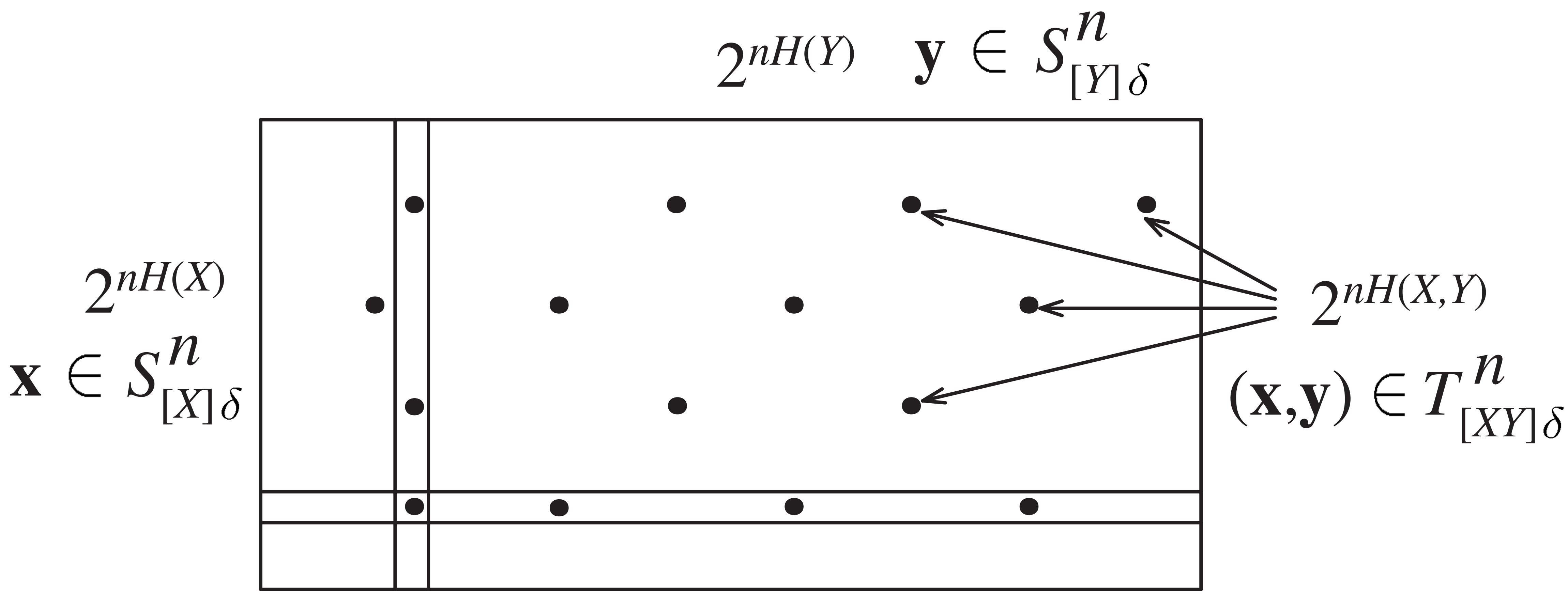
香港中文大學
The Chinese University of Hong Kong

