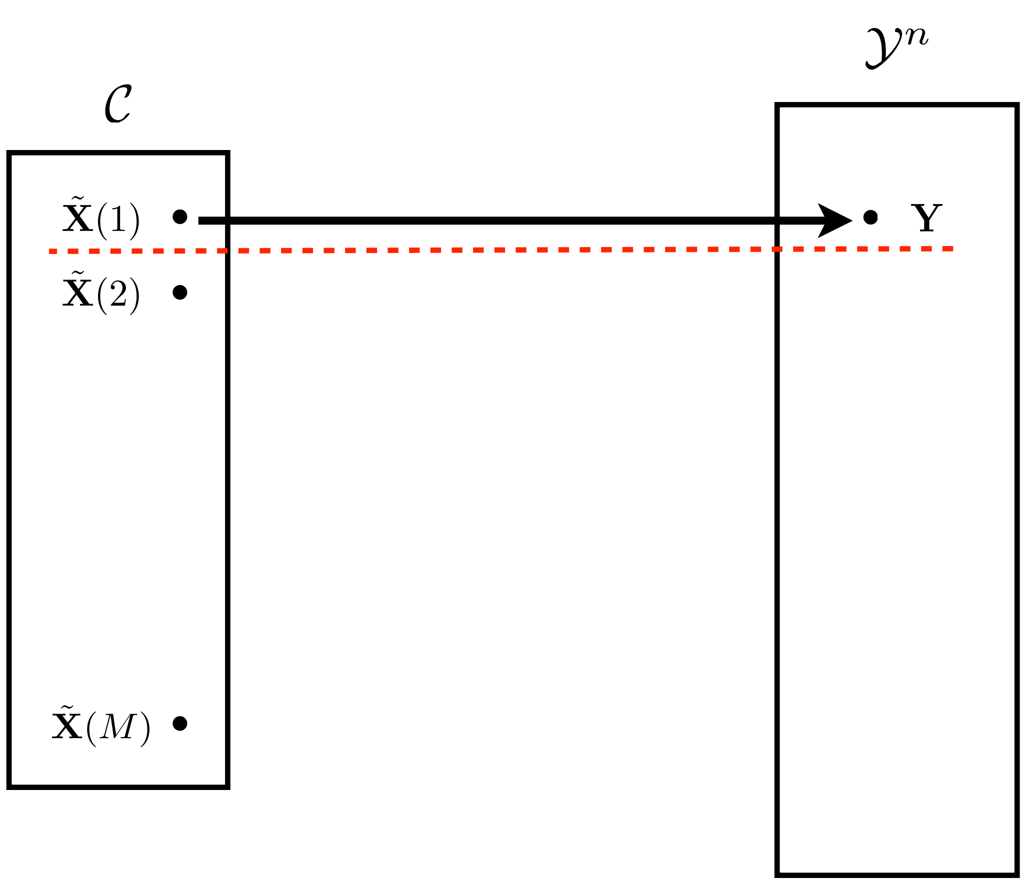

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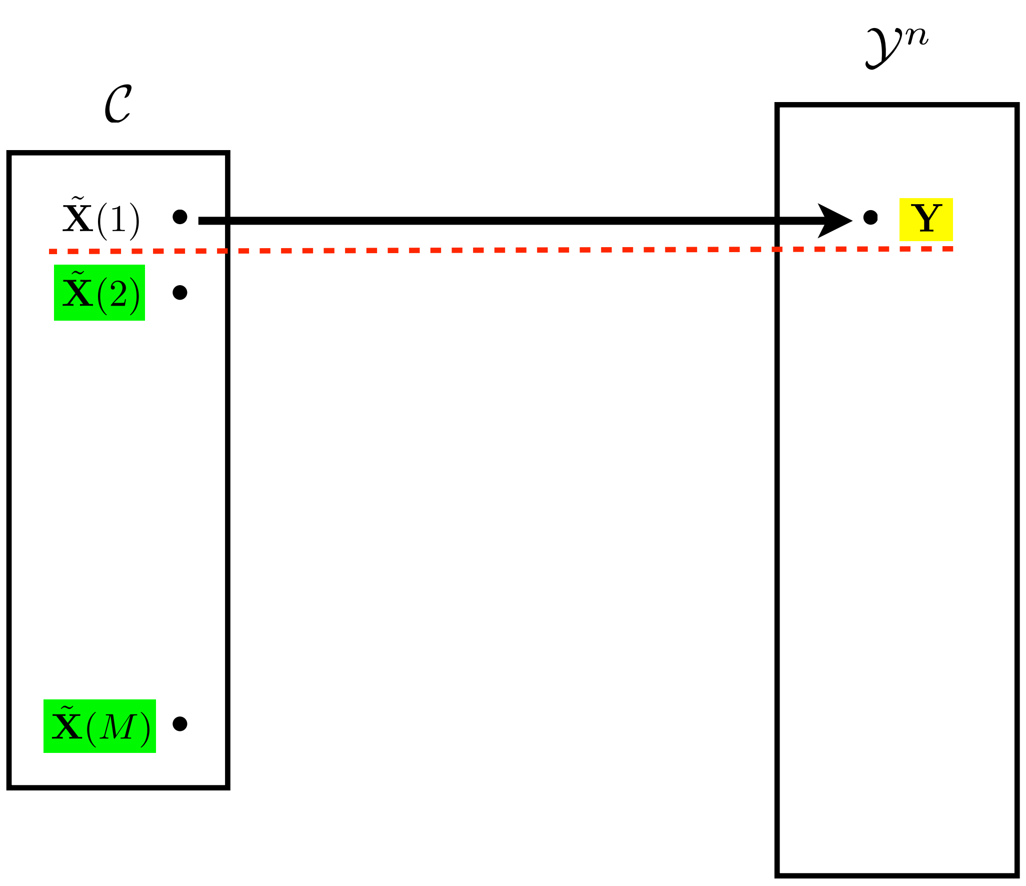

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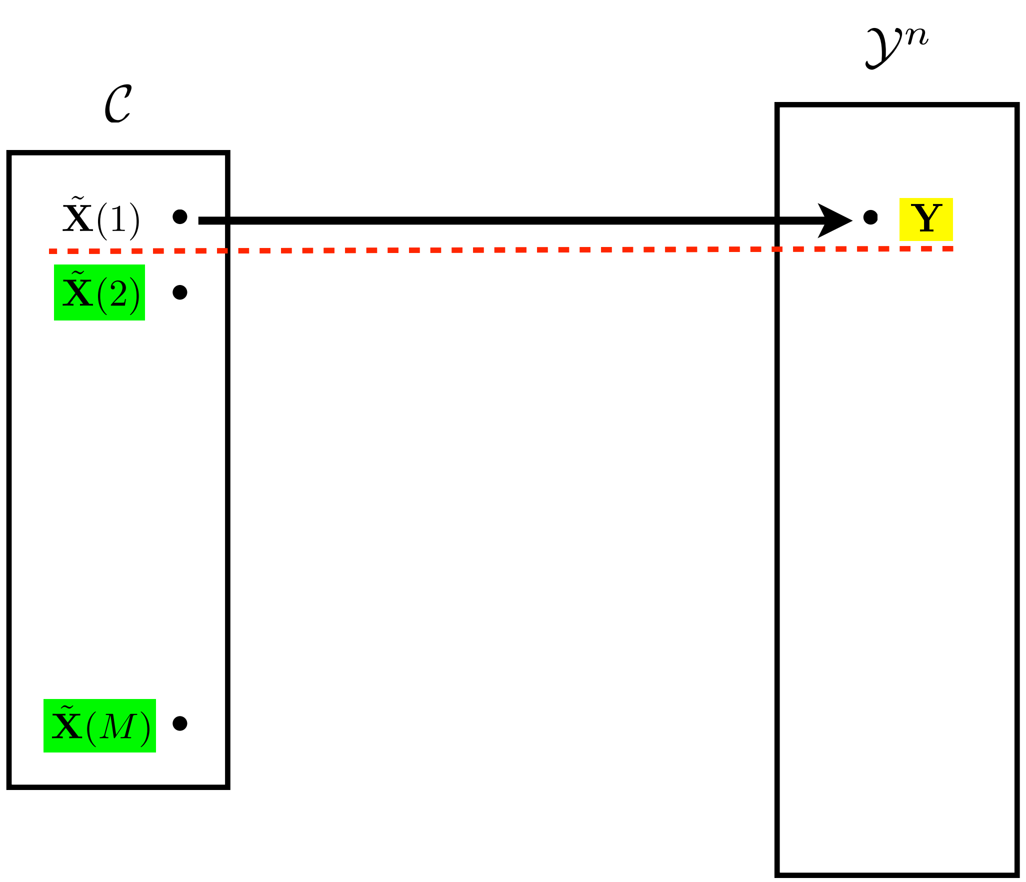

