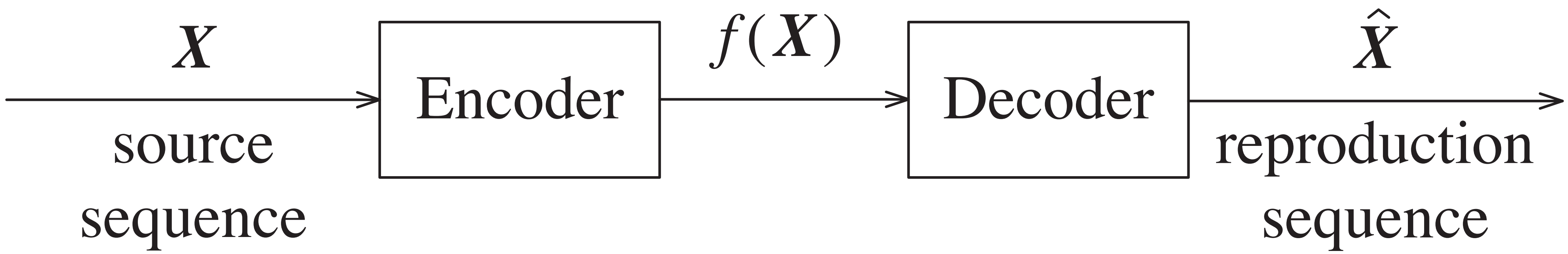
香港中文大學
The Chinese University of Hong Kong

5.2 The Source Coding Theorem

A Source Code



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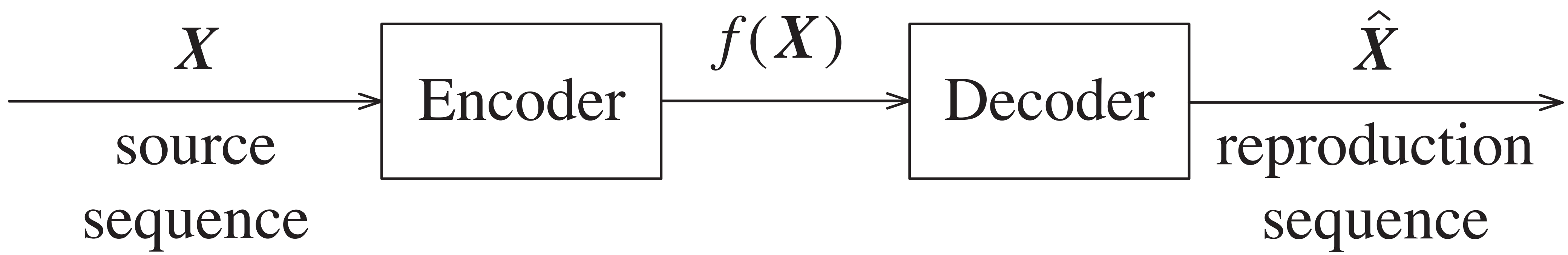

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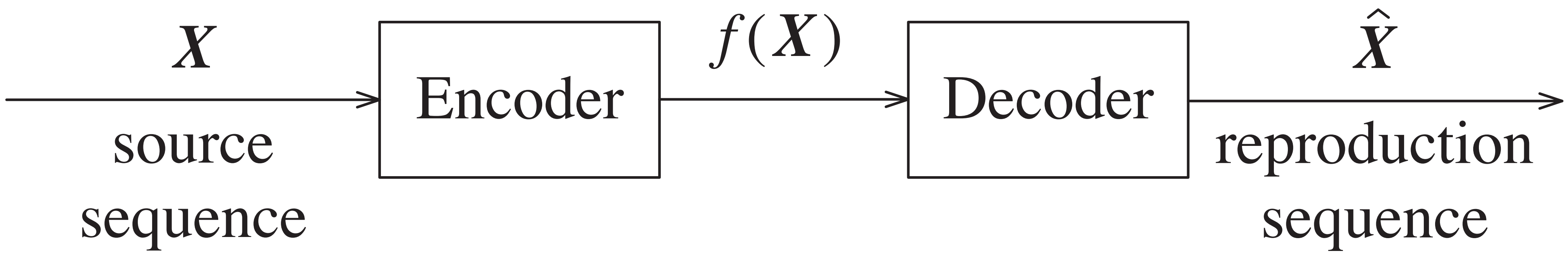

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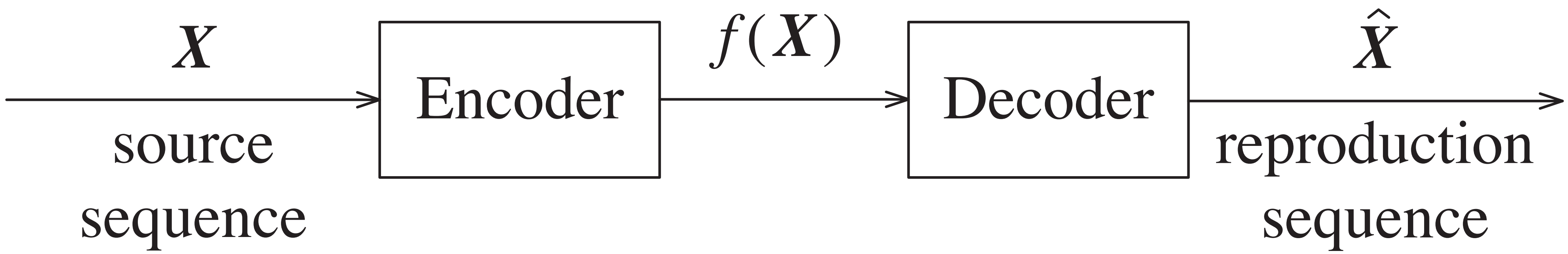

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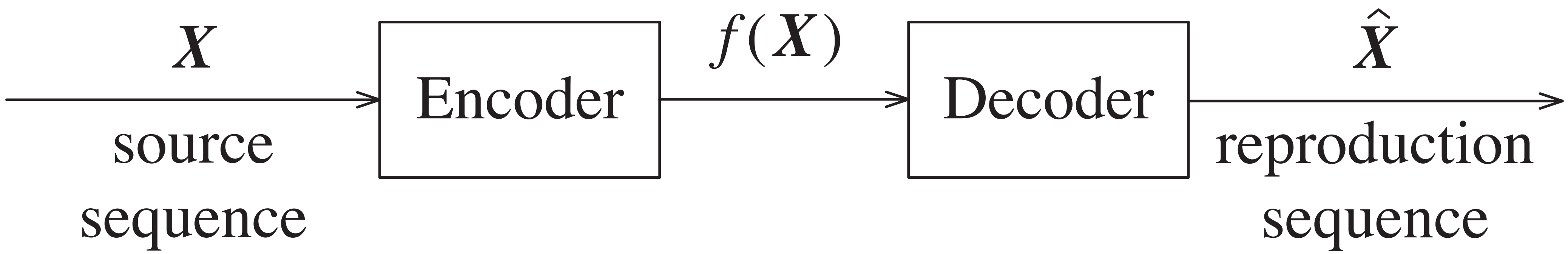

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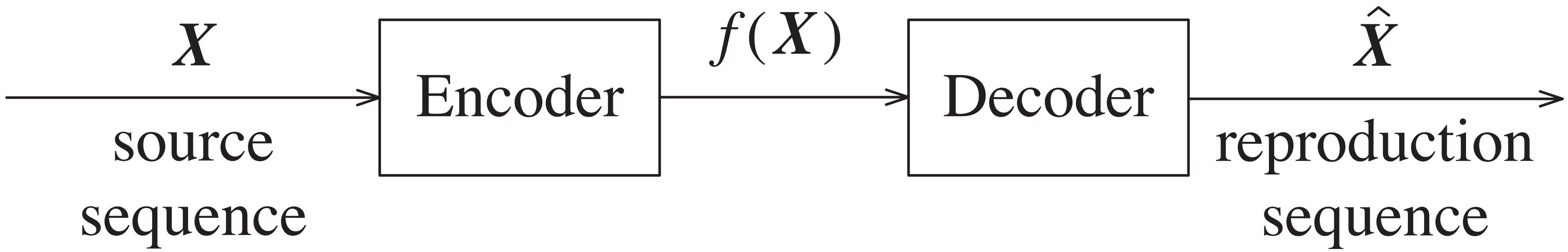
- The encoder sends $f(\mathbf{X})$ to the decoder through a noiseless channel.
- Based on the index, the decoder outputs $\hat{\mathbf{X}}$ as an estimate on $\mathbf{X}$.



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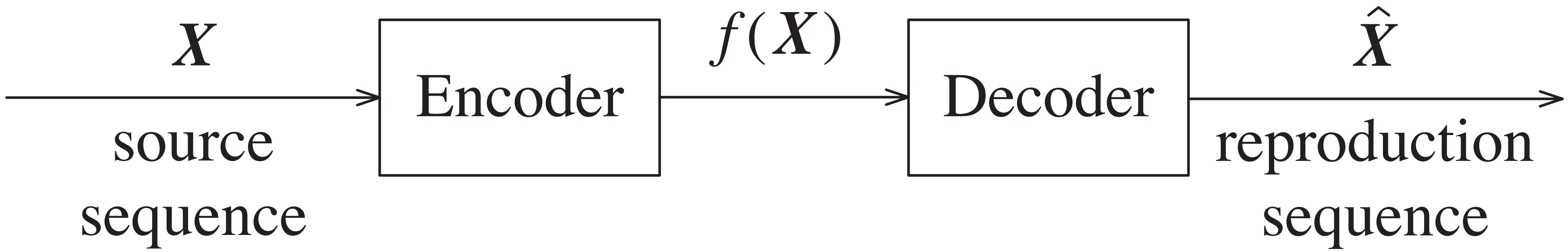

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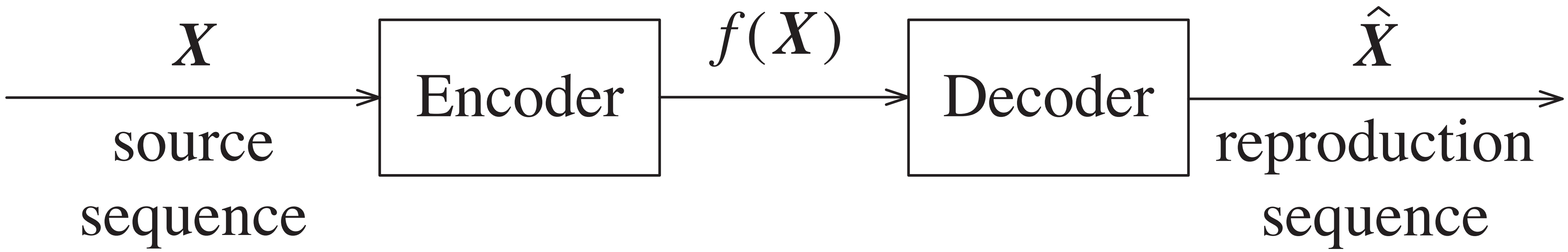

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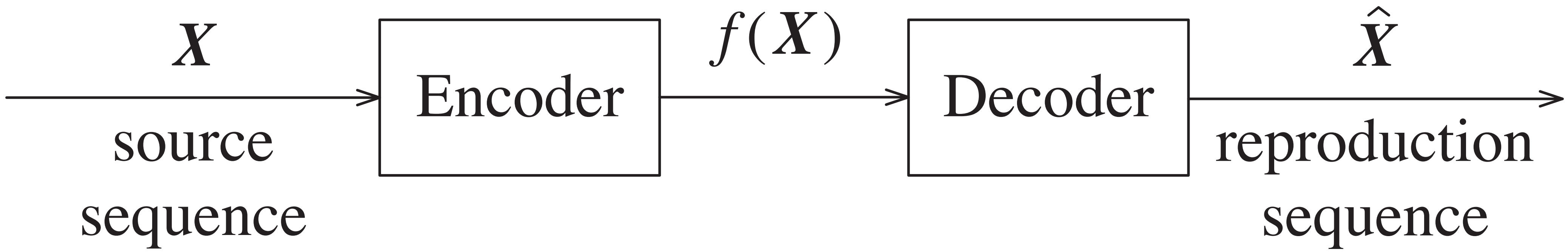

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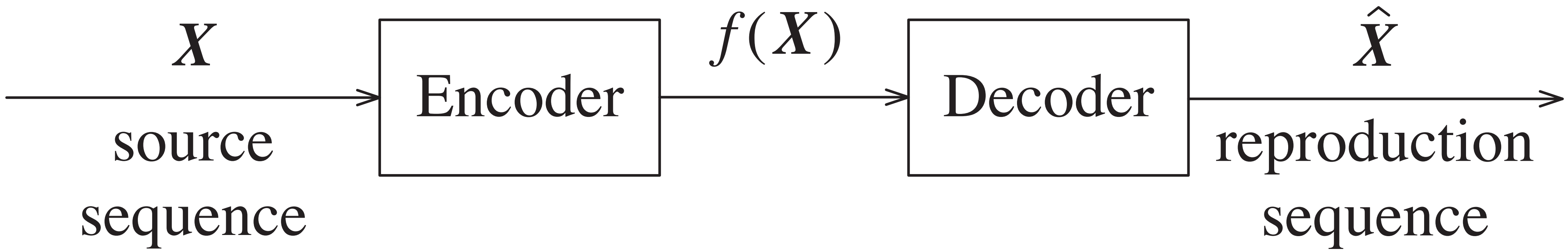

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- An error occurs if $\hat{\mathbf{X}} \neq \mathbf{X}$, and $P_e = \Pr\{\hat{\mathbf{X}} \neq \mathbf{X}\}$ is called the error probability.