6.4 An Interpretation of the Basic Inequalities



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$2^{nH(X)}$
$\mathbf{x} \in S^n_{[X]\delta}$
$2^{nH(X,Y)}$
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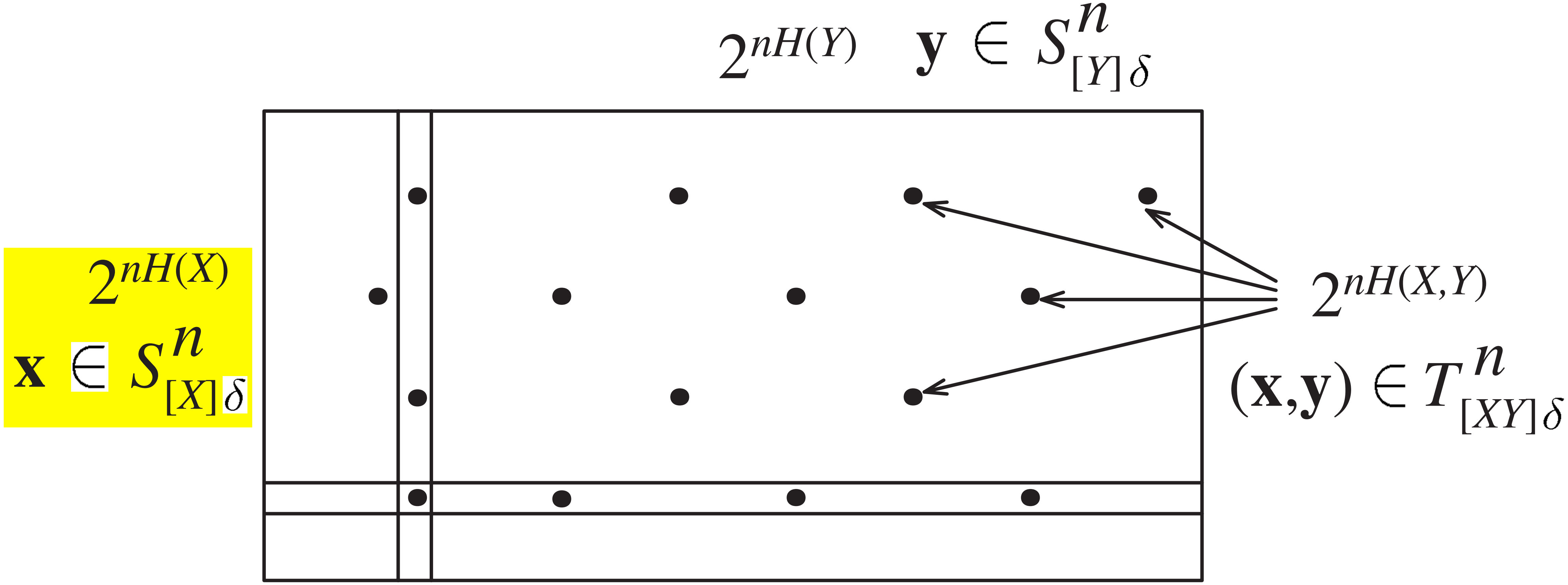

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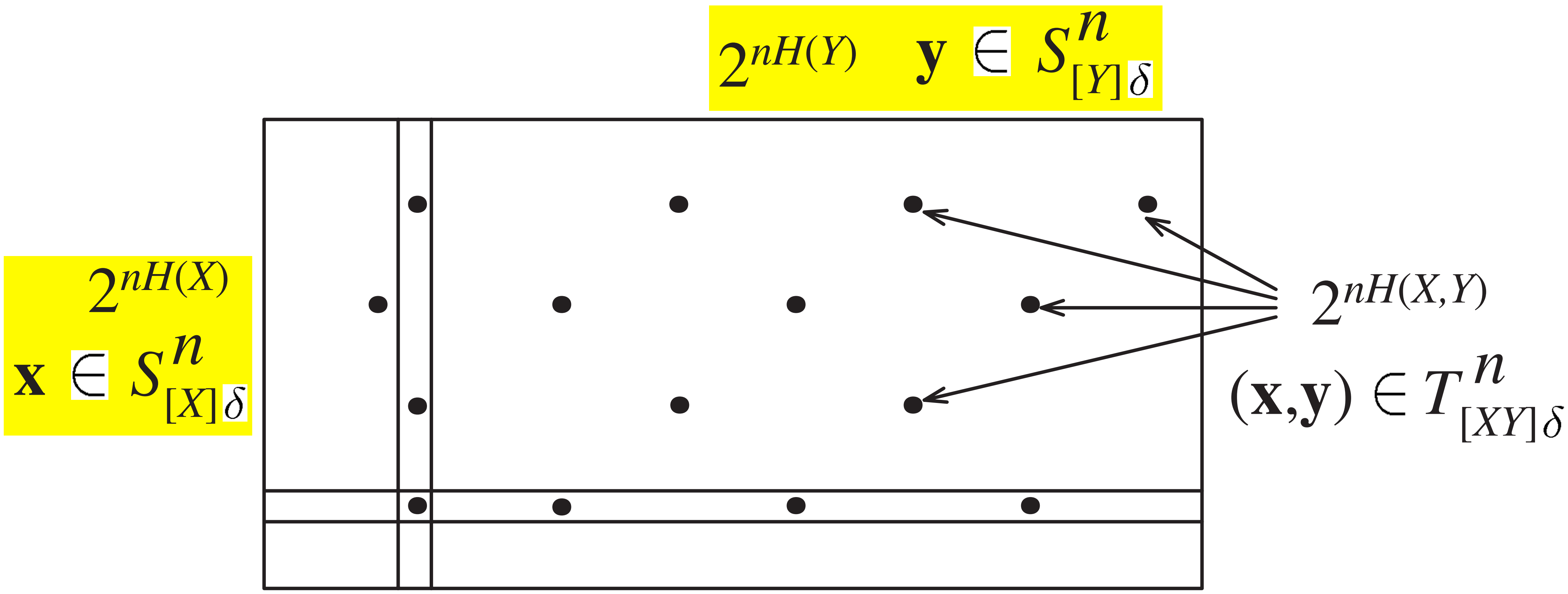