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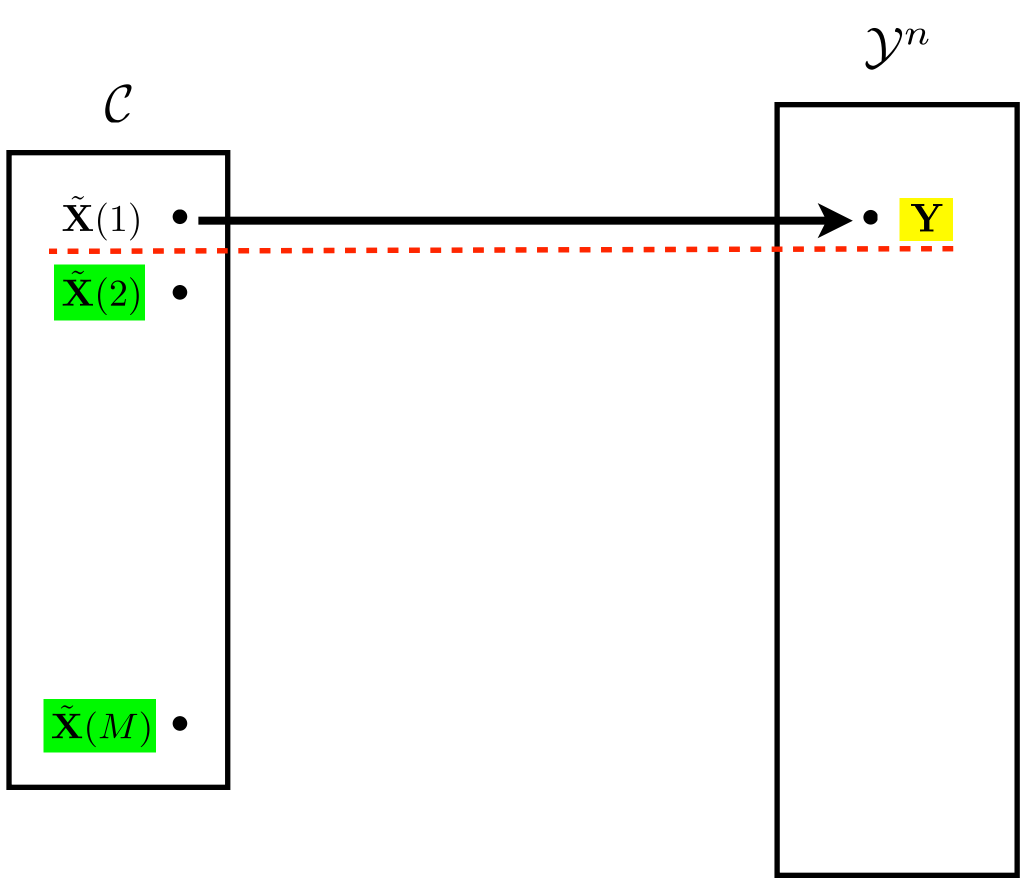

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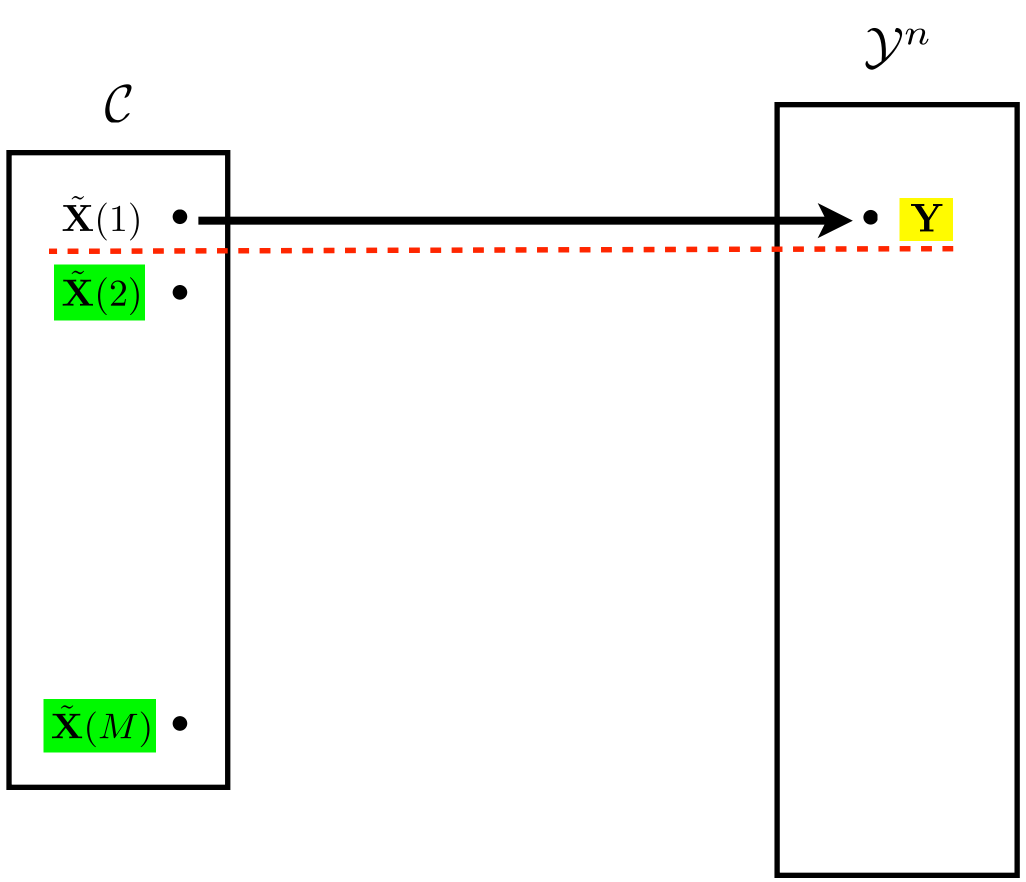

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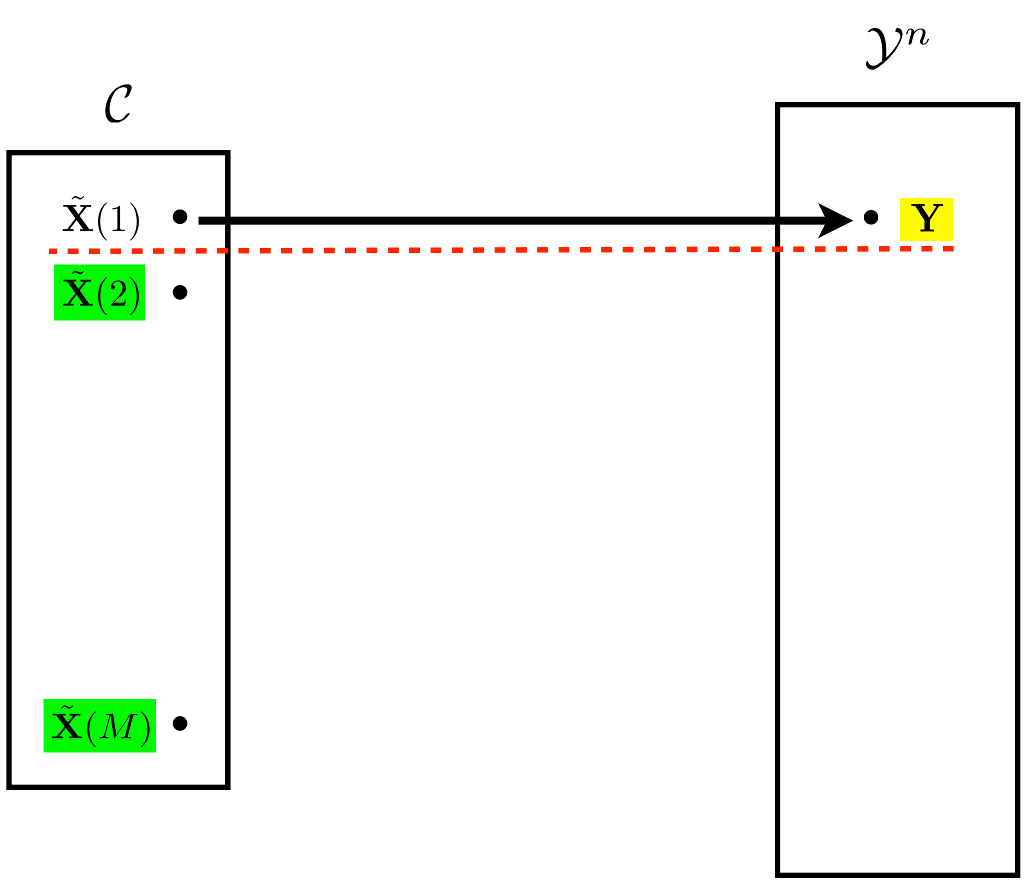

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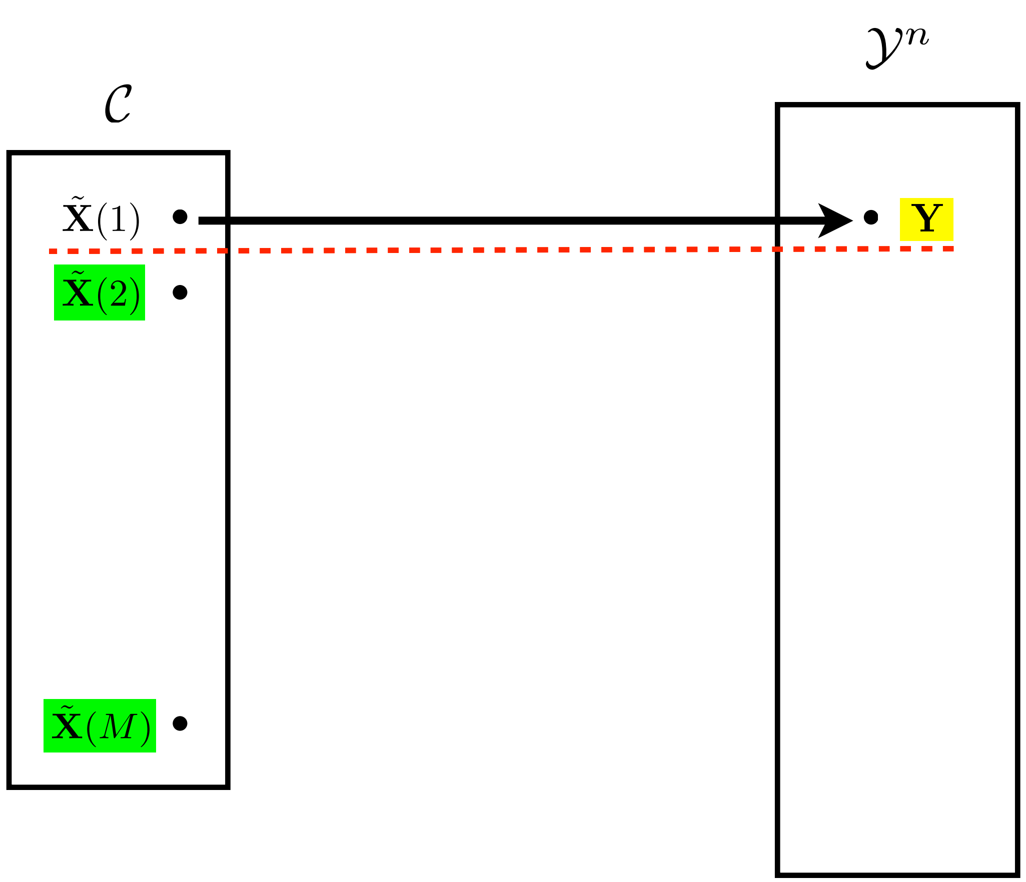