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- This part says that it is impossible to achieve reliable communication if the coding rate is less than $H(X)$.

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- We will consider a class of block codes with a particular structure.

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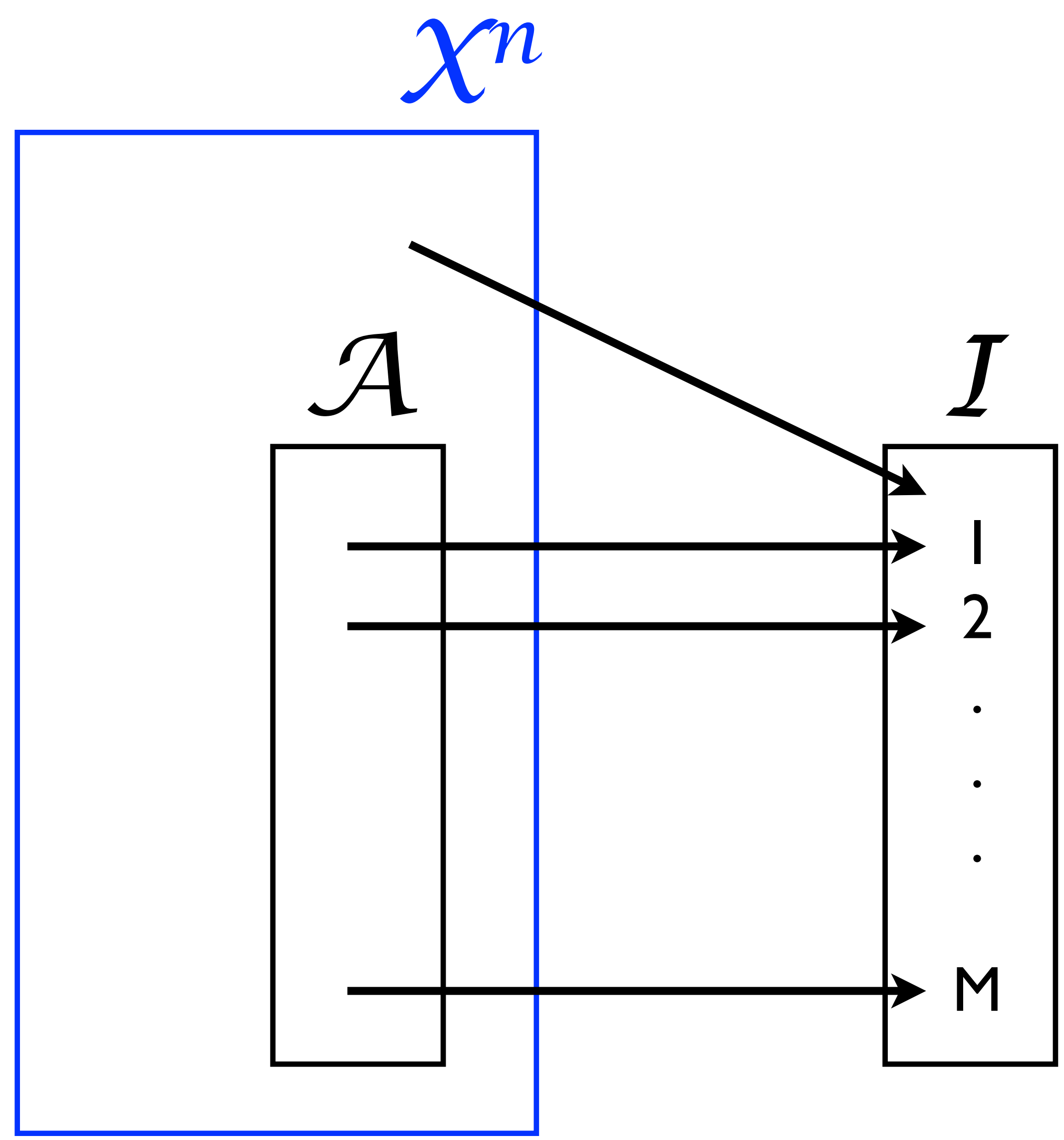

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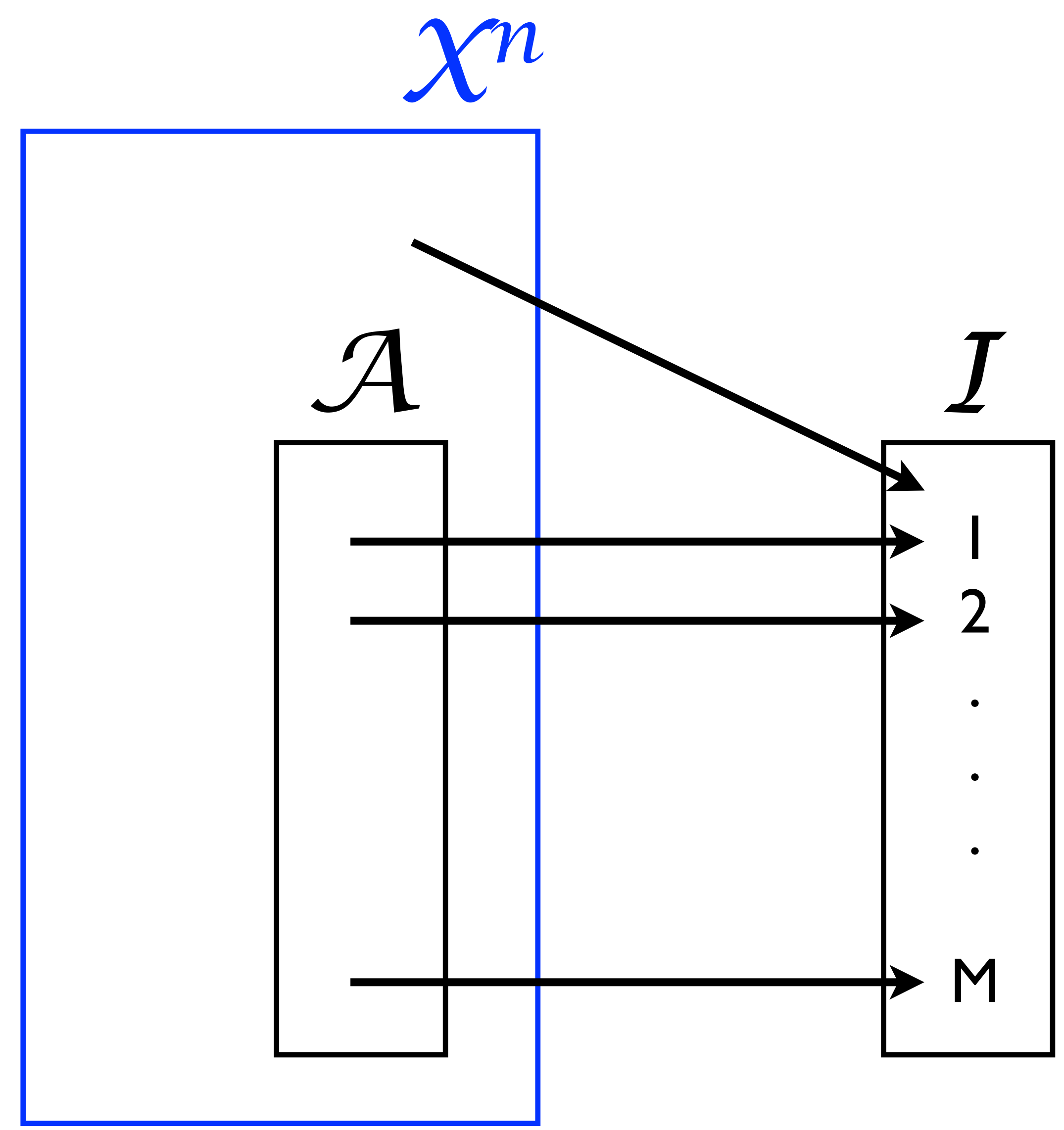

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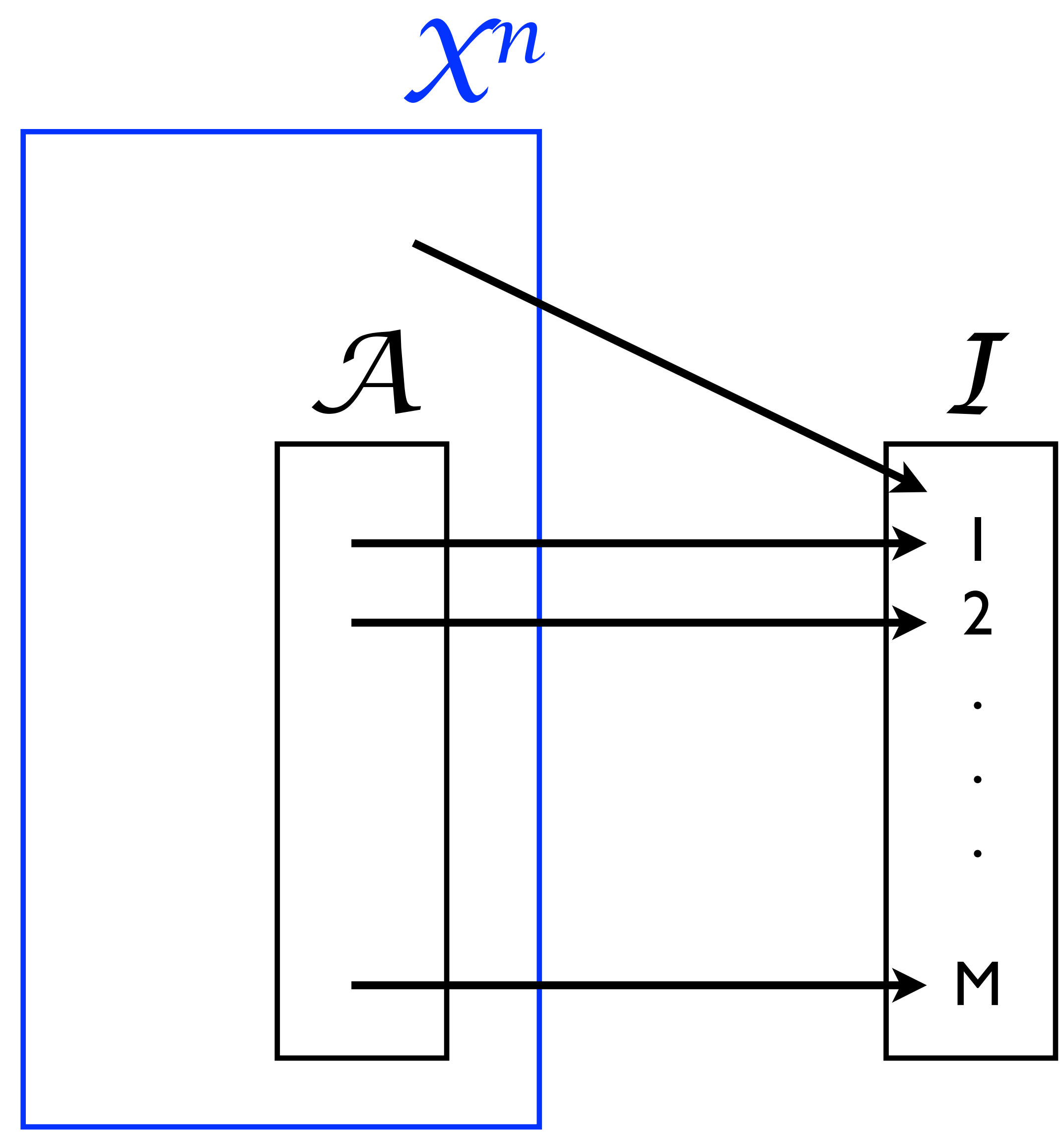