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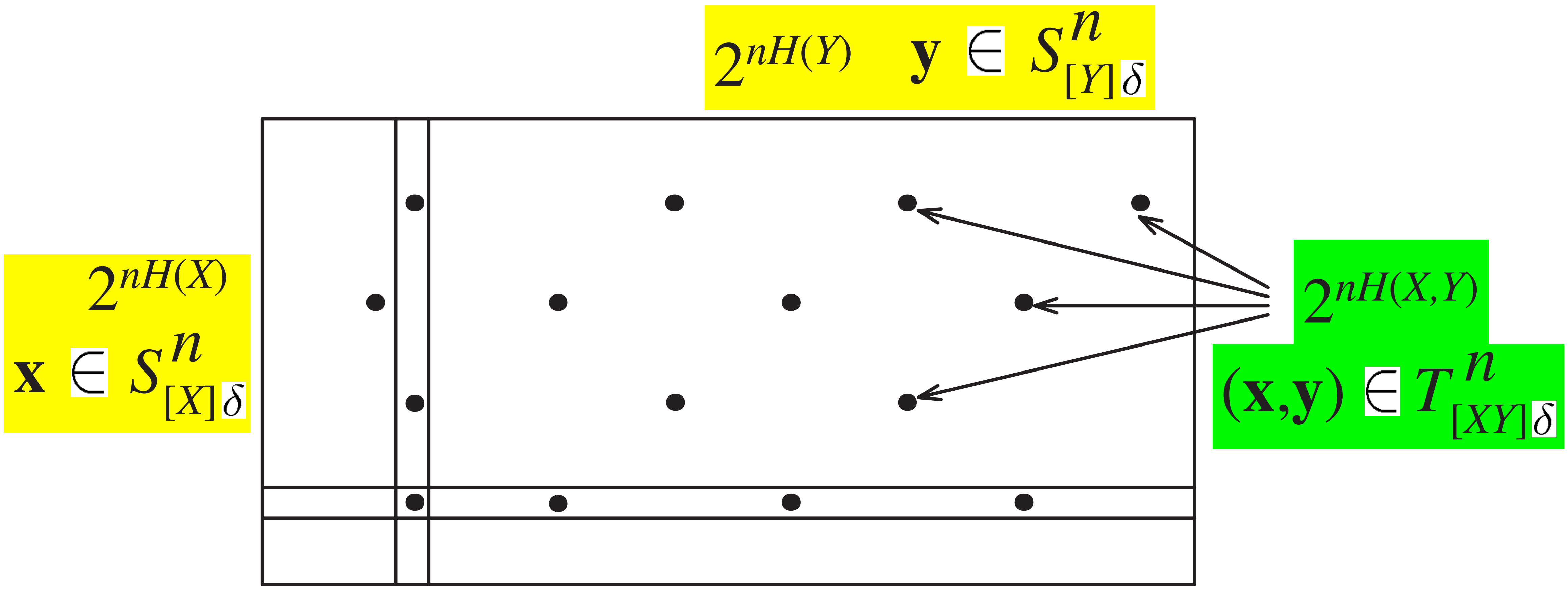

