香港中文大學
The Chinese University of Hong Kong

6.2 Strong Typicality vs Weak Typicality

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- Both have AEP, but strong typicality has stronger conditional asymptotic properties (Theorem 6.10).

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- Weak typicality: empirical entropy $\approx H(X)$
- Strong typicality: empirical distribution $\sim p(x)$
- Strong typicality $\Rightarrow$ weak typicality (Proposition 6.5)
- Weak typicality $\not\Rightarrow$ strong typicality (to be discussed)
- Both have AEP, but strong typicality has stronger conditional asymptotic properties (Theorem 6.10).
- Strong typicality works only for finite alphabet, i.e., $|\mathcal{X}| < \infty$, but weak typicality works for any countable alphabet.

Strong Typicality ⇒ Weak Typicality

Proposition 6.5 For any $\mathbf{x} \in \mathcal{X}^n$, if $\mathbf{x} \in T^n_{[X]\delta}$, then $\mathbf{x} \in W^n_{[X]\eta}$, where $\eta \to 0$ as $\delta \to 0$.

Proof Idea

- By strong AEP and the definition of weak typicality.

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Definition 5.2 The weakly typical set $W^n_{[X]\epsilon}$ with respect to $p(x)$ is the set of sequences $\mathbf{x} = (x_1, x_2, \cdots, x_n) \in \mathcal{X}^n$ such that

$$H(X) - \epsilon \leq -\frac{1}{n} \log p(\mathbf{x}) \leq H(X) + \epsilon,$$

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2. Consider a sequence $\mathbf{x}$ of length n and let $q(x) = n^{-1} N(x; \mathbf{x})$ be the relative frequency of occurrence of symbol x in $\mathbf{x}$, $x = 0, 1, 2$.

3. In order for the sequence $\mathbf{x}$ to be weakly typical, we need the empirical entropy to be close to $H(X)$:

$$
\begin{aligned}
& -\frac{1}{n} \log p(\mathbf{x}) \quad \text{(empirical entropy)} \\
= & \quad -\frac{1}{n} \log \prod_{k=1}^{n} p(x_k) \\
= & \quad -\frac{1}{n} \sum_{k=1}^{n} \log p(x_k) \\
= & \quad -\frac{1}{n} \left[N(0;\mathbf{x}) \log p(0) + N(1;\mathbf{x}) \log p(1) + N(2;\mathbf{x}) \log p(2) \right] \\
= & \quad -\frac{N(0;\mathbf{x})}{n} \log p(0) - \frac{N(1;\mathbf{x})}{n} \log p(1) - \frac{N(2;\mathbf{x})}{n} \log p(2) \\
= & \quad -q(0) \underline{\log 0.5} - q(1) \underline{\log 0.25} - q(2) \log 0.25 \qquad (1) \\
\approx & \quad H(X) \\
= & \quad -p(0) \log 0.5 - p(1) \log 0.25 - p(2) \log 0.25 \\
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&= -(0.5)\log 0.5 - (0.25)\log 0.25 - (0.25)\log 0.25. \qquad (2)
\end{aligned}$$

This can be satisfied by choosing $q(i) = p(i)$ for all i.

Weak Typicality $\not\Rightarrow$ Strong Typicality

1. Consider X with distribution p such that

$$p(0) = 0.5, \; p(1) = 0.25, \; p(2) = 0.25.$$

2. Consider a sequence $\mathbf{x}$ of length n and let $q(x) = n^{-1} N(x; \mathbf{x})$ be the relative frequency of occurrence of symbol x in $\mathbf{x}$, $x = 0, 1, 2$.