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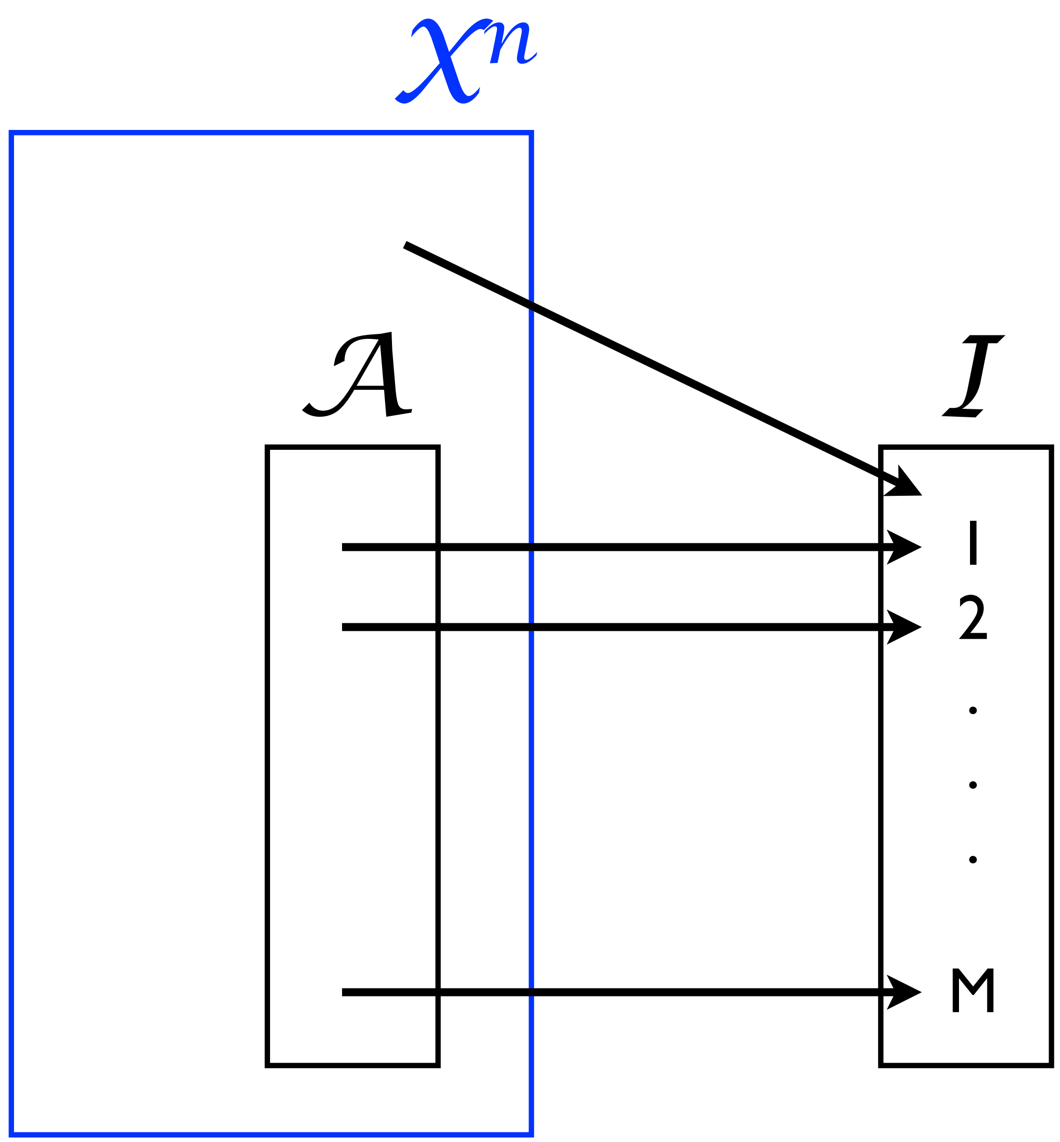

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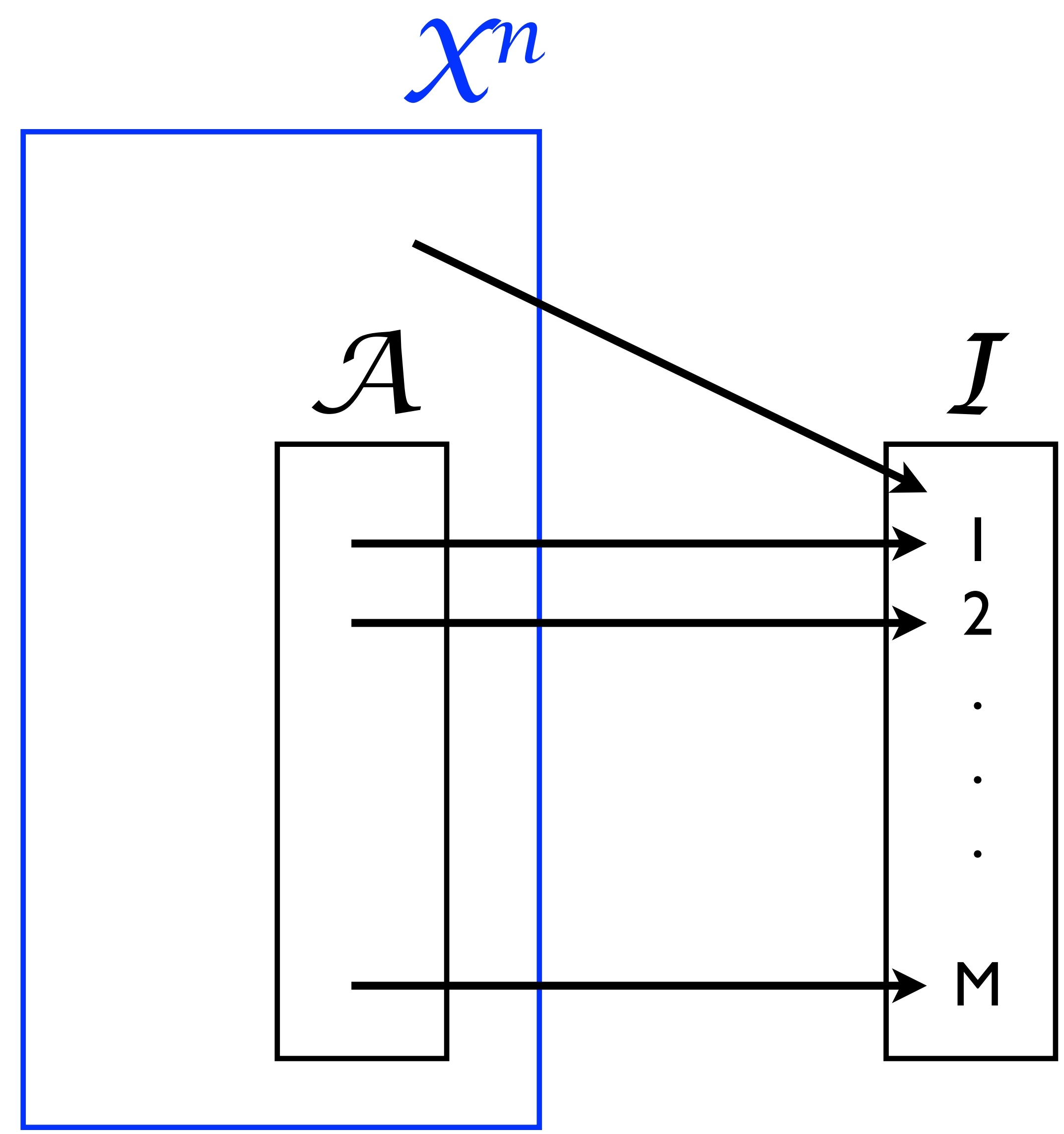

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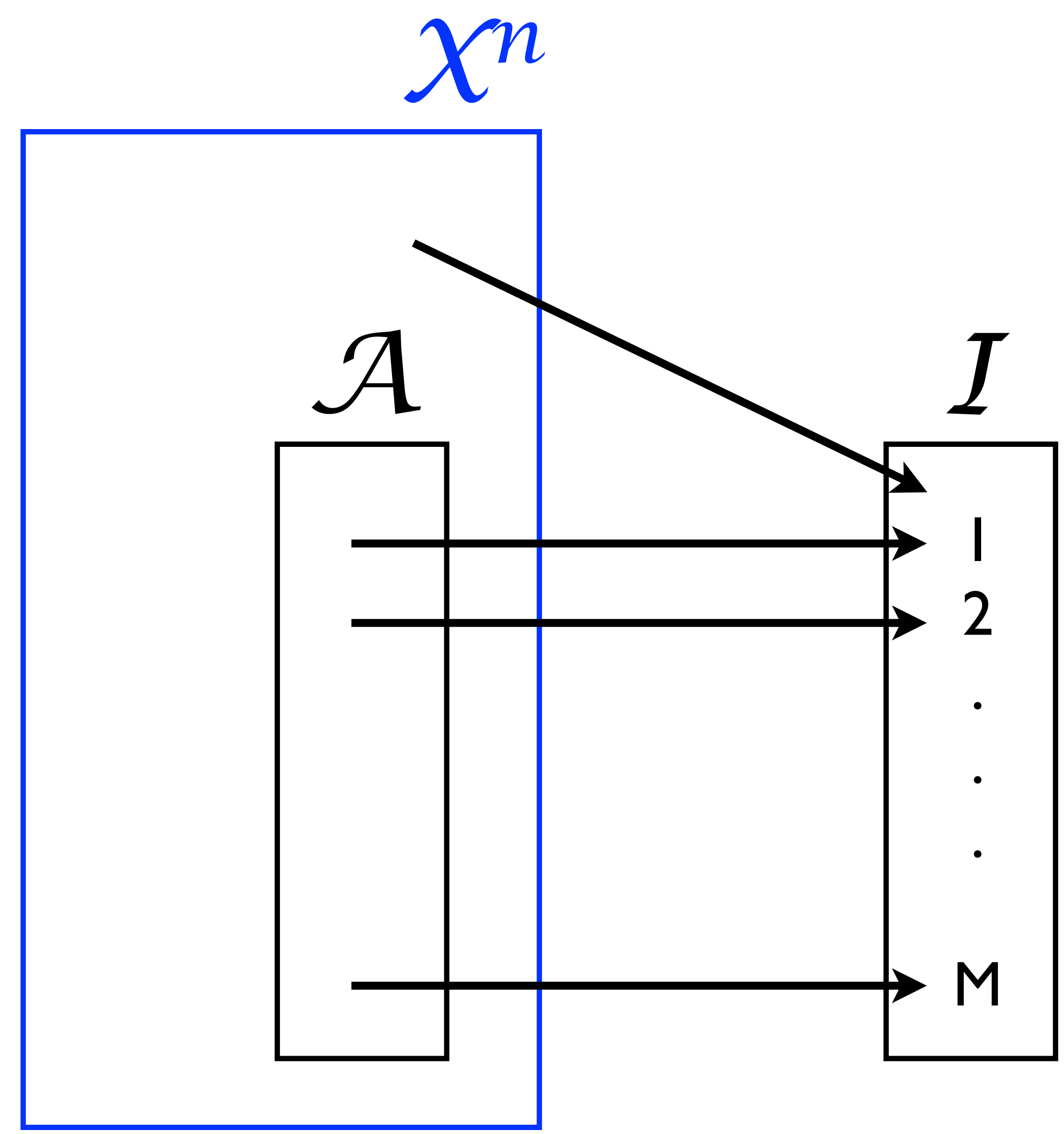

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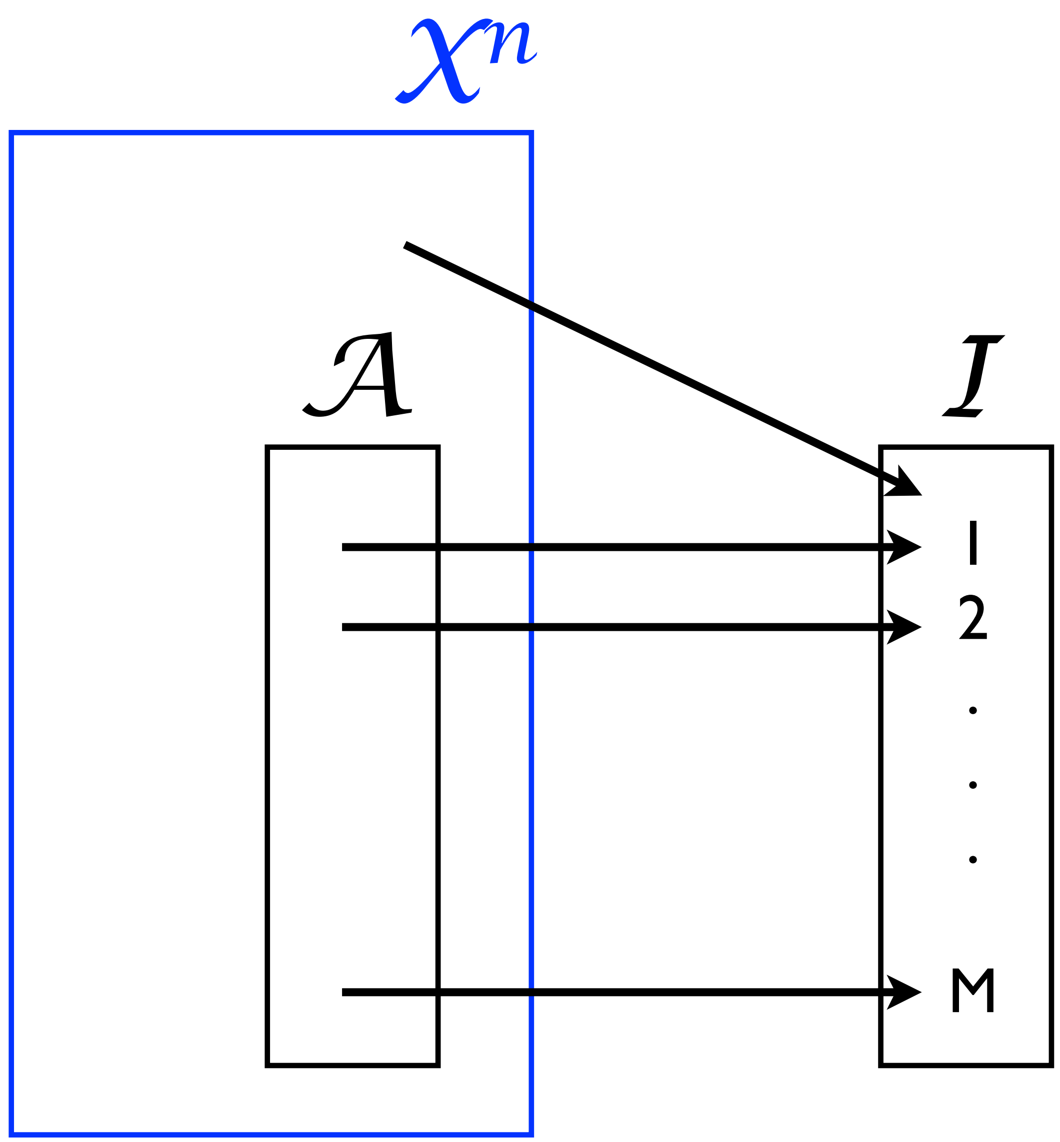