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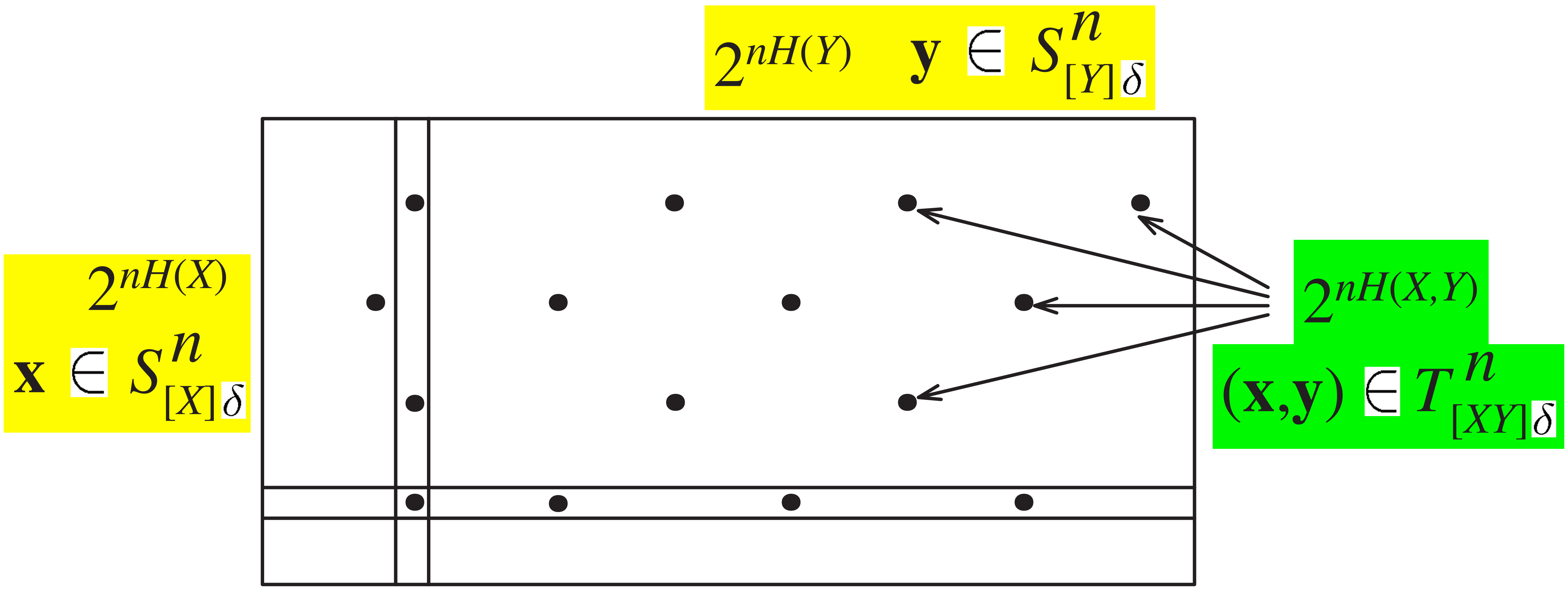
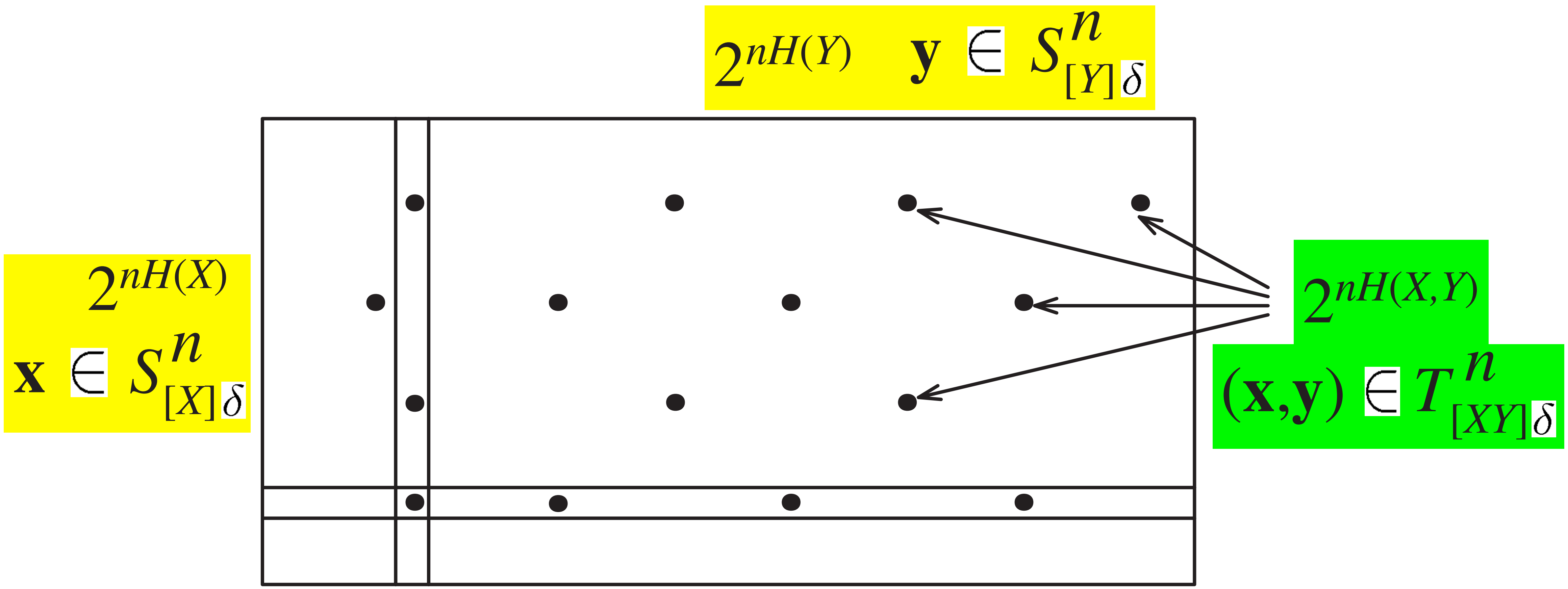