3. In order for the sequence $\mathbf{x}$ to be weakly typical, we need the empirical entropy to be close to $H(X)$:

$$
\begin{aligned}
& -\frac{1}{n} \log p(\mathbf{x}) \quad \text{(empirical entropy)} \\
= & \quad -\frac{1}{n} \log \prod_{k=1}^{n} p(x_k) \\
= & \quad -\frac{1}{n} \sum_{k=1}^{n} \log p(x_k) \\
= & \quad -\frac{1}{n} \left[N(0;\mathbf{x}) \log p(0) + N(1;\mathbf{x}) \log p(1) + N(2;\mathbf{x}) \log p(2) \right] \\
= & \quad -\frac{N(0;\mathbf{x})}{n} \log p(0) - \frac{N(1;\mathbf{x})}{n} \log p(1) - \frac{N(2;\mathbf{x})}{n} \log p(2) \\
= & \quad -q(0) \log 0.5 - q(1) \log 0.25 - q(2) \log 0.25 \qquad (1) \\
\approx & \quad H(X) \\
= & \quad -p(0) \log 0.5 - p(1) \log 0.25 - p(2) \log 0.25 \\
= & \quad -(0.5) \log 0.5 - (0.25) \log 0.25 - (0.25) \log 0.25. \qquad (2)
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&= -\frac{1}{n} [N(0;\mathbf{x}) \log p(0) + N(1;\mathbf{x}) \log p(1) + N(2;\mathbf{x}) \log p(2)] \\
&= -\frac{N(0;\mathbf{x})}{n} \log p(0) - \frac{N(1;\mathbf{x})}{n} \log p(1) - \frac{N(2;\mathbf{x})}{n} \log p(2) \\
&= -q(0) \log 0.5 - q(1) \log 0.25 - q(2) \log 0.25 \qquad (1) \\
&\approx H(X) \\
&= -p(0) \log 0.5 - p(1) \log 0.25 - p(2) \log 0.25 \\
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&= -\frac{1}{n}\log \prod_{k=1}^{n} p(x_k) \\
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&= -\frac{1}{n}\left[N(0;\mathbf{x})\log p(0) + N(1;\mathbf{x})\log p(1) + N(2;\mathbf{x})\log p(2)\right] \\
&= -\frac{N(0;\mathbf{x})}{n}\log p(0) - \frac{N(1;\mathbf{x})}{n}\log p(1) - \frac{N(2;\mathbf{x})}{n}\log p(2) \\
&= -q(0)\log 0.5 - q(1)\log 0.25 - q(2)\log 0.25 \qquad (1) \\
&\approx H(X) \\
&= -p(0)\log 0.5 - p(1)\log 0.25 - p(2)\log 0.25 \\
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&= -\frac{1}{n} \log \prod_{k=1}^{n} p(x_k) \\
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&= -\frac{N(0;\mathbf{x})}{n} \log p(0) - \frac{N(1;\mathbf{x})}{n} \log p(1) - \frac{N(2;\mathbf{x})}{n} \log p(2) \\
&= -q(0) \log 0.5 - q(1) \log 0.25 - q(2) \log 0.25 \qquad (1) \\
&\approx H(X) \\
&= -p(0) \log 0.5 - p(1) \log 0.25 - p(2) \log 0.25 \\
&= -(0.5) \log 0.5 - (0.25) \log 0.25 - (0.25) \log 0.25. \qquad (2)
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\begin{aligned}
& -\frac{1}{n} \log p(\mathbf{x}) \quad \text{(empirical entropy)} \\
= \quad & -\frac{1}{n} \log \prod_{k=1}^{n} p(x_k) \\
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= \quad & -\frac{1}{n} [N(0; \mathbf{x}) \log p(0) + N(1; \mathbf{x}) \log p(1) + N(2; \mathbf{x}) \log p(2)] \\
= \quad & -\frac{N(0; \mathbf{x})}{n} \log p(0) - \frac{N(1; \mathbf{x})}{n} \log p(1) - \frac{N(2; \mathbf{x})}{n} \log p(2) \\
= \quad & -q(0) \log 0.5 - q(1) \log 0.25 - q(2) \log 0.25 \qquad (1) \\
\approx \quad & H(X) \\
= \quad & -p(0) \log 0.5 - p(1) \log 0.25 - p(2) \log 0.25 \\
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&= -\frac{N(0;\mathbf{x})}{n} \log p(0) - \frac{N(1;\mathbf{x})}{n} \log p(1) - \frac{N(2;\mathbf{x})}{n} \log p(2) \\
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&\approx H(X) \\
&= -p(0) \log 0.5 - p(1) \log 0.25 - p(2) \log 0.25 \\
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= \;& -q(0) \log 0.5 - q(1) \log 0.25 - q(2) \log 0.25 \qquad (1) \\
\approx \;& H(X) \\
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&= -\overset{0.5}{q(0)} \log 0.5 - q(1) \log 0.25 - q(2) \log 0.25 \qquad (1) \\
&\approx H(X) \\
&= -p(0) \log 0.5 - p(1) \log 0.25 - p(2) \log 0.25 \\
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= \quad & -\frac{1}{n} \left[N(0; \mathbf{x}) \log p(0) + N(1; \mathbf{x}) \log p(1) + N(2; \mathbf{x}) \log p(2) \right] \\
= \quad & -\frac{N(0; \mathbf{x})}{n} \log p(0) - \frac{N(1; \mathbf{x})}{n} \log p(1) - \frac{N(2; \mathbf{x})}{n} \log p(2) \\
= \quad & -\overset{0.5}{q(0)} \log 0.5 - \overset{0.5}{q(1)} \log 0.25 - q(2) \log 0.25 \qquad (1) \\
\approx \quad & H(X) \\
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&= -\frac{N(0;\mathbf{x})}{n}\log p(0) - \frac{N(1;\mathbf{x})}{n}\log p(1) - \frac{N(2;\mathbf{x})}{n}\log p(2) \\
&= -\overset{0.5}{q(0)}\log 0.5 - \overset{0.5}{q(1)}\log 0.25 - \overset{0}{q(2)}\log 0.25 \qquad (1) \\
&\approx H(X) \\
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\approx & \quad H(X) \\
= & \quad -p(0) \log 0.5 - p(1) \log 0.25 - p(2) \log 0.25 \\
= & \quad -(0.5) \log 0.5 - (0.25) \log 0.25 - (0.25) \log 0.25. \qquad (2)
\end{aligned}
$$

This can be satisfied by choosing $q(i) = p(i)$ for all i.