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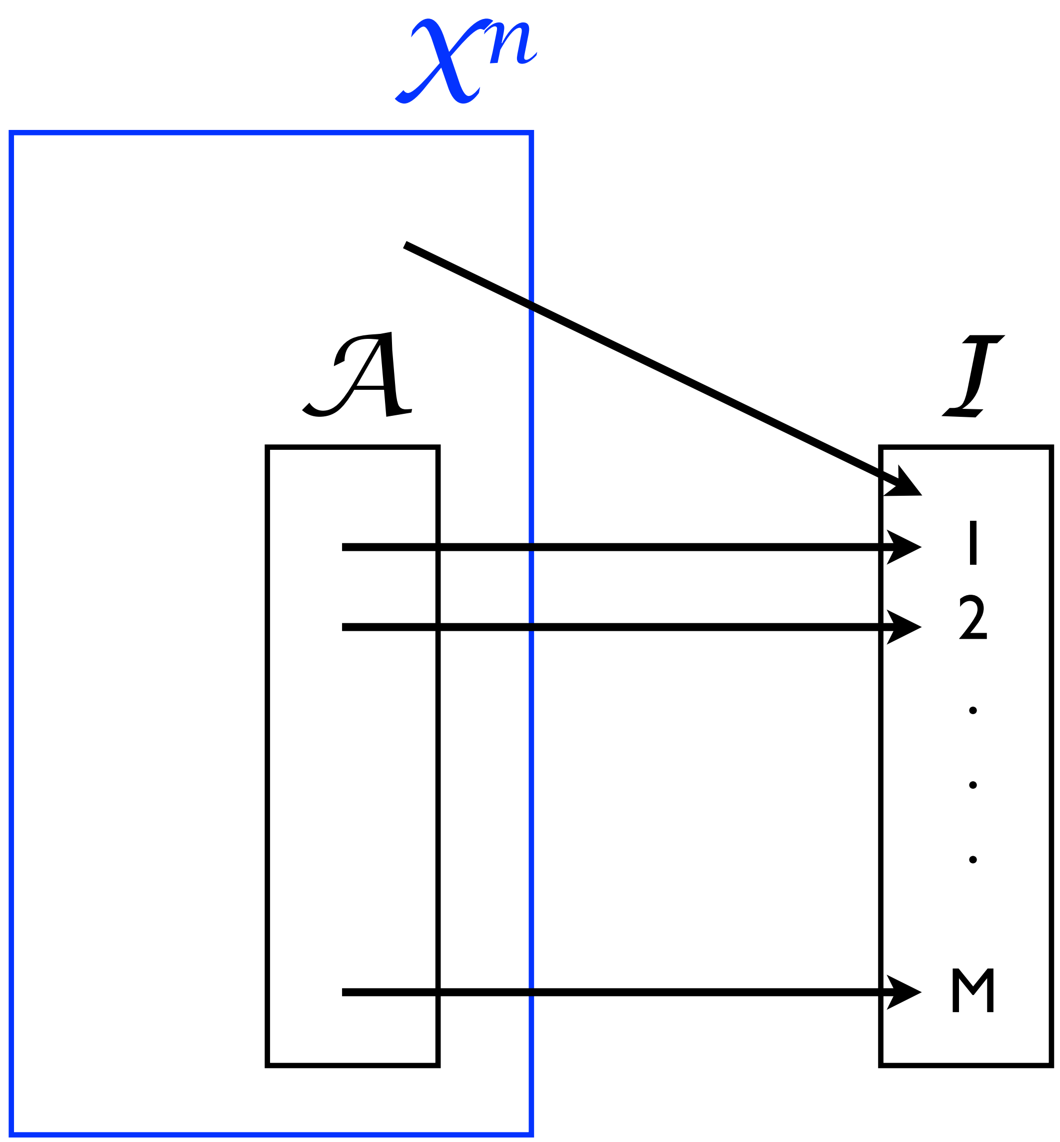
4. Thus $P_e = \Pr\{\mathbf{X} \notin \mathcal{A}\}$.



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For arbitrarily small P_e, there exists a block code whose coding rate is arbitrarily close to $H(X)$ when n is sufficiently large.

Proof

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1) If $\mathbf{x} \in W^n_{[X]\epsilon}$, then

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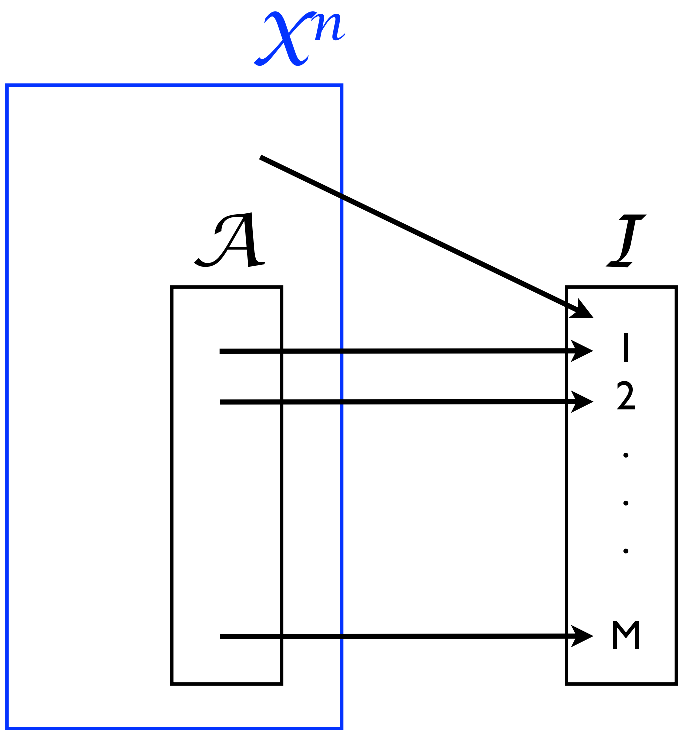

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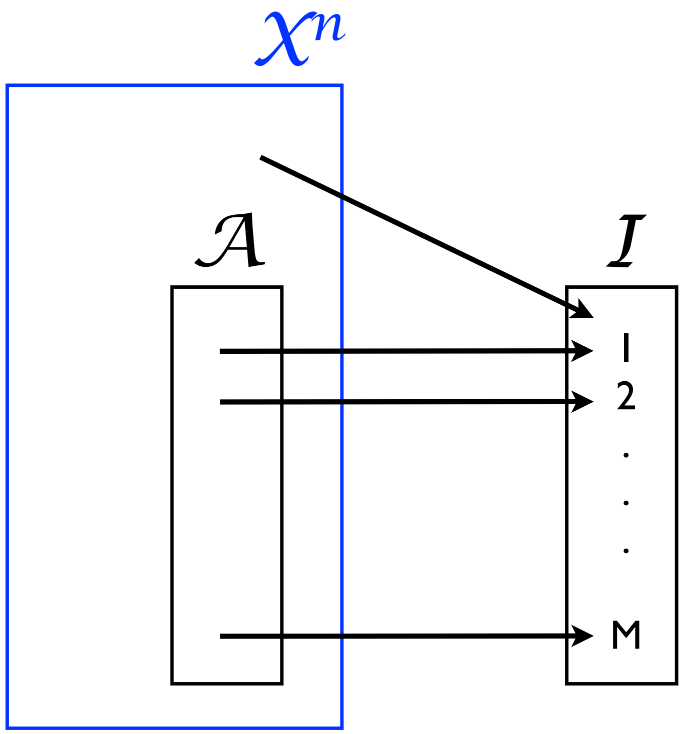

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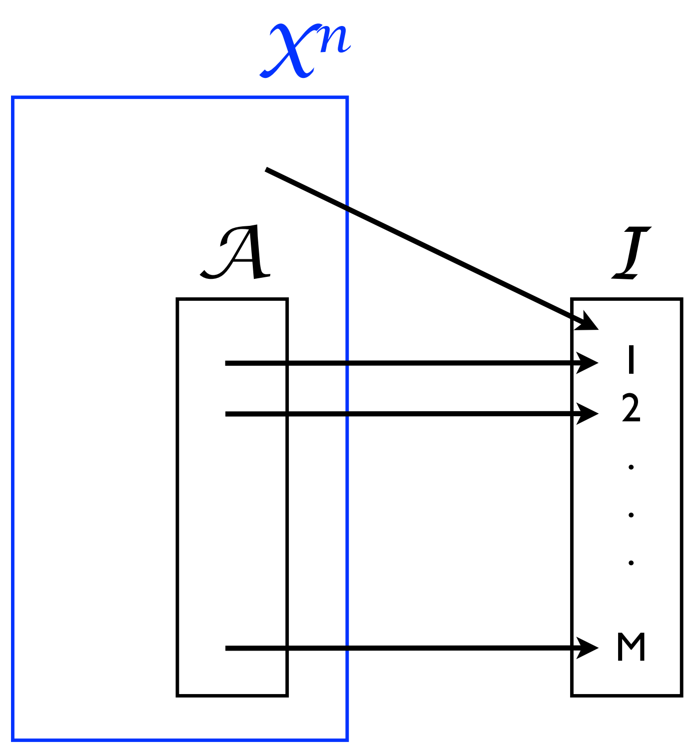

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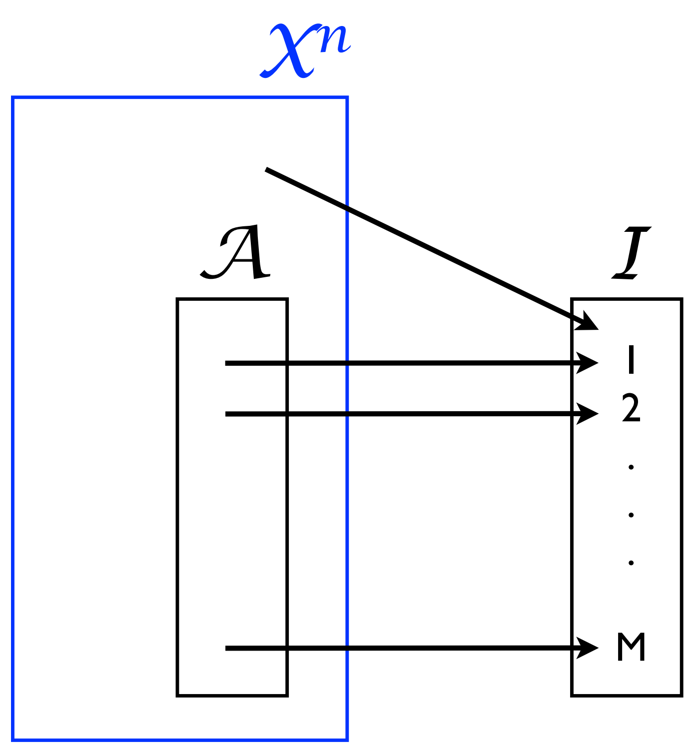

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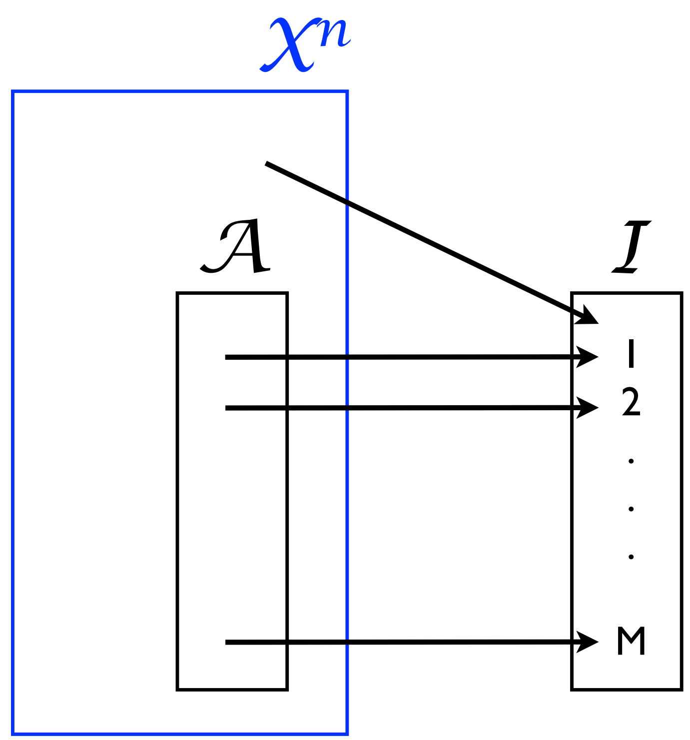

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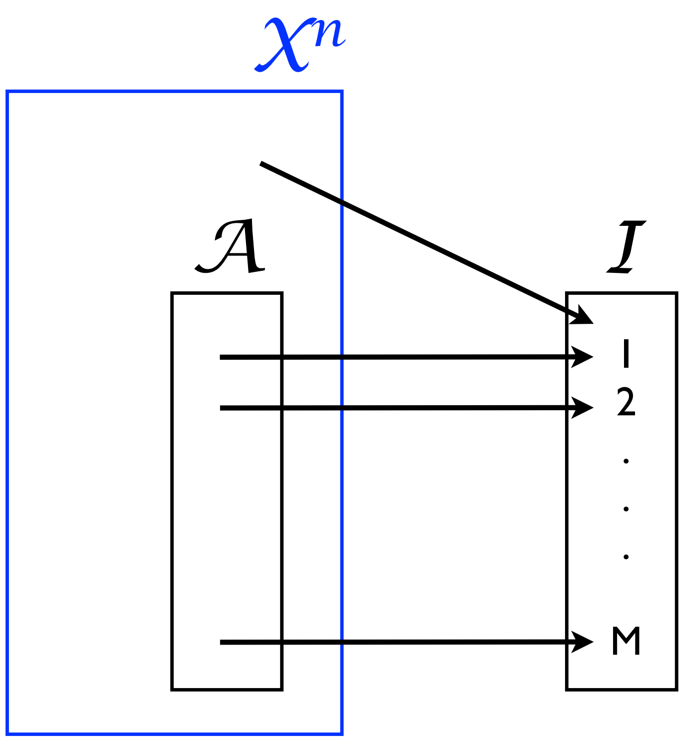