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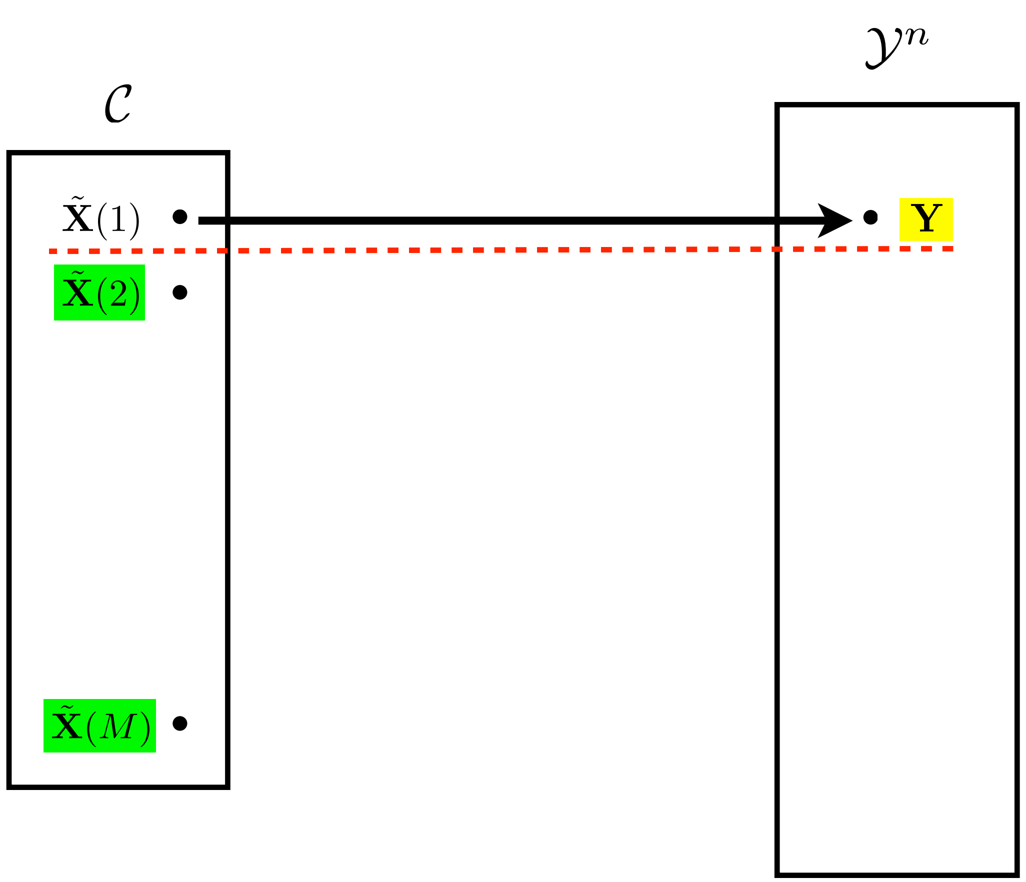

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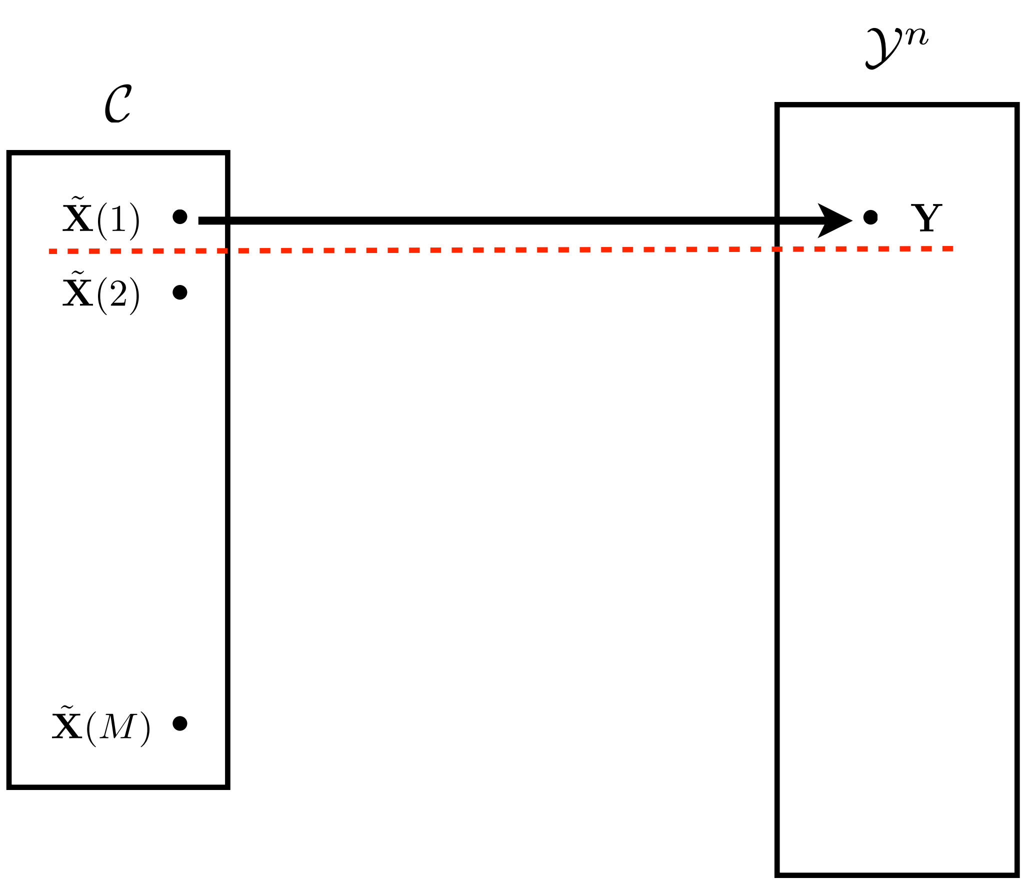

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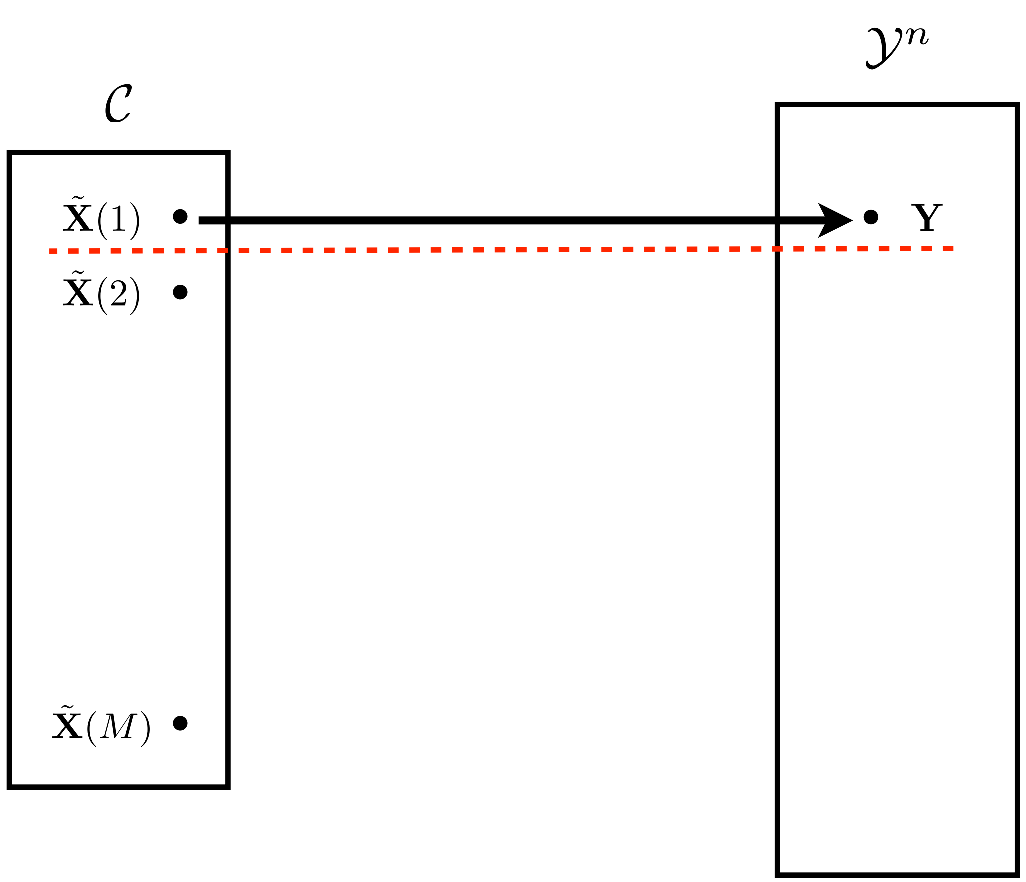