- Evidently,

$$\# \text{ of dots} \leq \# \text{ of cells}$$



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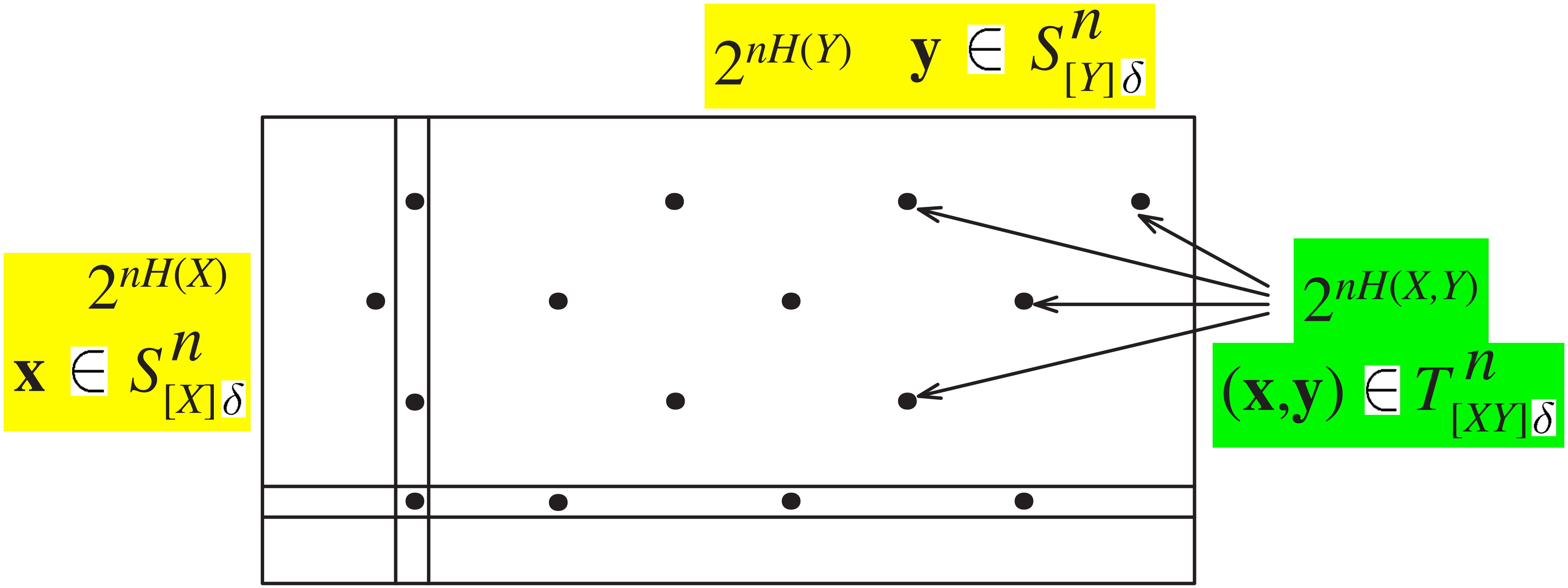