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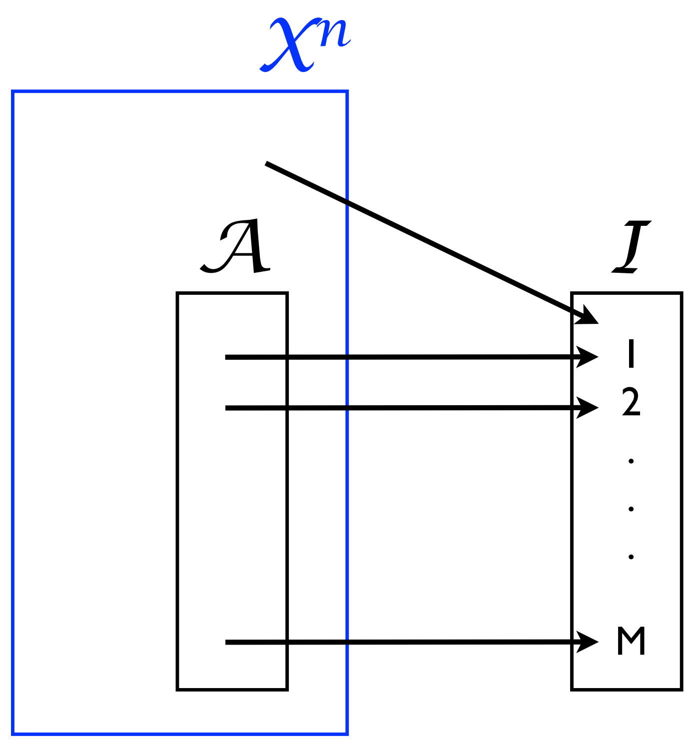

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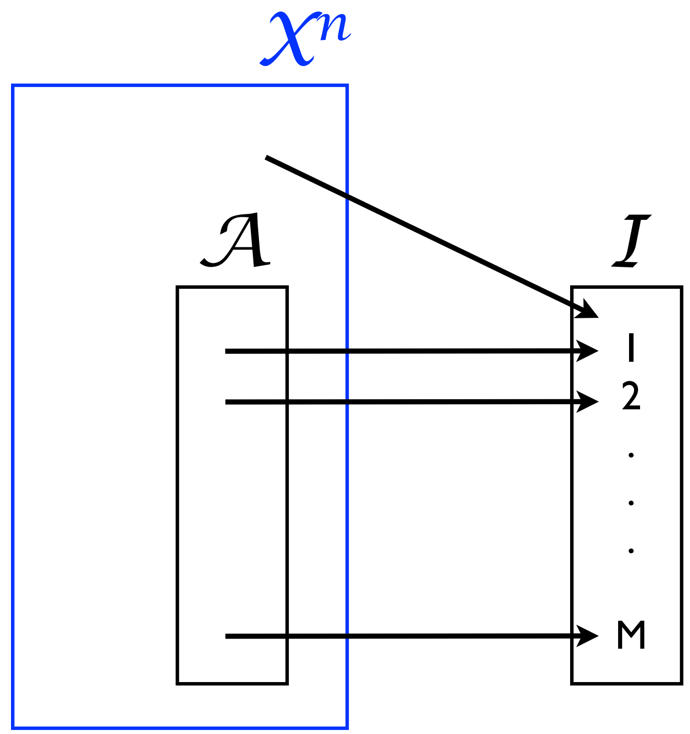

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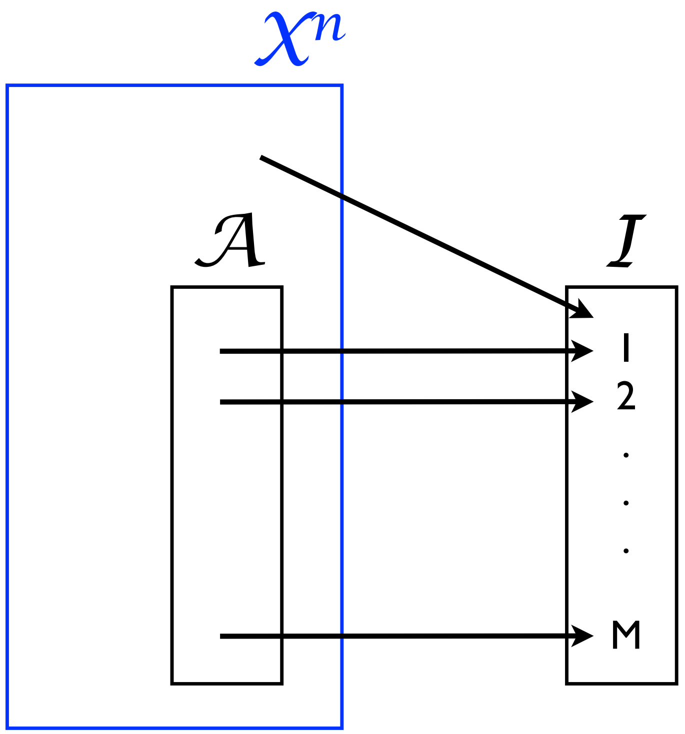

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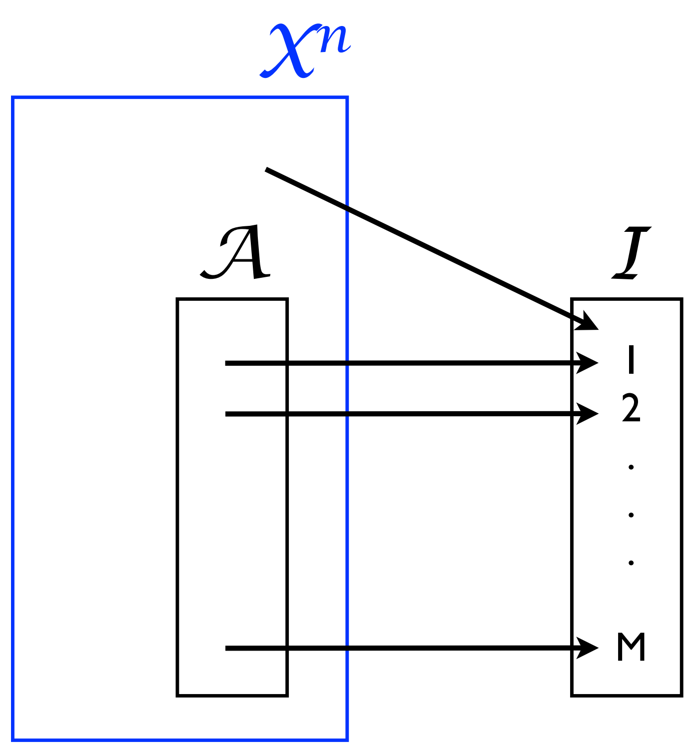

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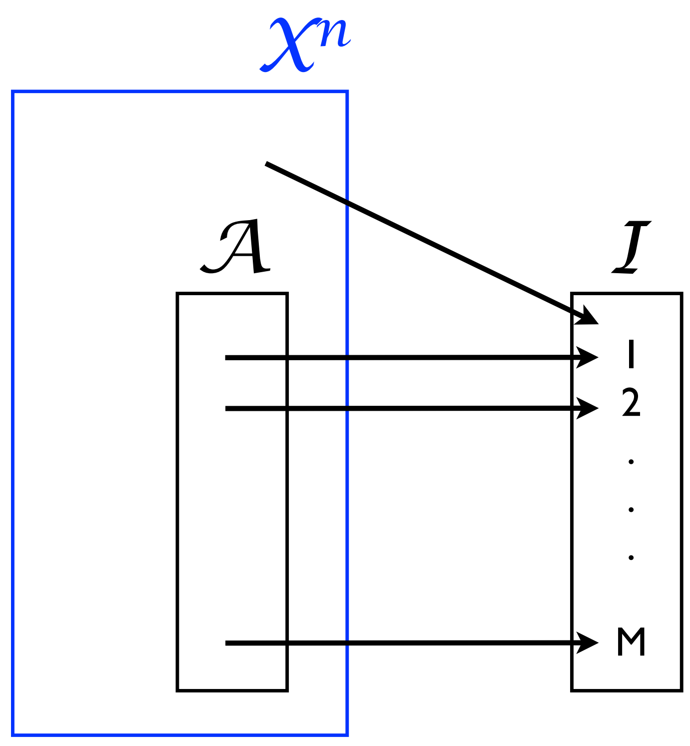

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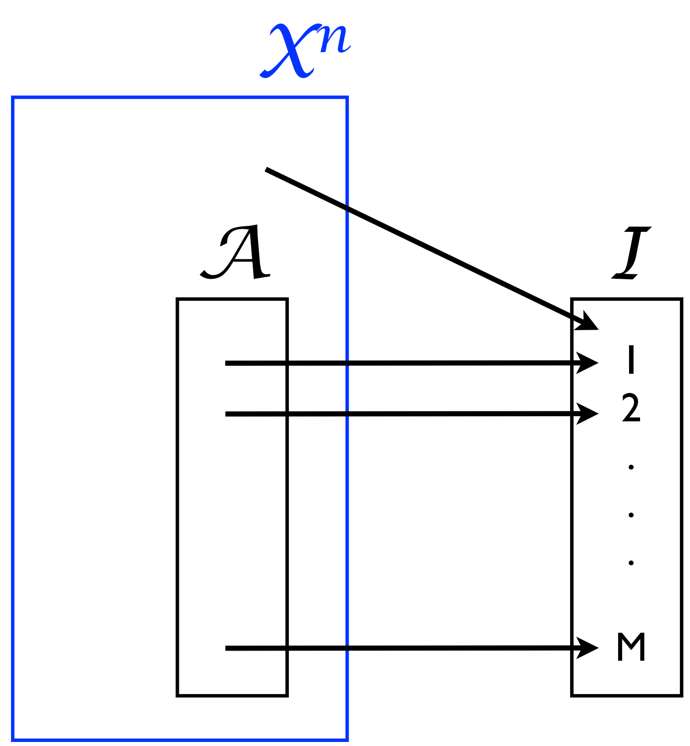

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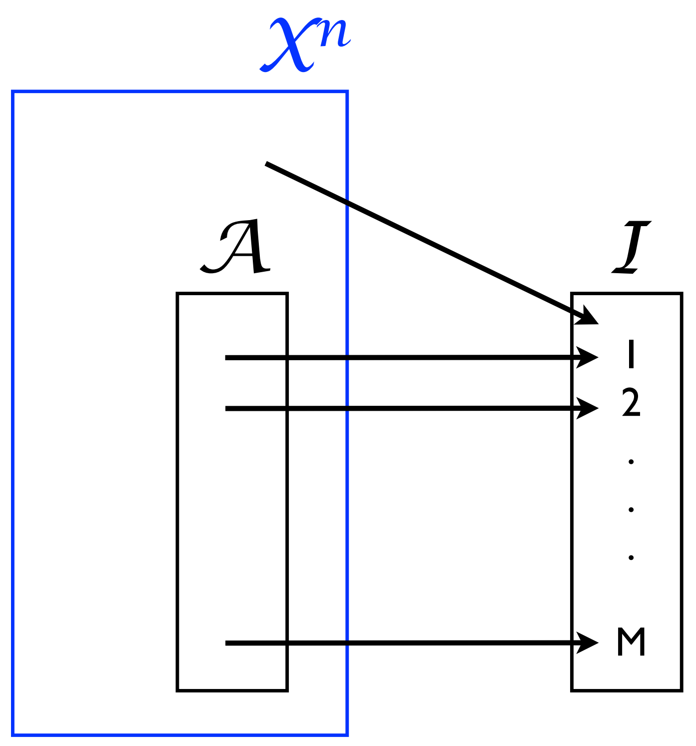

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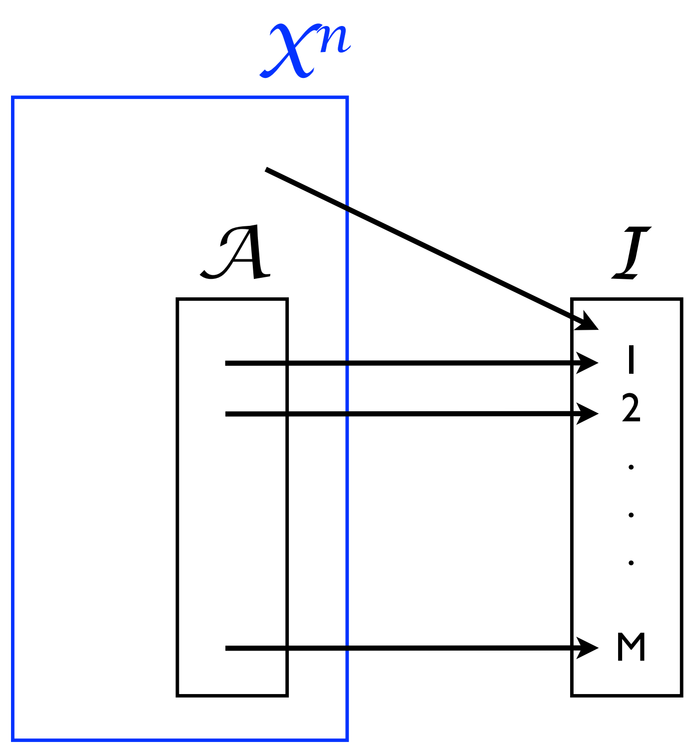