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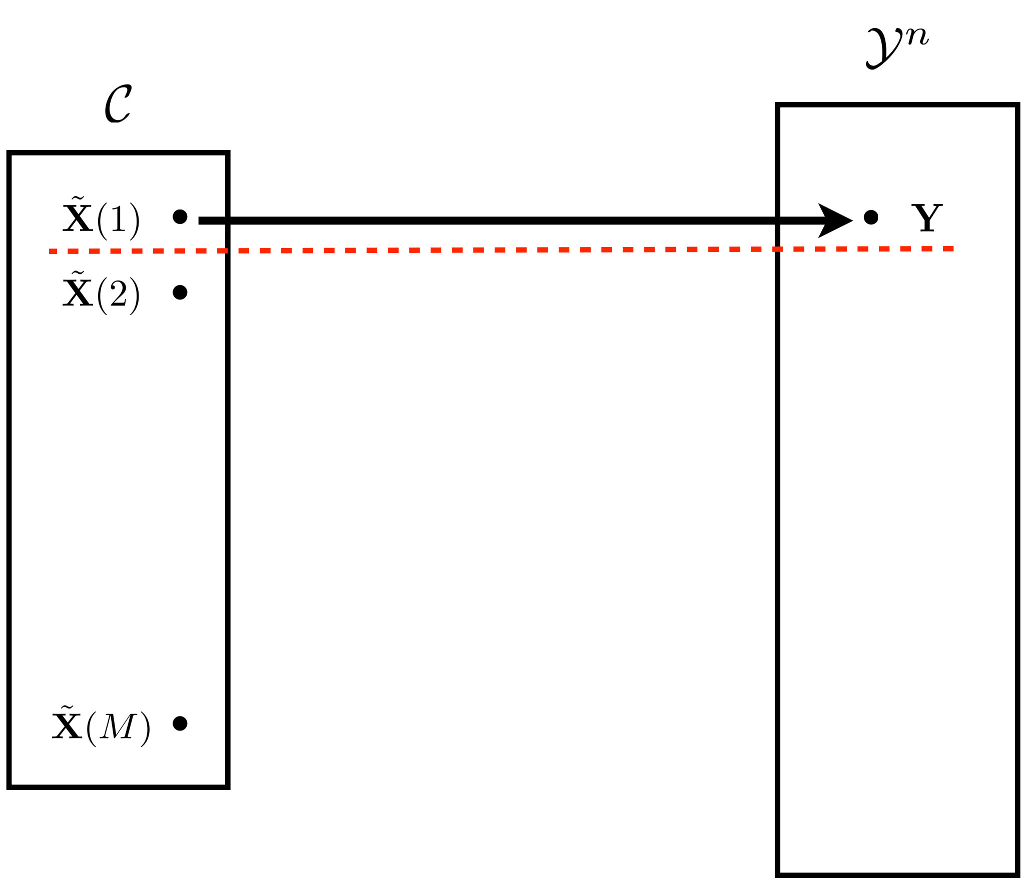

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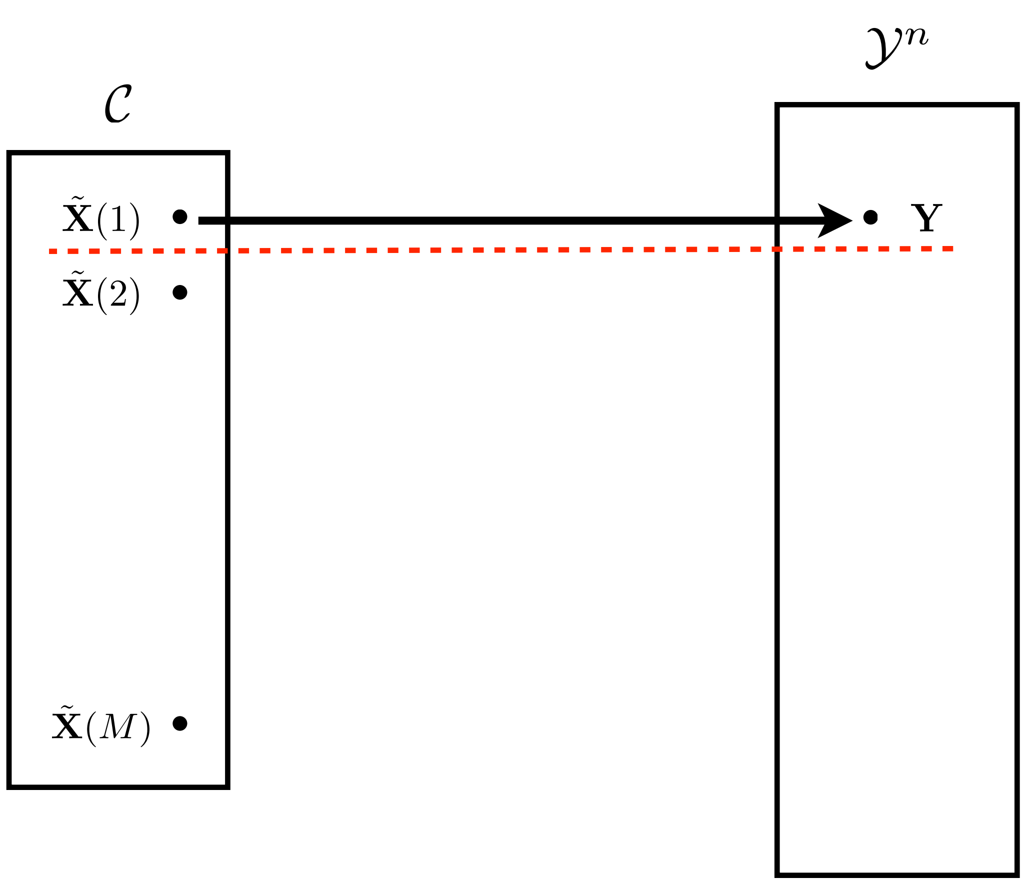

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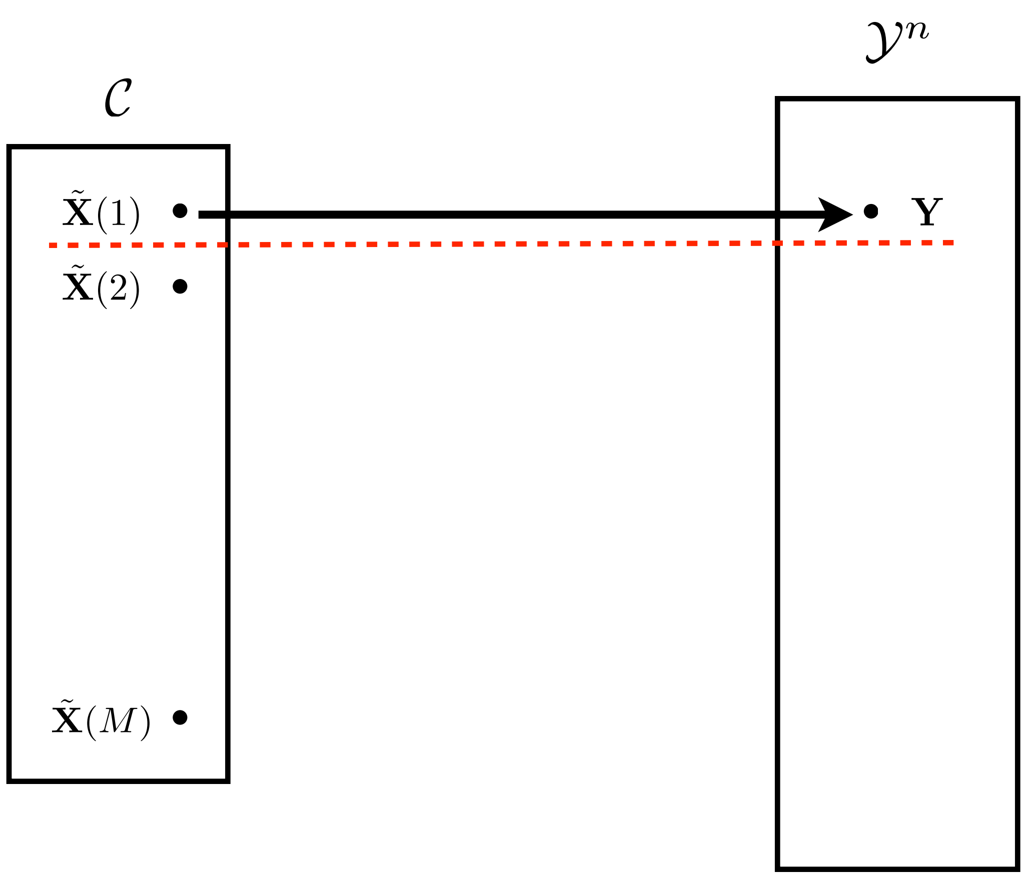

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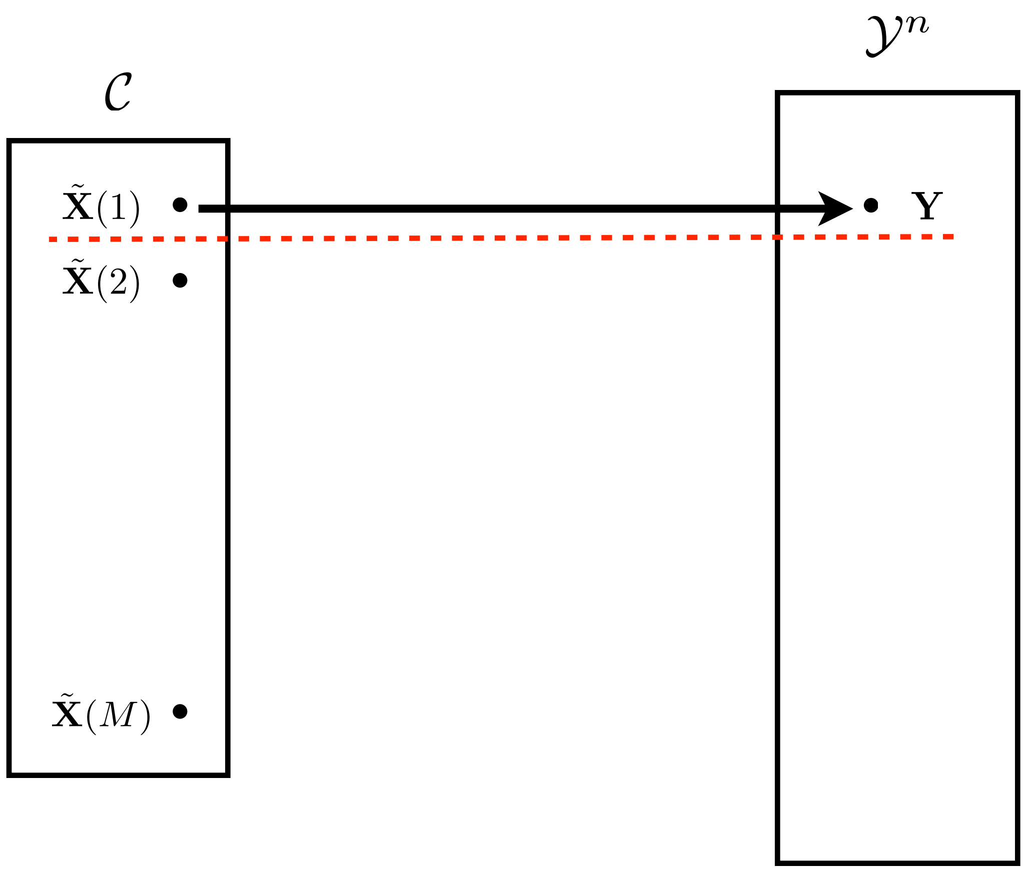