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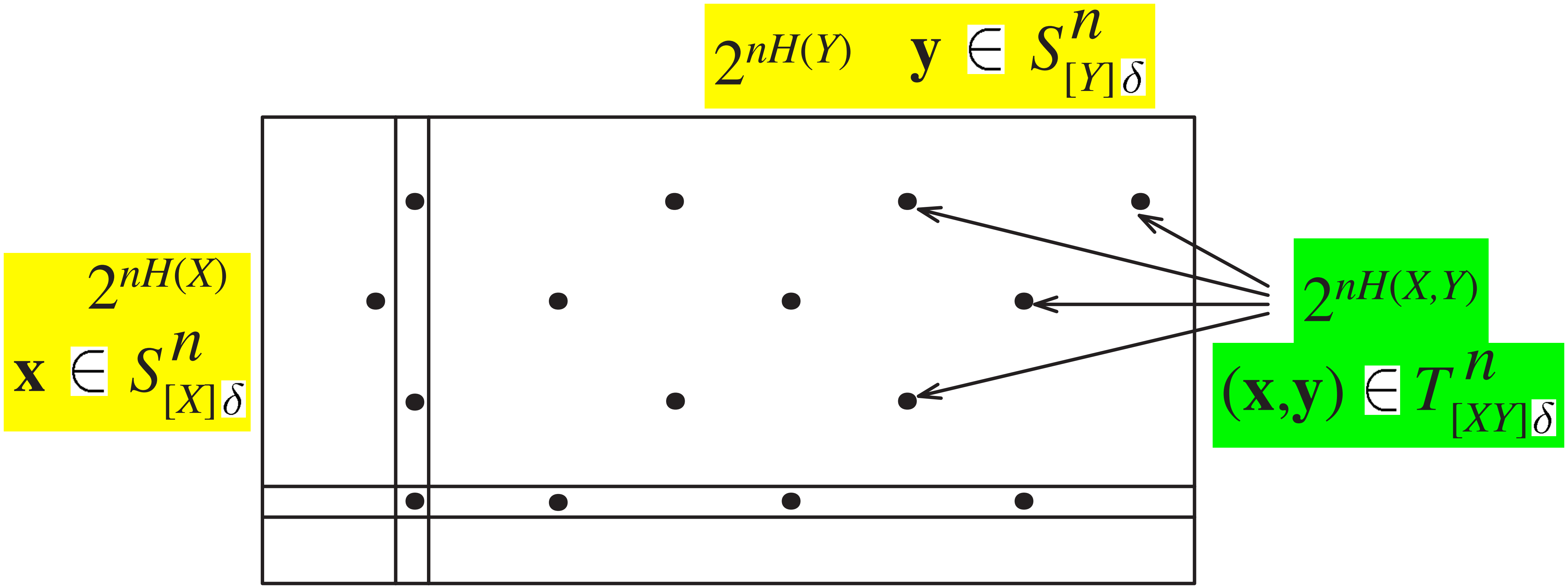

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- Therefore,

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- Taking logarithm and dividing by n, we have

$$H(X,Y) \leq H(X) + H(Y)$$

or

$$I(X;Y) \geq 0.$$

An Interpretation of $I(X;Y|Z) \geq 0$

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5. This is equivalent to $I(X;Y|Z) \geq 0$.

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- Tutorial: R. W. Yeung, "Facets of entropy"
  - `http://www.inc.cuhk.edu.hk/EII2013/entropy.pdf` (paper)
    IEEE Information Theory Society Newsletter, vol. 62, no. 8, Dec 2012
  - `http://iest2.ie.cuhk.edu.hk/~whyeung/entropy/slides.pdf` (slides)
  - `http://iest2.ie.cuhk.edu.hk/~whyeung/entropy/video.mp4` (video)