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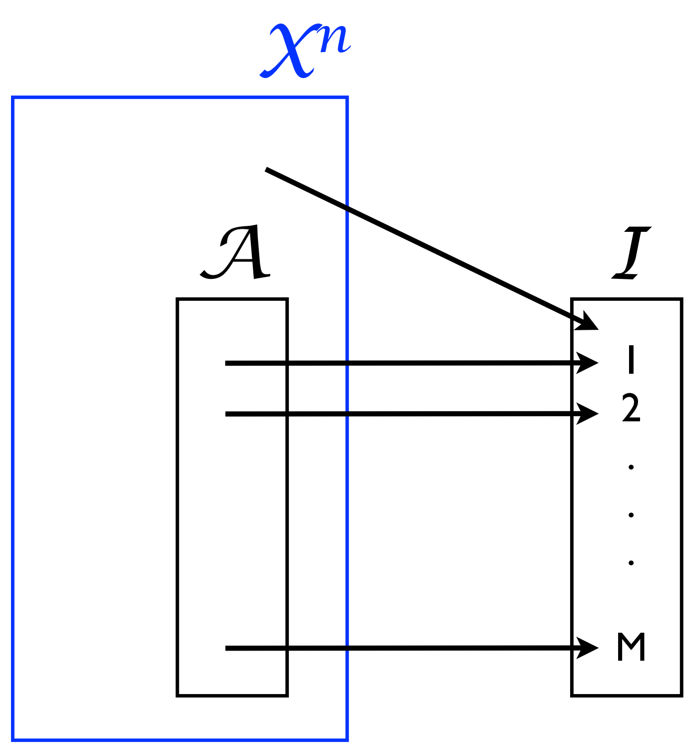

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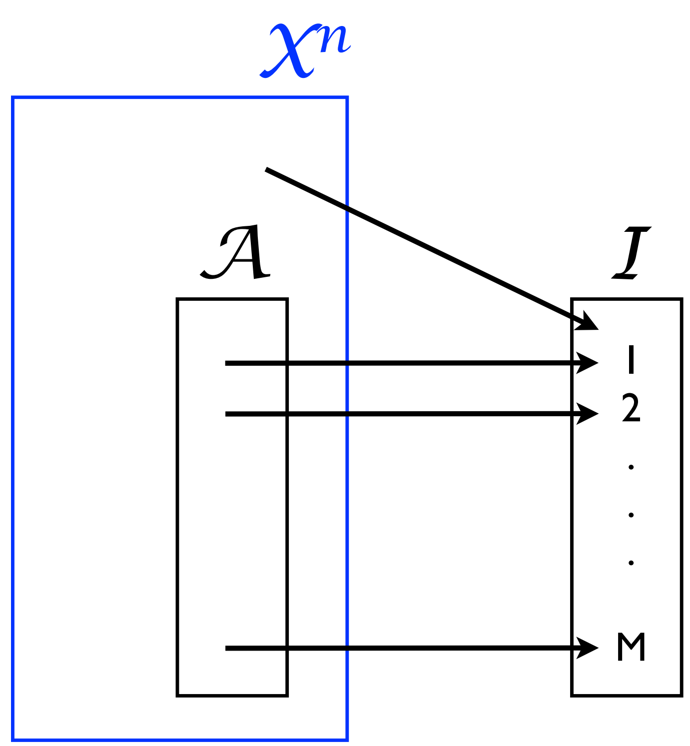

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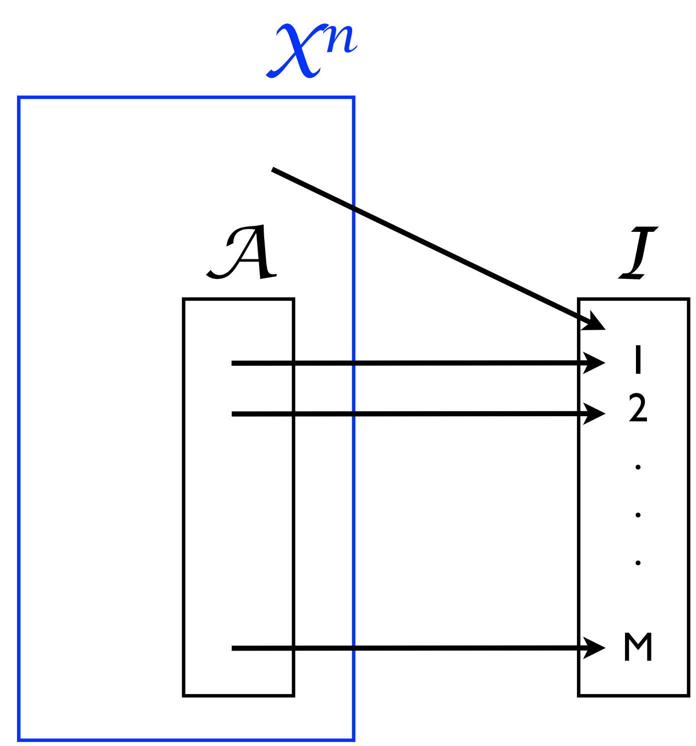

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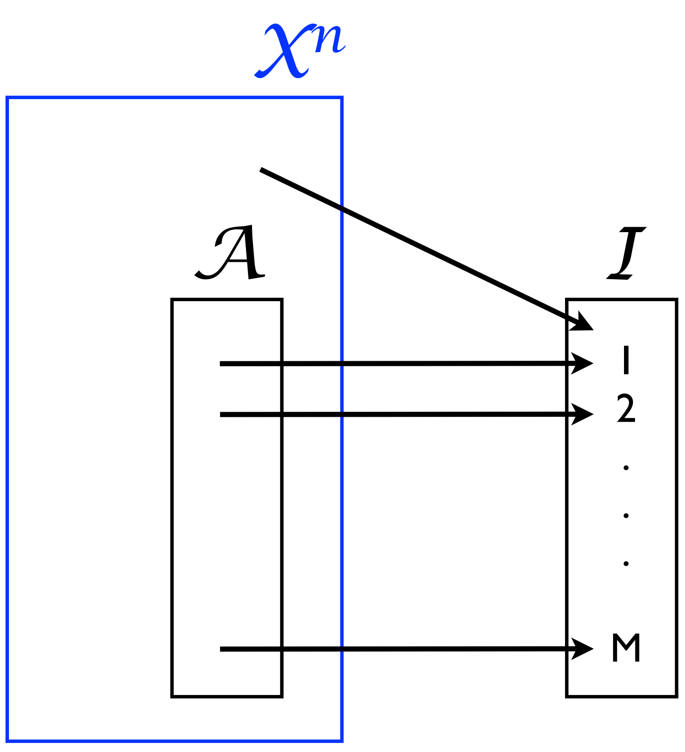

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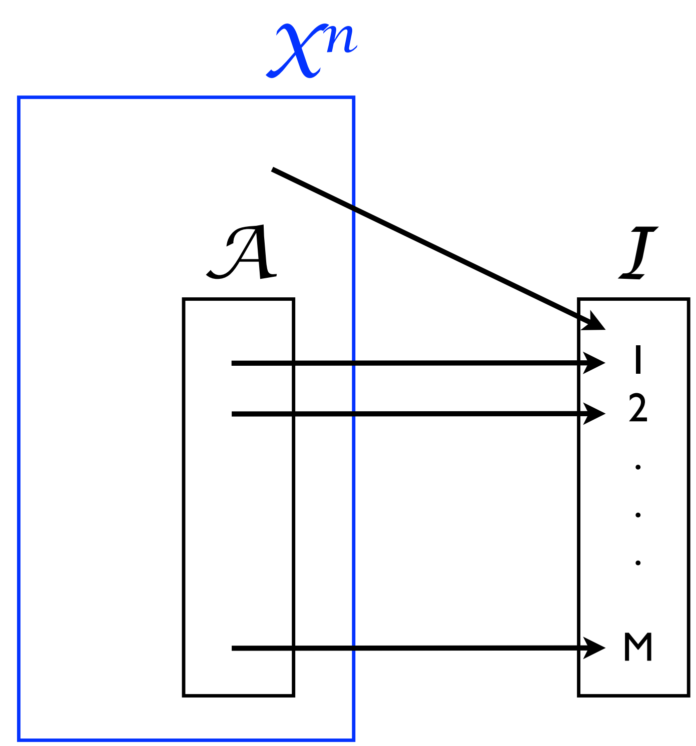

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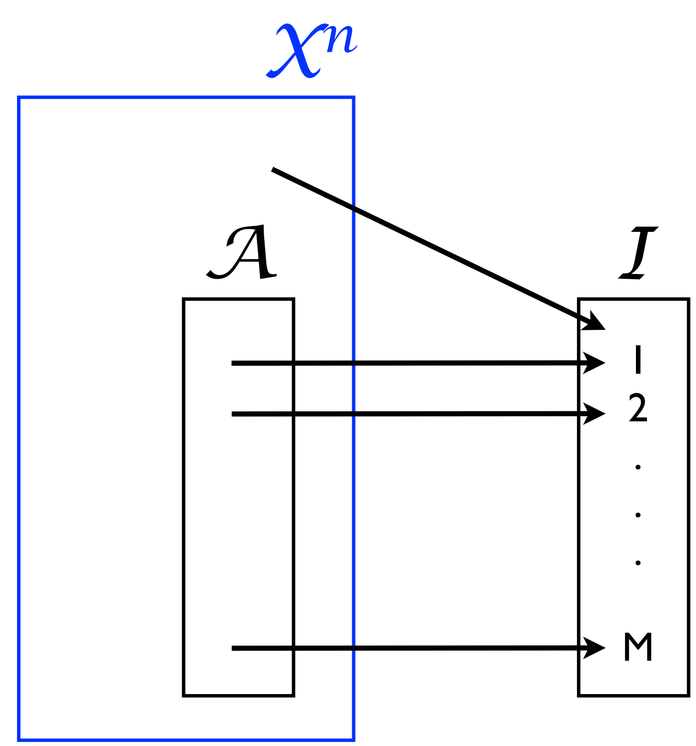

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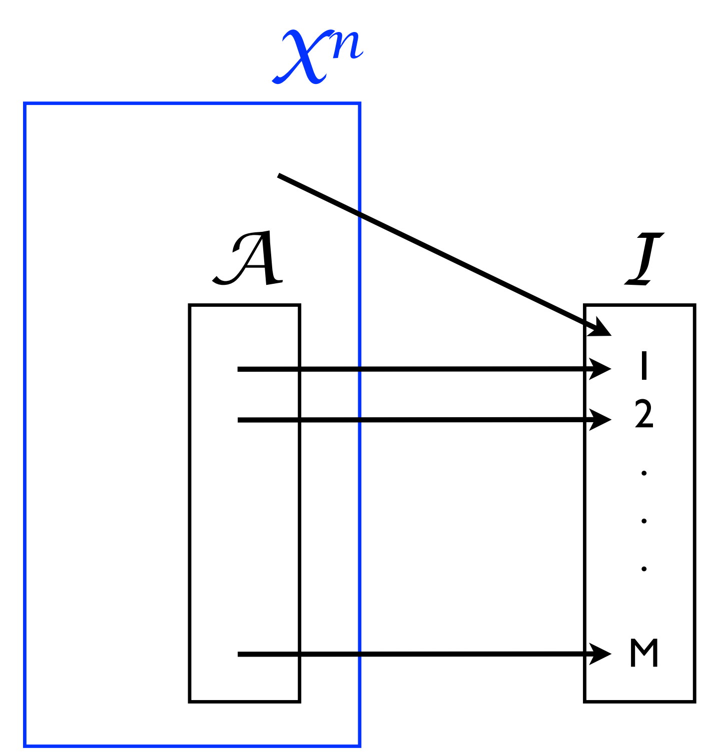

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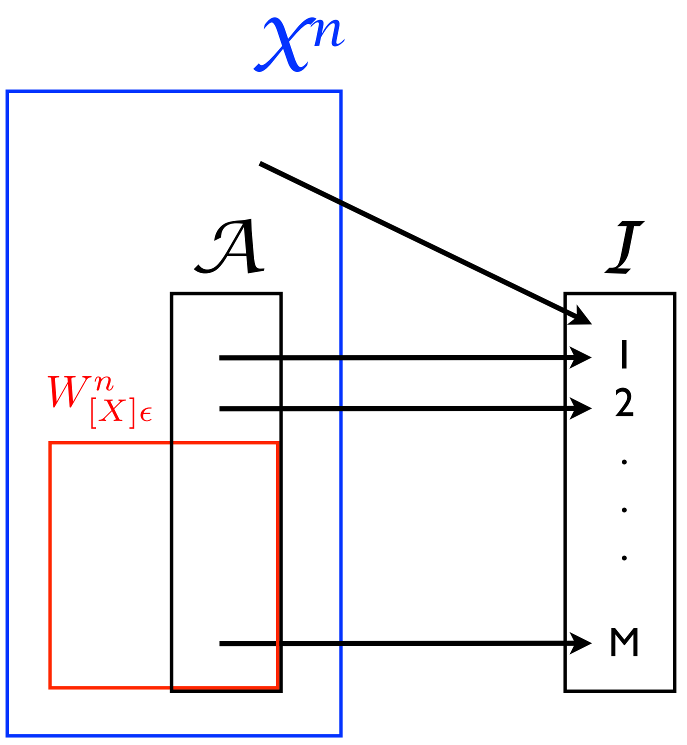