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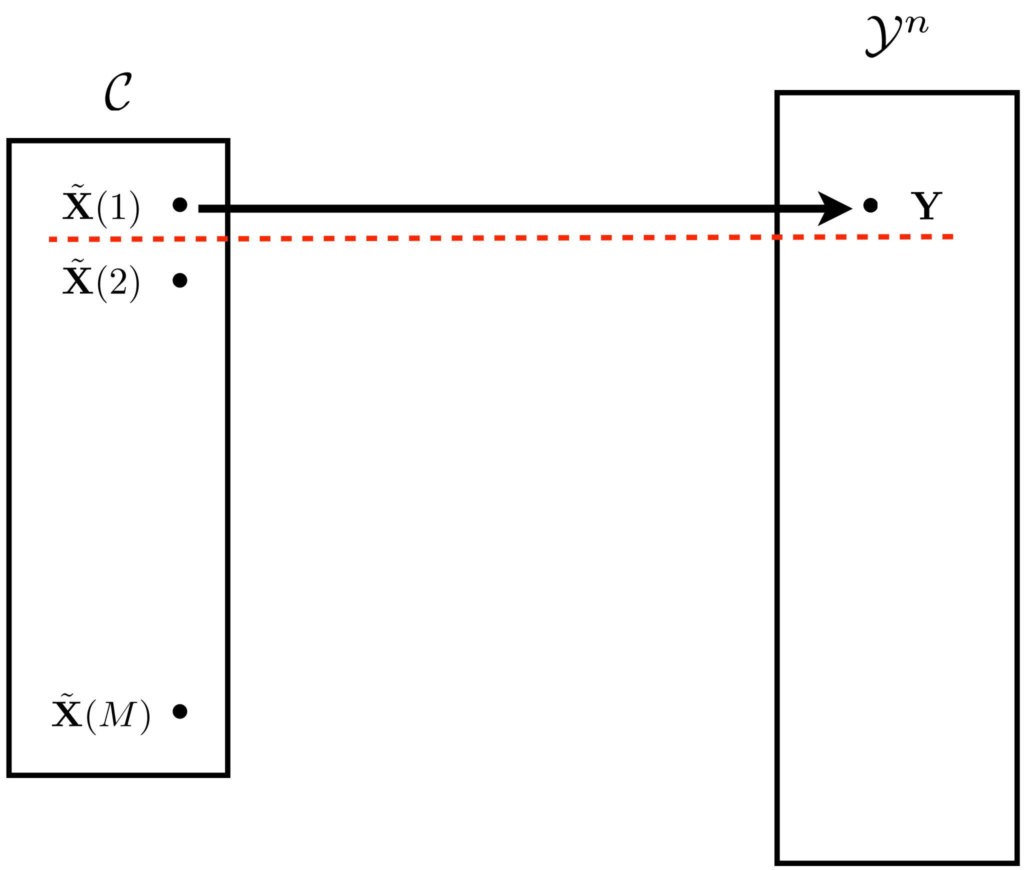

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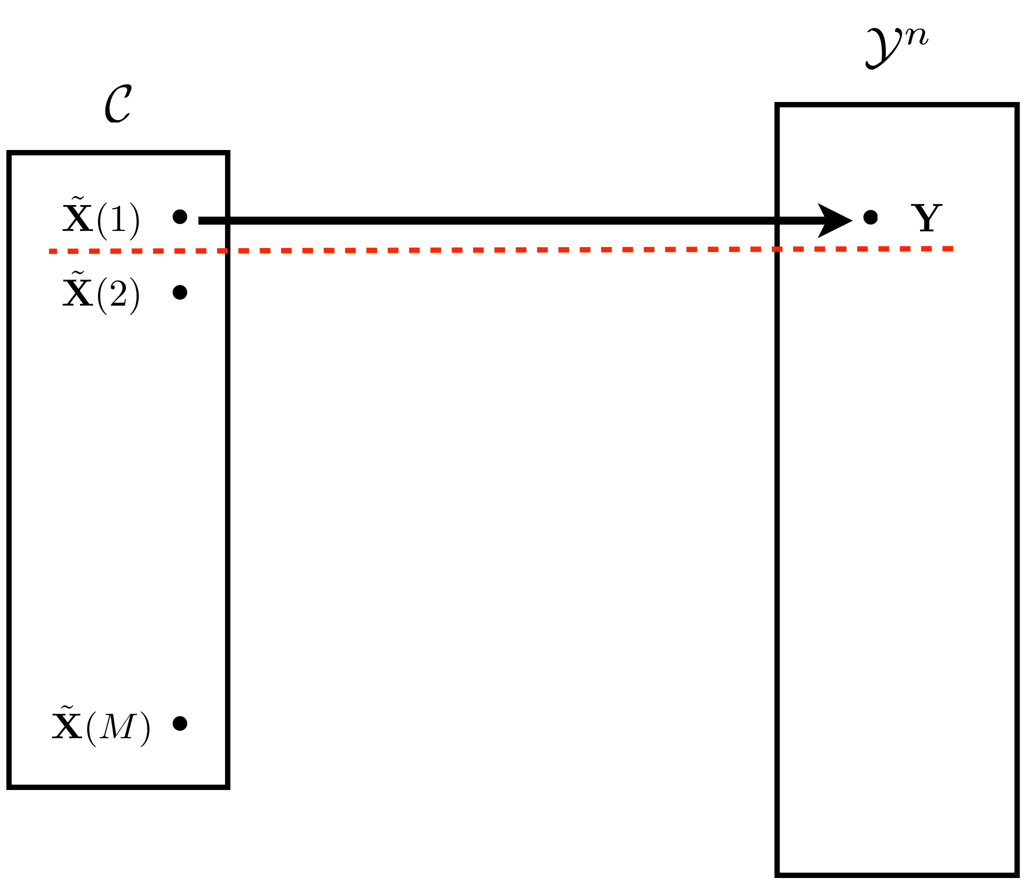

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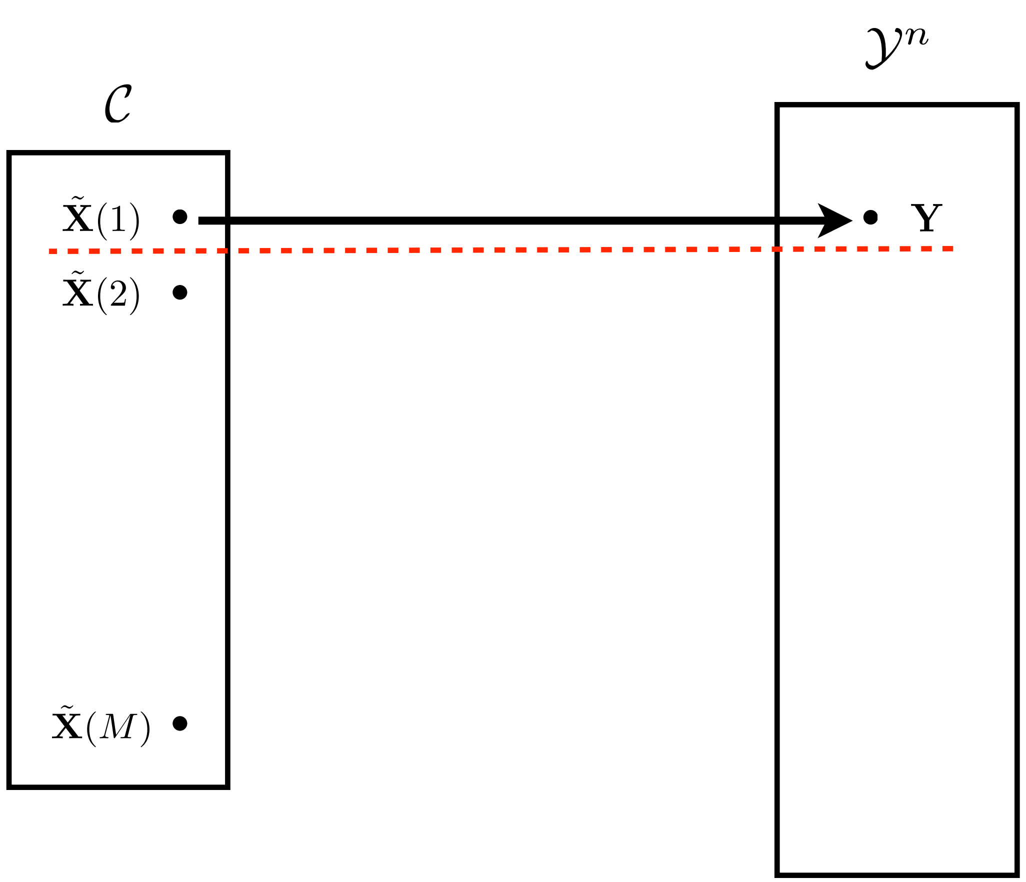

