



香港中文大學  
The Chinese University of Hong Kong

## 6.2 Strong Typicality vs Weak Typicality

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- Weak typicality  $\not\Rightarrow$  strong typicality (to be discussed)
- Both have AEP, but strong typicality has stronger conditional asymptotic properties (Theorem 6.10).
- Strong typicality works only for finite alphabet, i.e.,  $|\mathcal{X}| < \infty$ , but weak typicality works for any countable alphabet.

# Strong Typicality $\Rightarrow$ Weak Typicality

**Proposition 6.5** For any  $\mathbf{x} \in \mathcal{X}^n$ , if  $\mathbf{x} \in T_{[X]\delta}^n$ , then  $\mathbf{x} \in W_{[X]\eta}^n$ , where  $\eta \rightarrow 0$  as  $\delta \rightarrow 0$ .

## Proof Idea

- By strong AEP and the definition of weak typicality.



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**Definition 5.2** The weakly typical set  $W_{[X]\epsilon}^n$  with respect to  $p(x)$  is the set of sequences  $\mathbf{x} = (x_1, x_2, \dots, x_n) \in \mathcal{X}^n$  such that

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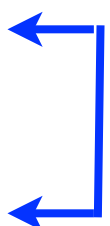
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# Weak Typicality $\not\Rightarrow$ Strong Typicality

1. Consider  $X$  with distribution  $p$  such that

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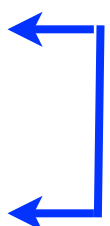
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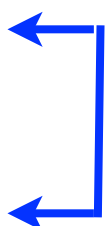
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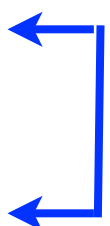
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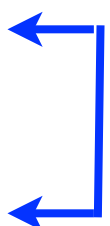
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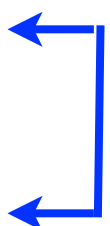
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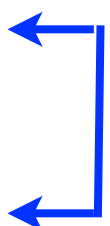
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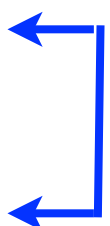
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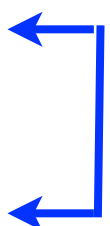
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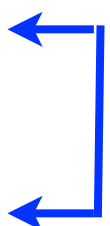
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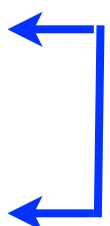
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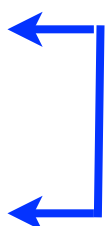
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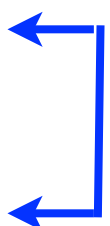
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3. In order for the sequence  $\mathbf{x}$  to be weakly typical, we need the empirical entropy to be close to  $H(X)$ :

$$\begin{aligned} & -\frac{1}{n} \log p(\mathbf{x}) \quad (\text{empirical entropy}) \\ &= -\frac{1}{n} \log \prod_{k=1}^n p(x_k) \\ &= -\frac{1}{n} \sum_{k=1}^n \log p(x_k) \\ &= -\frac{1}{n} [N(0; \mathbf{x}) \log p(0) + N(1; \mathbf{x}) \log p(1) + N(2; \mathbf{x}) \log p(2)] \\ &= -\frac{N(0; \mathbf{x})}{n} \log p(0) - \frac{N(1; \mathbf{x})}{n} \log p(1) - \frac{N(2; \mathbf{x})}{n} \log p(2) \\ &= -\frac{0.5}{n} \log 0.5 - \frac{0.5}{n} \log 0.25 - \frac{0}{n} \log 0.25 \quad (1) \\ &\approx H(X) \\ &= -p(0) \log 0.5 - p(1) \log 0.25 - p(2) \log 0.25 \\ &= -0.5 \log 0.5 - 0.25 \log 0.25 - 0.25 \log 0.25. \quad (2) \end{aligned}$$

4. Alternatively, we can choose

$$q(0) = 0.5, \quad q(1) = 0.5, \quad q(2) = 0.$$

This can be satisfied by choosing  $q(i) = p(i)$  for all  $i$ .

# Weak Typicality $\not\Rightarrow$ Strong Typicality

1. Consider  $X$  with distribution  $p$  such that

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4. Alternatively, we can choose

$$q(0) = 0.5, \quad q(1) = 0.5, \quad q(2) = 0.$$

5. With such a choice of  $\{q(i)\}$ , the sequence  $\mathbf{x}$  is weakly typical with respect to  $p$  because (1) and (2) are evaluated to the same value which implies

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(1) ←

(2) ←

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but obviously not strongly typical with respect to  $p$ ,

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