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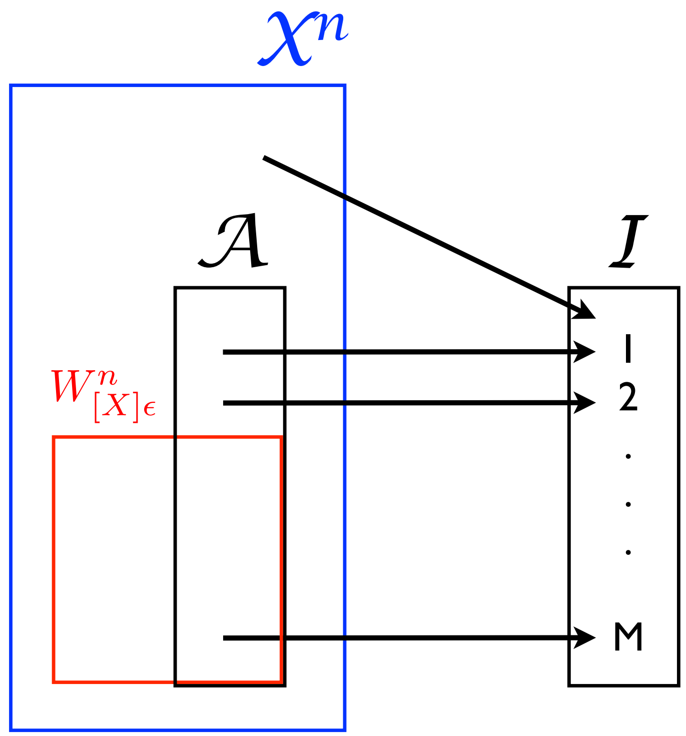

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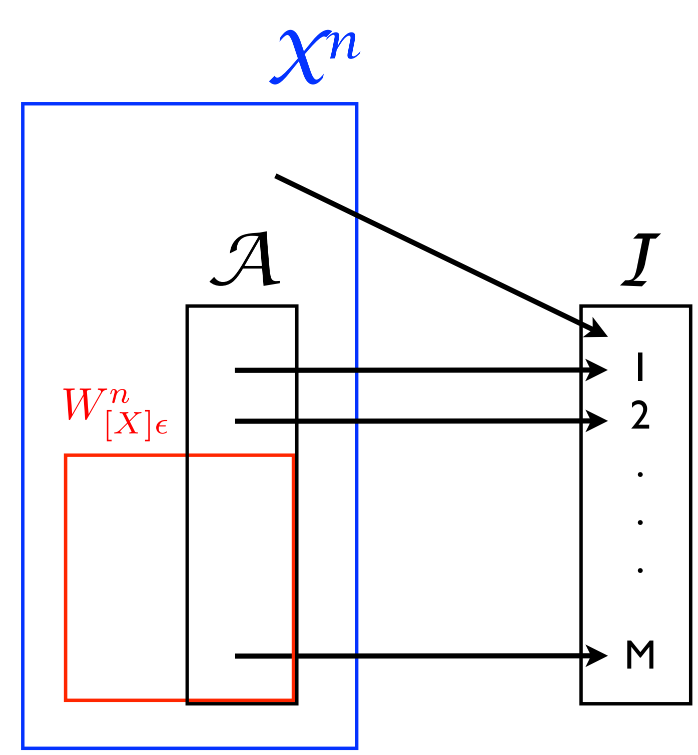

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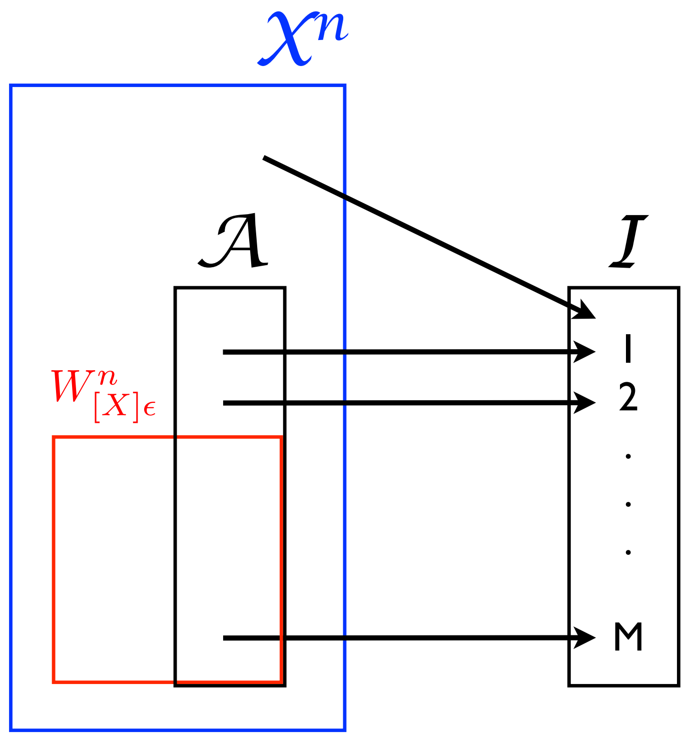

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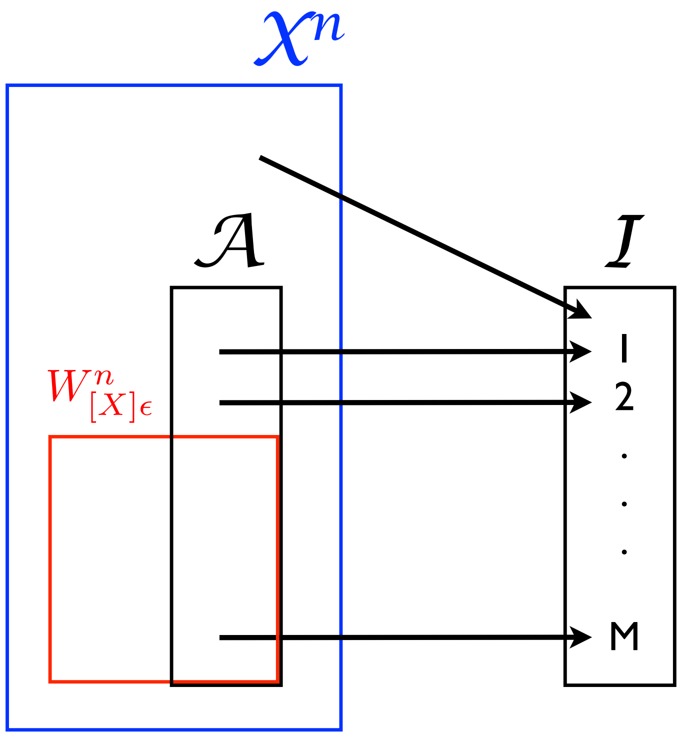

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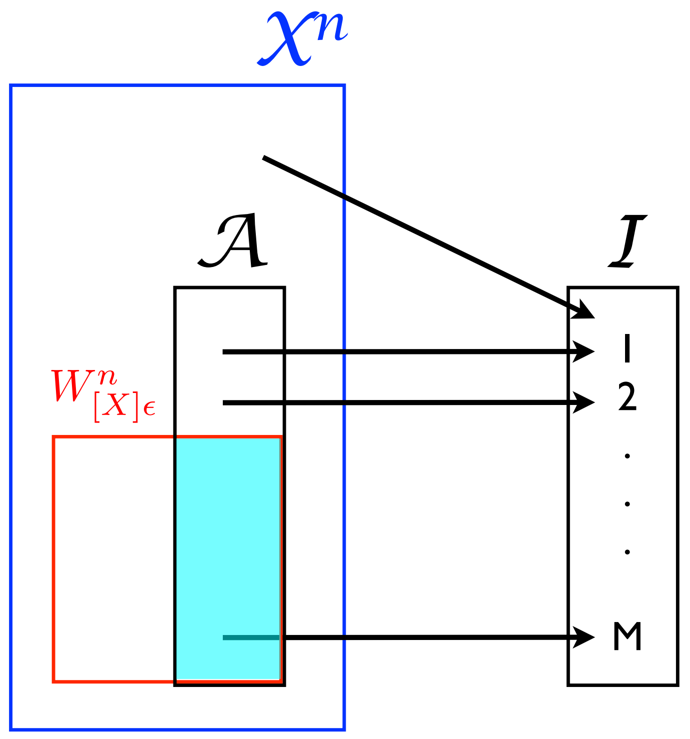

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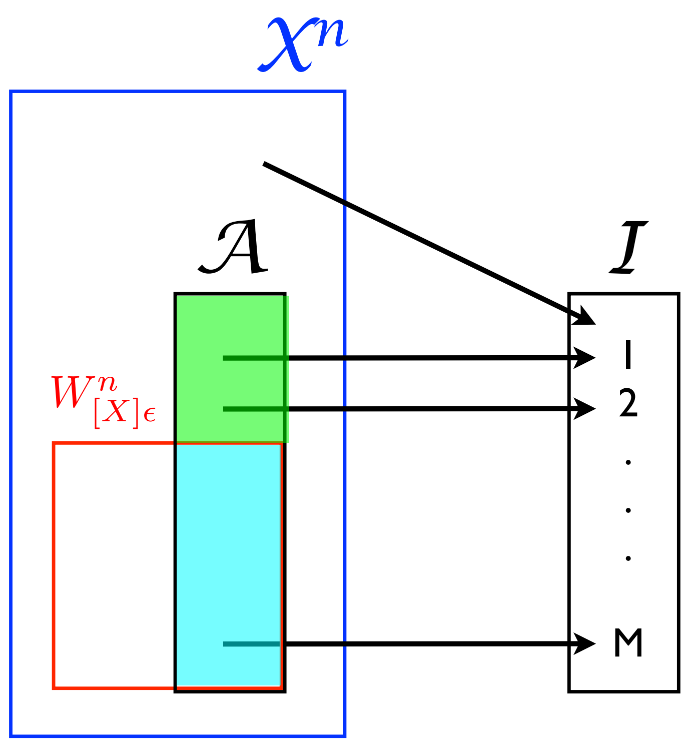

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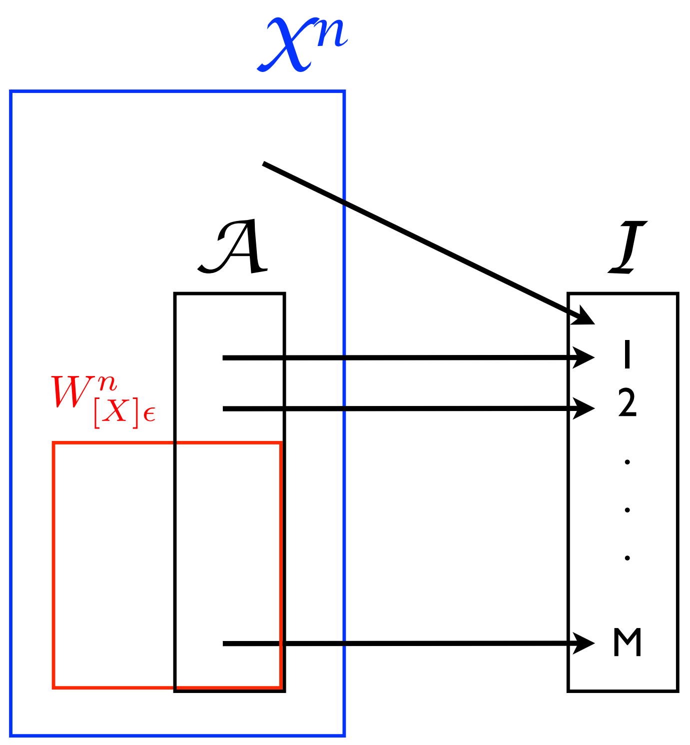

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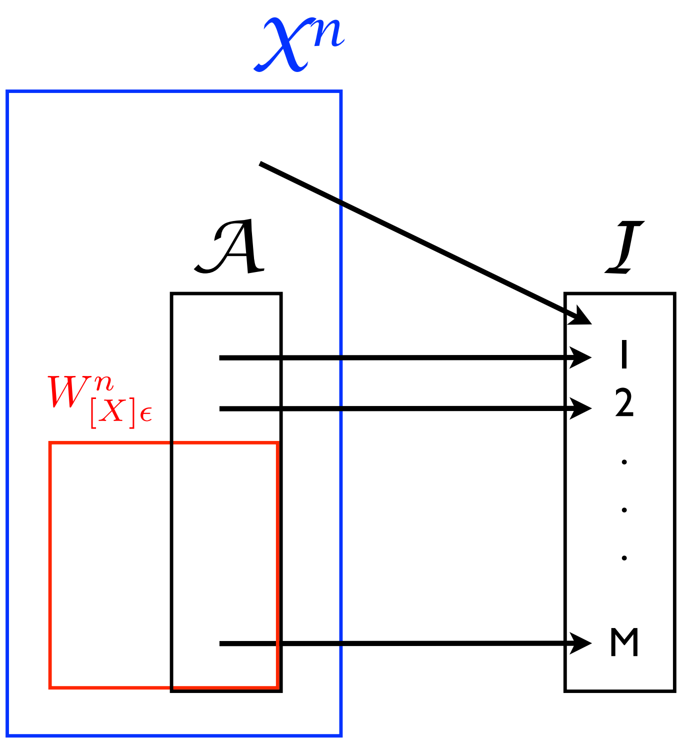