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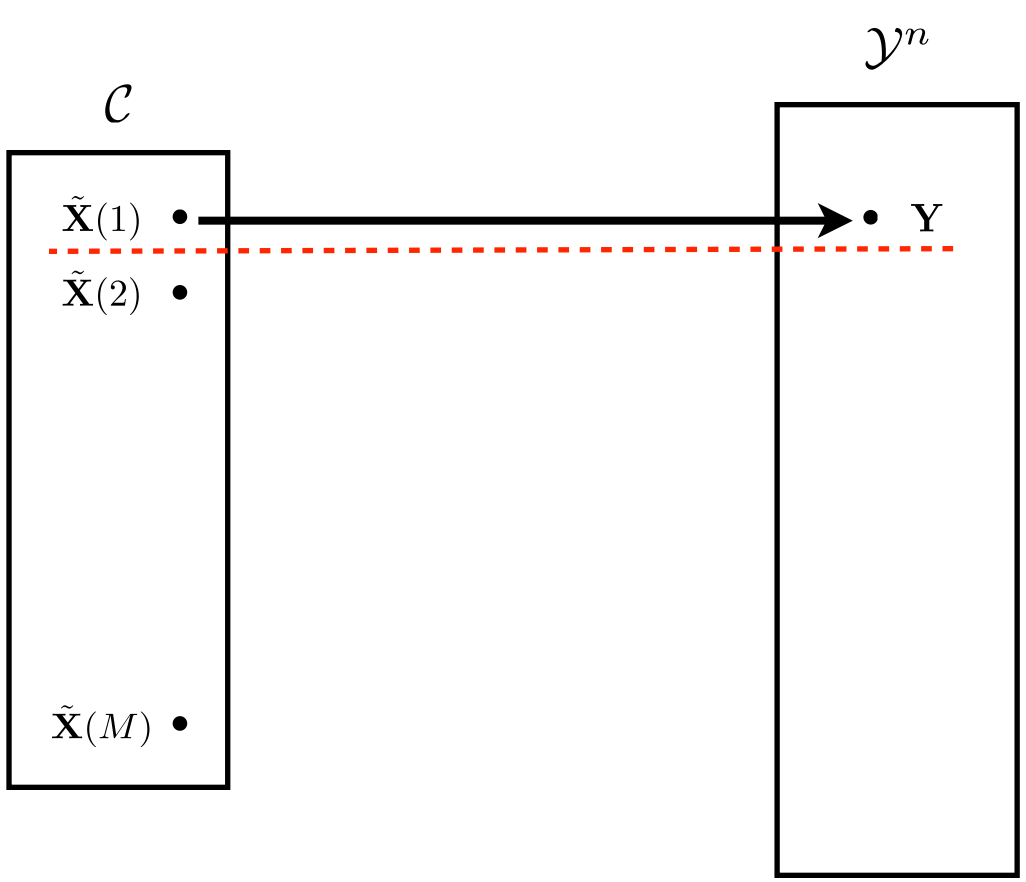

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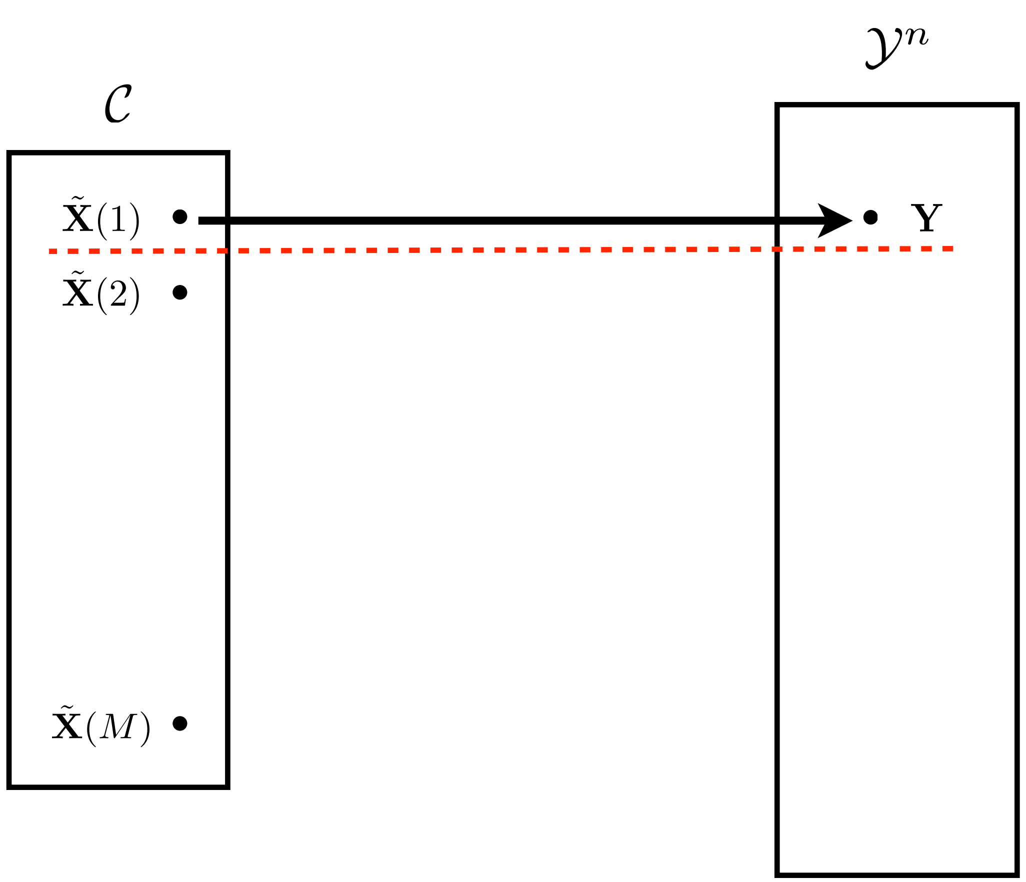

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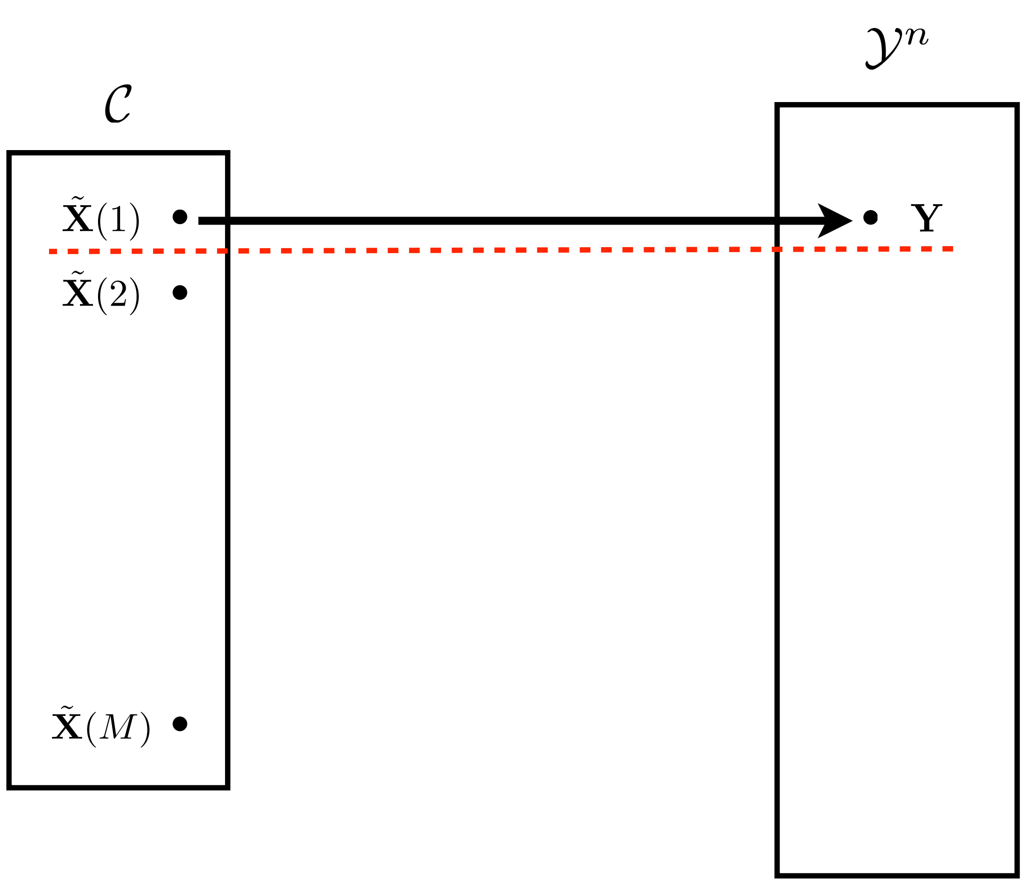