4. Alternatively, we can choose

$$q(0) = 0.5, \ q(1) = 0.5, \ q(2) = 0.$$

5. With such a choice of $\{q(i)\}$, the sequence $\mathbf{x}$ is weakly typical with respect to p because (1) and (2) are evaluated to the same value which implies

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1. Consider X with distribution p such that

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&= -\frac{1}{n}[N(0;\mathbf{x})\log p(0) + N(1;\mathbf{x})\log p(1) + N(2;\mathbf{x})\log p(2)] \\
&= -\frac{N(0;\mathbf{x})}{n}\log p(0) - \frac{N(1;\mathbf{x})}{n}\log p(1) - \frac{N(2;\mathbf{x})}{n}\log p(2) \\
&= -\overset{0.5}{q(0)}\log 0.5 - \overset{0.5}{q(1)}\log 0.25 - \overset{0}{q(2)}\log 0.25 \qquad (1) \\
&\approx H(X) \\
&= -p(0)\log 0.5 - p(1)\log 0.25 - p(2)\log 0.25 \\
&= -(0.5)\log 0.5 - (0.25)\log 0.25 - (0.25)\log 0.25. \qquad (2)
\end{aligned}$$

This can be satisfied by choosing $q(i) = p(i)$ for all i.

4. Alternatively, we can choose

$$q(0) = 0.5, \ q(1) = 0.5, \ q(2) = 0.$$

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$$\text{empirical entropy} \approx H(X),$$

Weak Typicality $\not\Rightarrow$ Strong Typicality

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&= -\frac{1}{n} \left[N(0;\mathbf{x}) \log p(0) + N(1;\mathbf{x}) \log p(1) + N(2;\mathbf{x}) \log p(2) \right] \\
&= -\frac{N(0;\mathbf{x})}{n} \log p(0) - \frac{N(1;\mathbf{x})}{n} \log p(1) - \frac{N(2;\mathbf{x})}{n} \log p(2) \\
&= -\overset{0.5}{q(0)} \log 0.5 - \overset{0.5}{q(1)} \log 0.25 - \overset{0}{q(2)} \log 0.25 \qquad (1) \\
&\approx H(X) \\
&= -p(0) \log 0.5 - p(1) \log 0.25 - p(2) \log 0.25 \\
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This can be satisfied by choosing $q(i) = p(i)$ for all i.

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5. With such a choice of $\{q(i)\}$, the sequence $\mathbf{x}$ is weakly typical with respect to p because (1) and (2) are evaluated to the same value which implies

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but obviously not strongly typical with respect to p,

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$$p(0) = 0.5, \quad p(1) = 0.25, \quad p(2) = 0.25.$$

2. Consider a sequence $\mathbf{x}$ of length n and let $q(x) = n^{-1}N(x;\mathbf{x})$ be the relative frequency of occurrence of symbol x in $\mathbf{x}$, $x = 0, 1, 2$.

3. In order for the sequence $\mathbf{x}$ to be weakly typical, we need the empirical entropy to be close to $H(X)$:

$$-\frac{1}{n}\log p(\mathbf{x}) \quad \text{(empirical entropy)}$$

$$\begin{aligned}
&= -\frac{1}{n}\log \prod_{k=1}^{n} p(x_k) \\
&= -\frac{1}{n}\sum_{k=1}^{n} \log p(x_k) \\
&= -\frac{1}{n}\left[N(0;\mathbf{x})\log p(0) + N(1;\mathbf{x})\log p(1) + N(2;\mathbf{x})\log p(2)\right] \\
&= -\frac{N(0;\mathbf{x})}{n}\log p(0) - \frac{N(1;\mathbf{x})}{n}\log p(1) - \frac{N(2;\mathbf{x})}{n}\log p(2) \\
&= -\overset{0.5}{q(0)}\log 0.5 - \overset{0.5}{q(1)}\log 0.25 - \overset{0}{q(2)}\log 0.25 \qquad (1) \\
&\approx H(X) \\
&= -p(0)\log 0.5 - p(1)\log 0.25 - p(2)\log 0.25 \\
&= -(0.5)\log 0.5 - (0.25)\log 0.25 - (0.25)\log 0.25. \qquad (2)
\end{aligned}$$

This can be satisfied by choosing $q(i) = p(i)$ for all i.

4. Alternatively, we can choose

$$q(0) = 0.5, \quad q(1) = 0.5, \quad q(2) = 0.$$

5. With such a choice of $\{q(i)\}$, the sequence $\mathbf{x}$ is weakly typical with respect to p because (1) and (2) are evaluated to the same value which implies

$$\text{empirical entropy} \approx H(X),$$

but obviously not strongly typical with respect to p, because

$$p \not\approx q.$$