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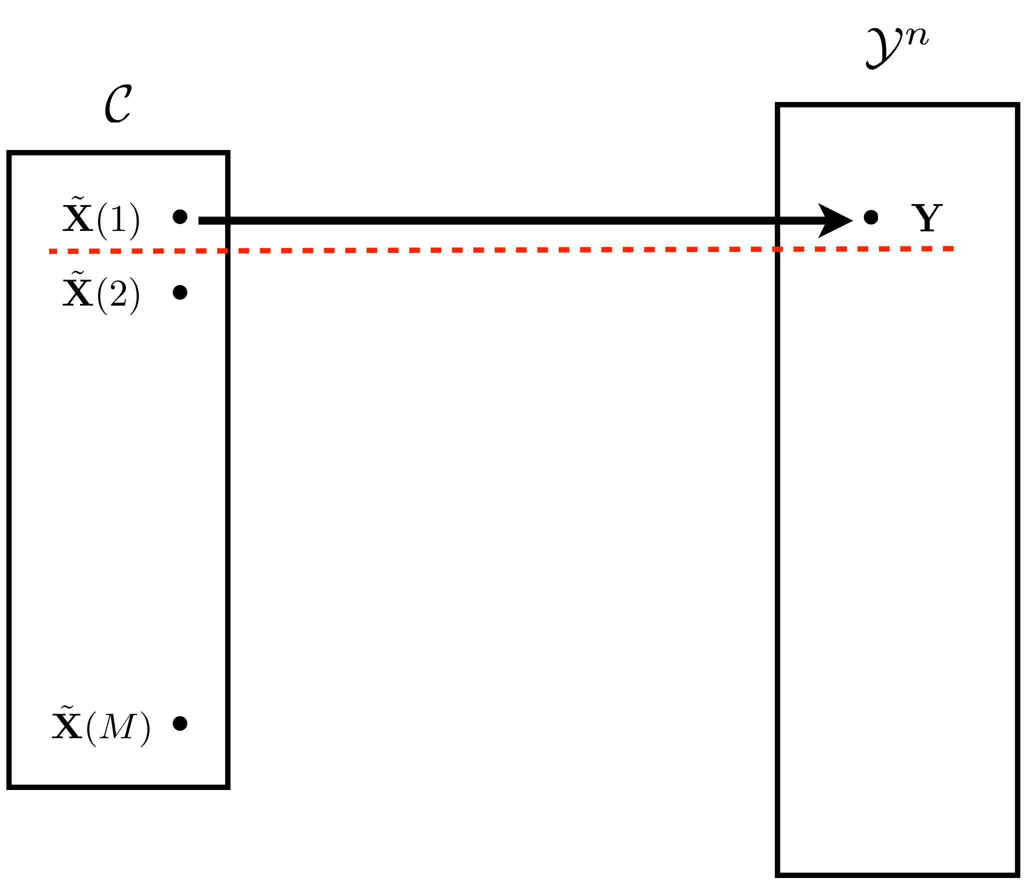

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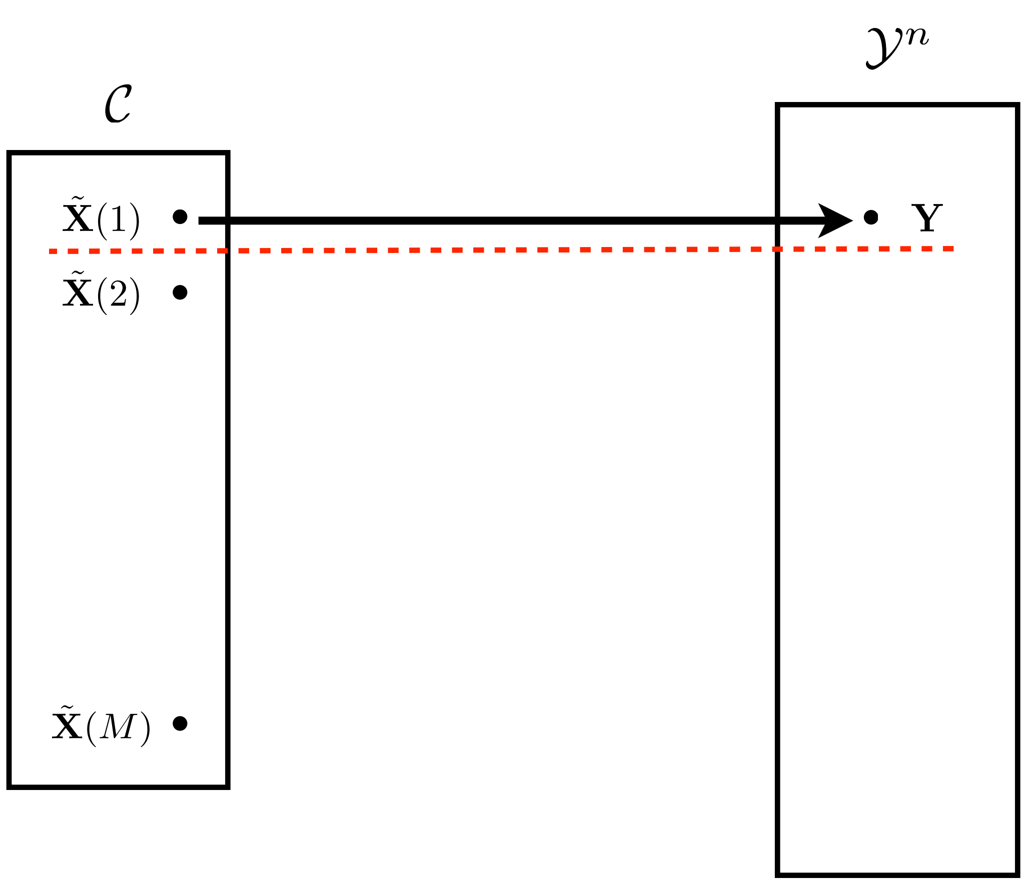

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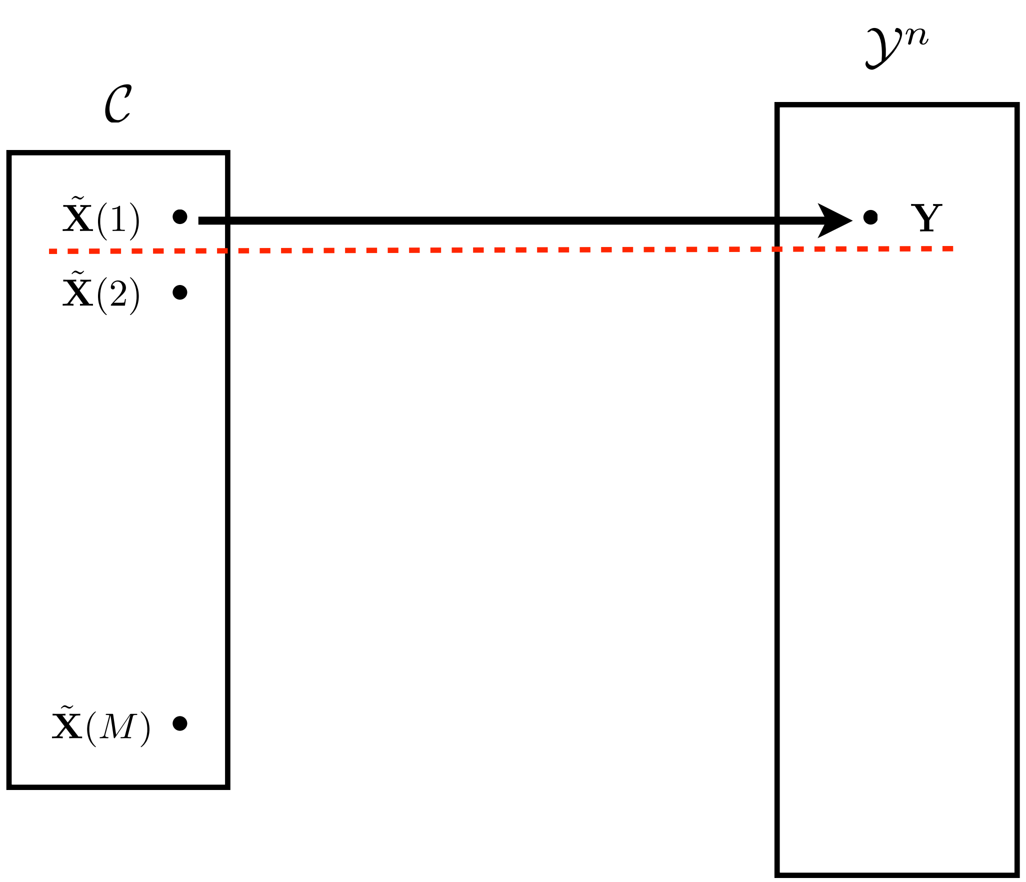

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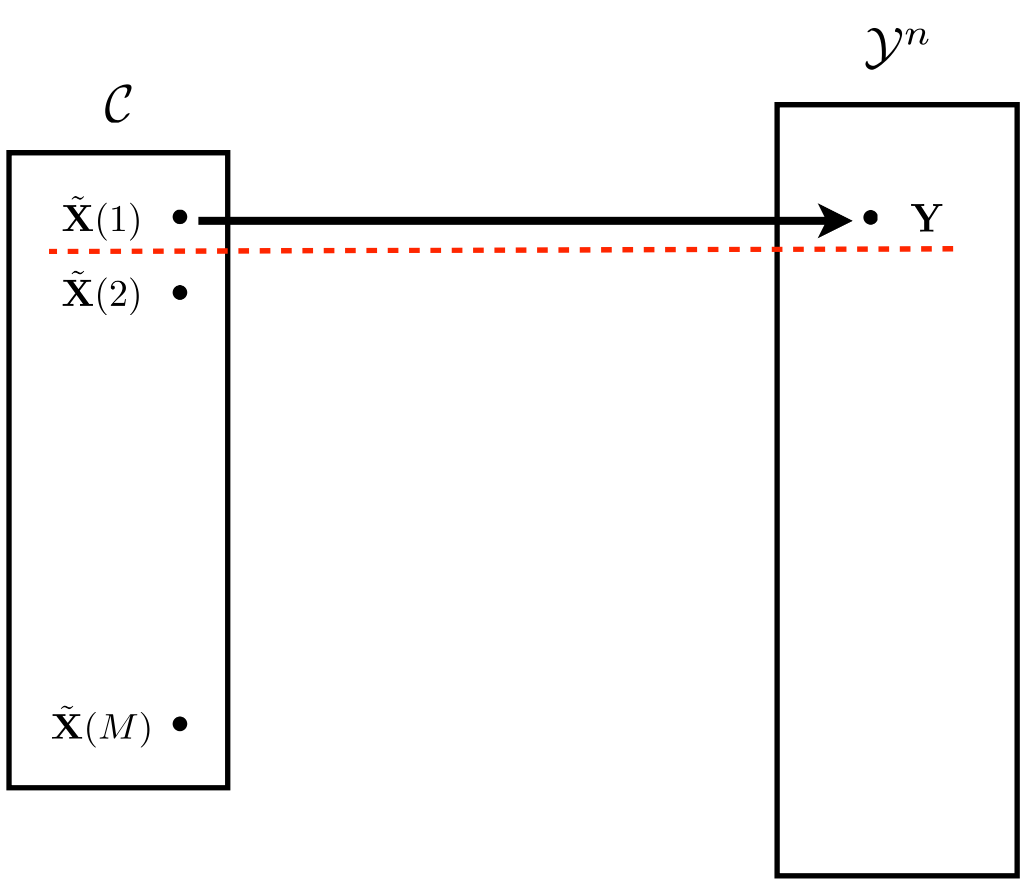

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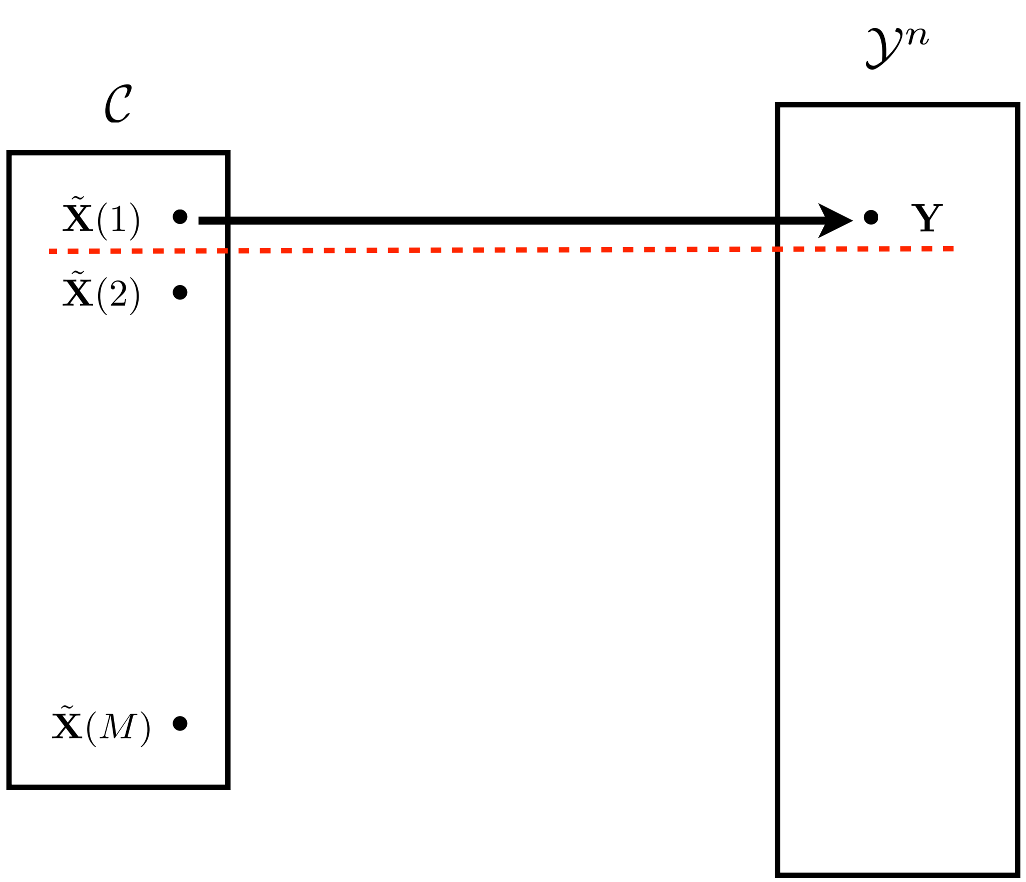

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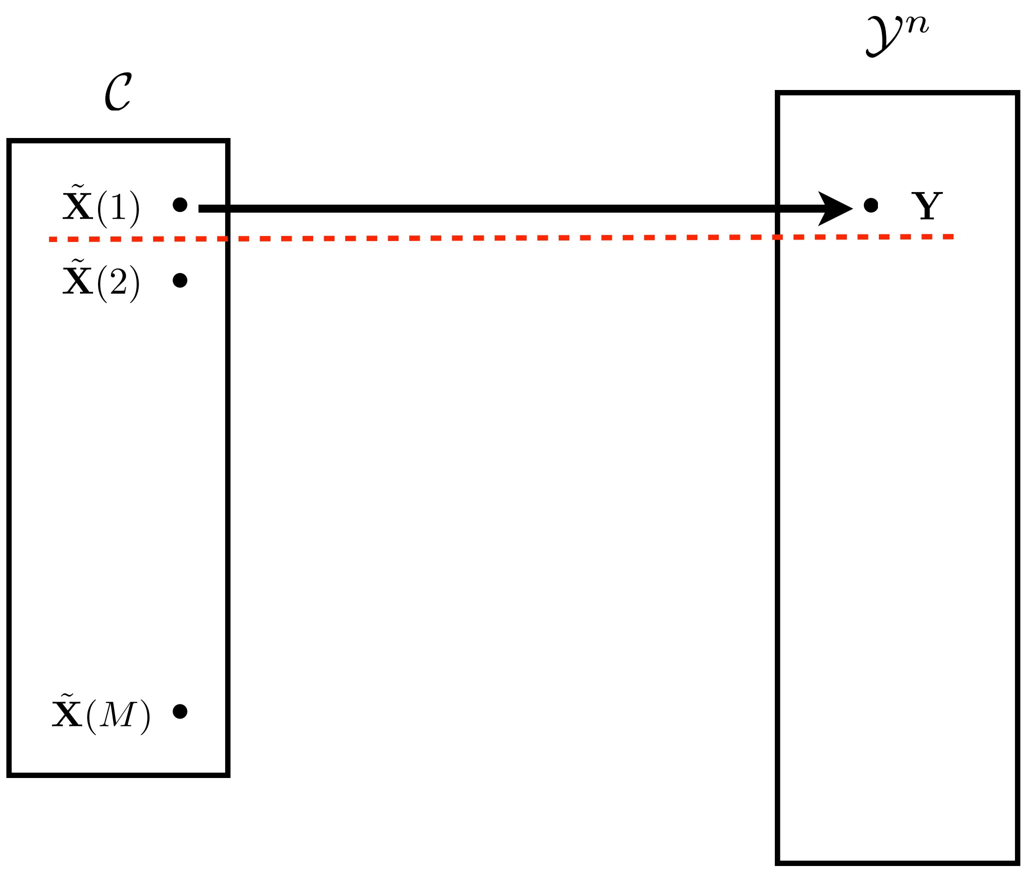

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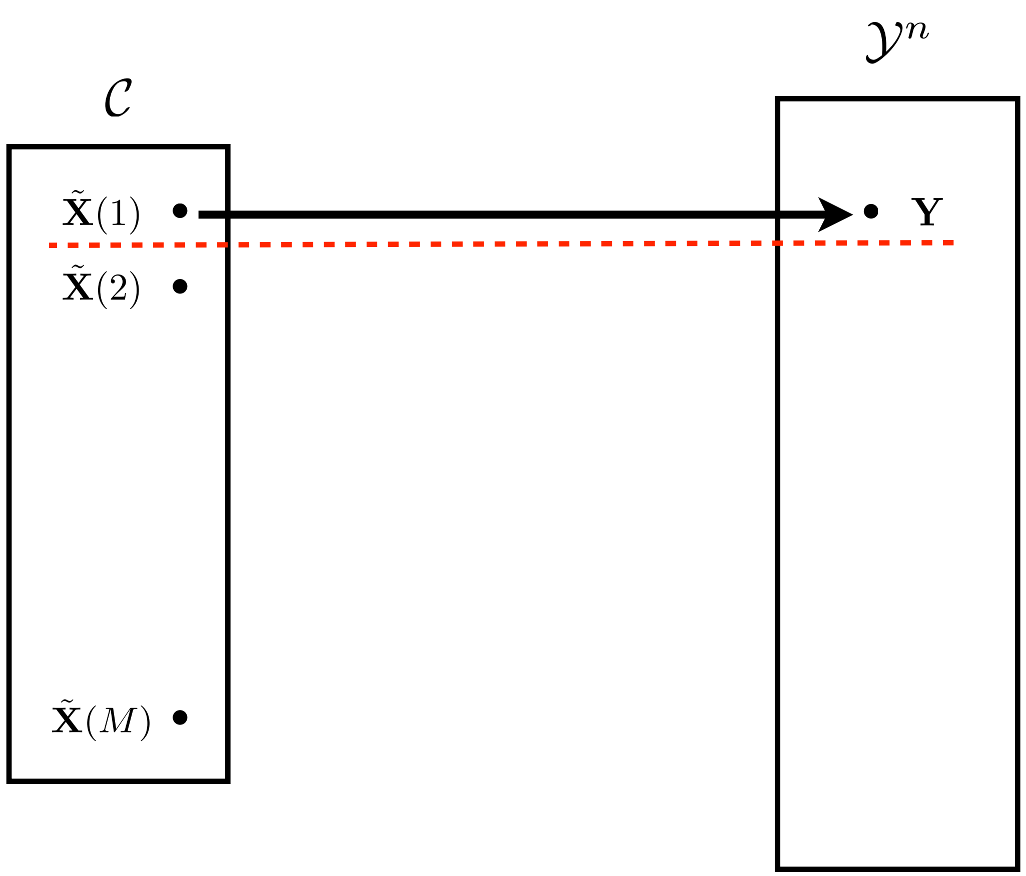

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Lemma 7.17

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- Conclusion: If $P_e < \epsilon/2$, then $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_{M/2} < \epsilon$.
- Discard the worst half of the codewords in $\mathcal{C}^*$ to achieve $\lambda_{max} < \epsilon$.

- After discarding the worse half of $\mathcal{C}^*$, the rate of the code becomes

$$\begin{aligned}\frac{1}{n}\log\frac{M}{2} &= \frac{1}{n}\log M - \frac{1}{n}\\ &> \left(I(X;Y) - \frac{\epsilon}{2}\right) - \frac{1}{n}\\ &> I(X;Y) - \epsilon\end{aligned}$$

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- Here we assume that the decoding function is unchanged, so that deletion of worst half of the codewords does not affect the conditional probabilities of error of the remaining codewords.