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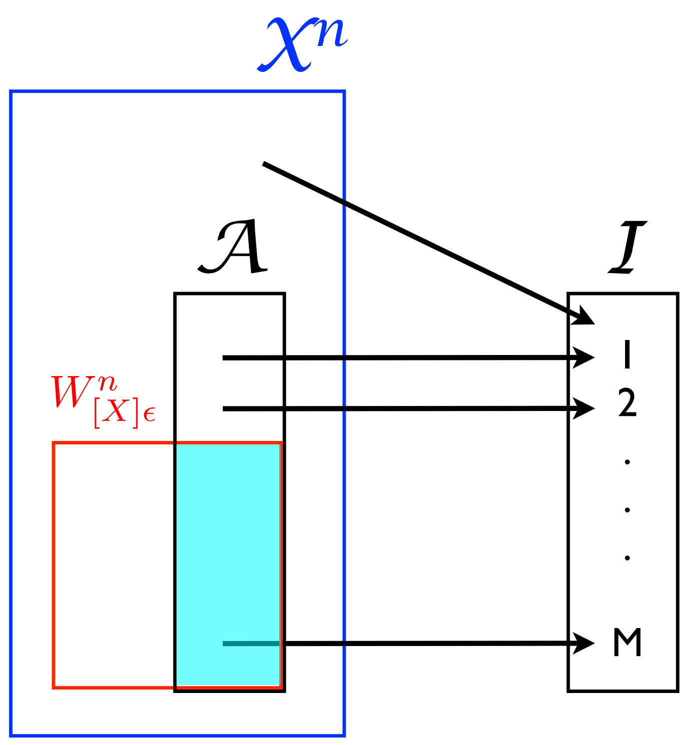

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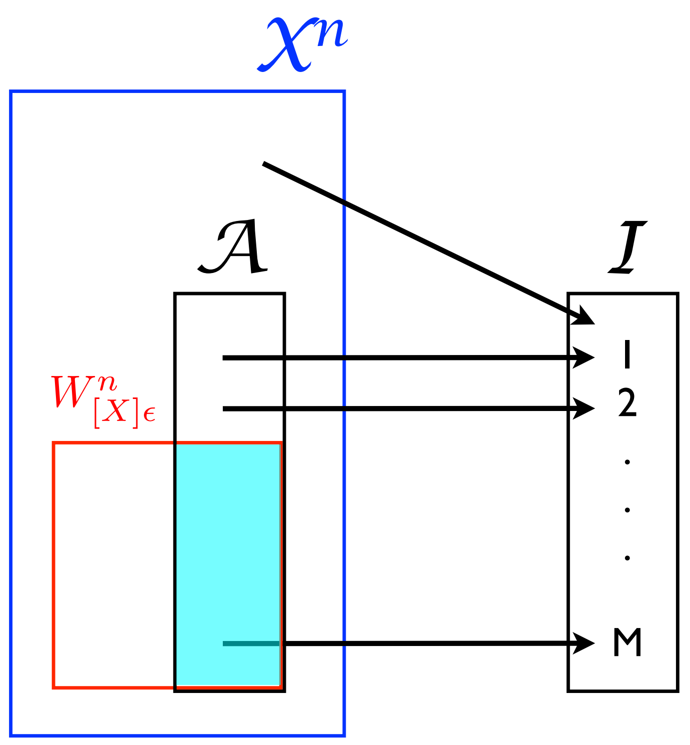

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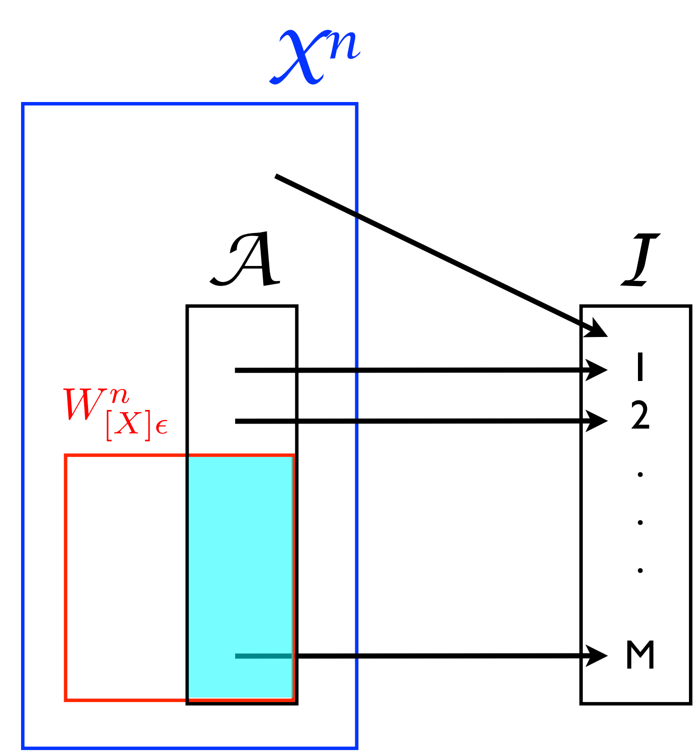

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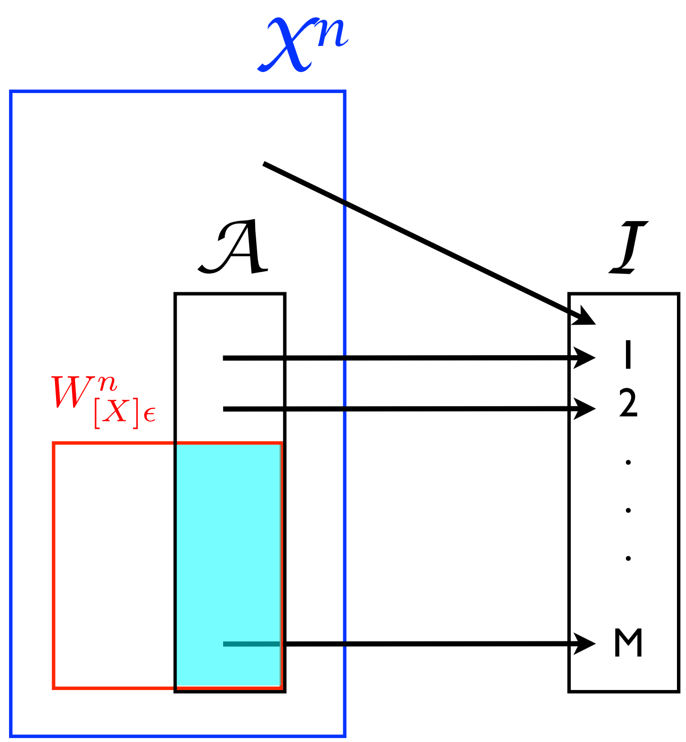

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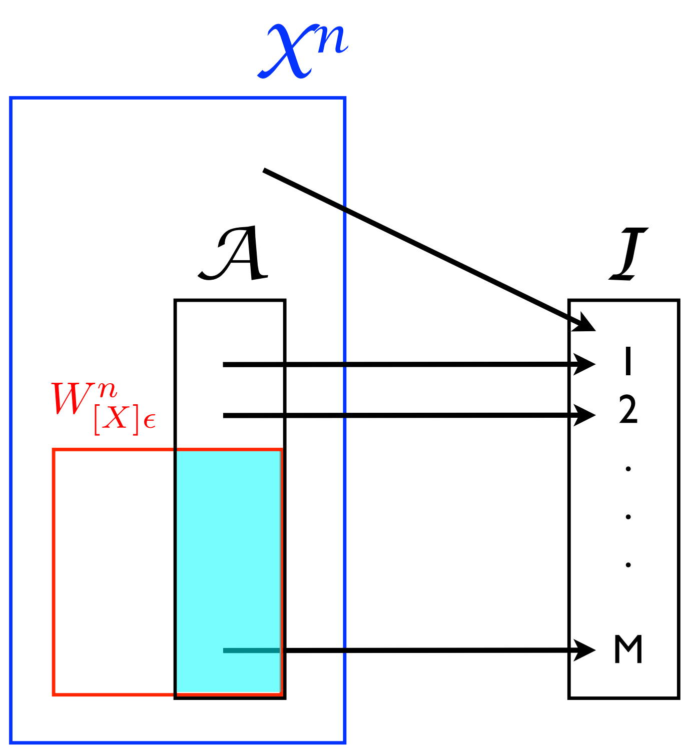

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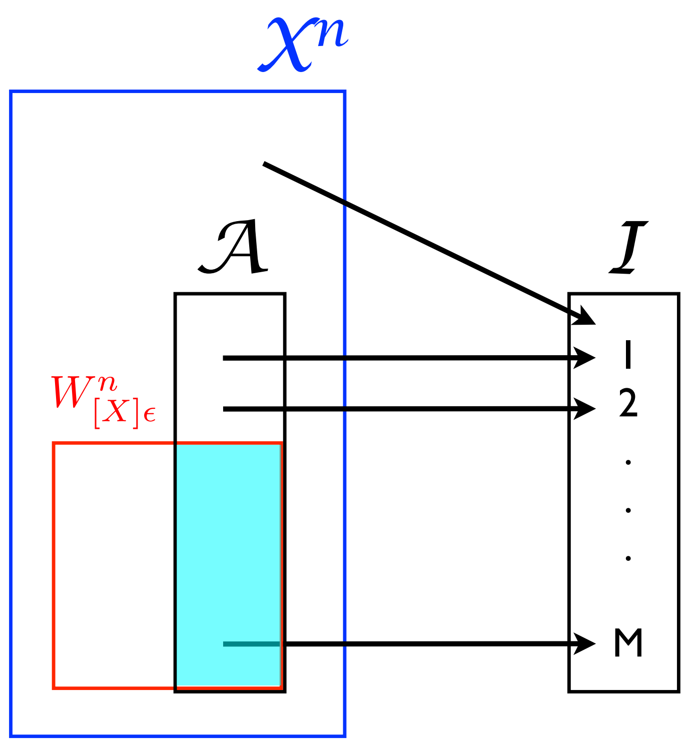

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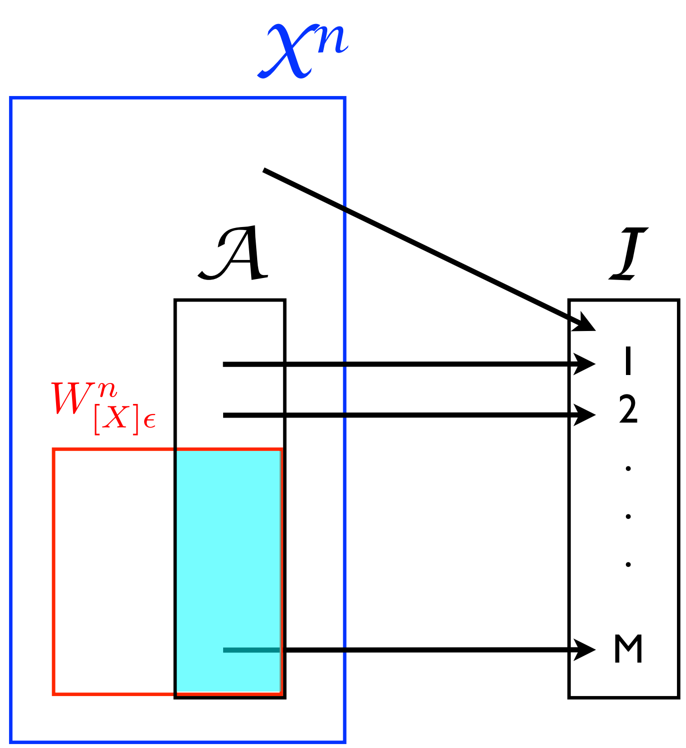

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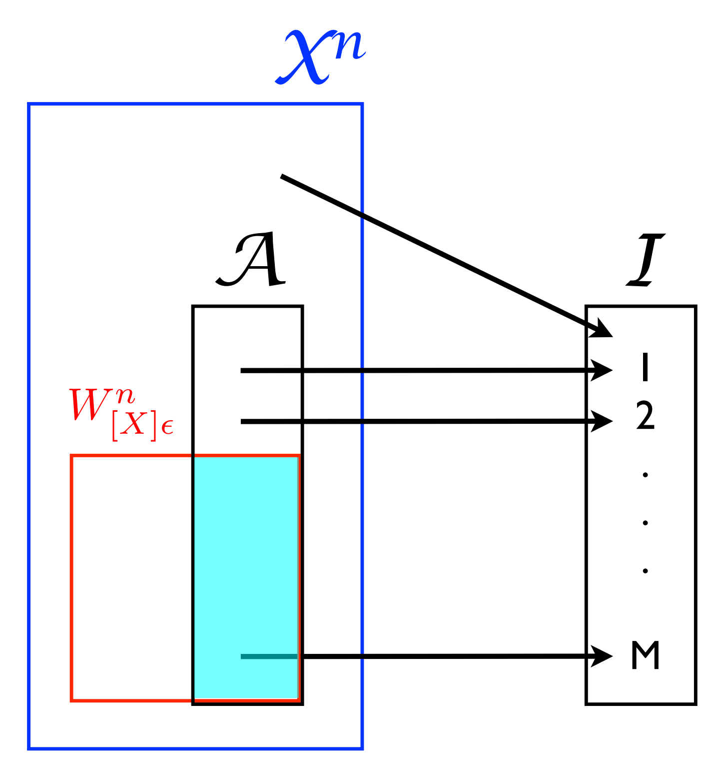

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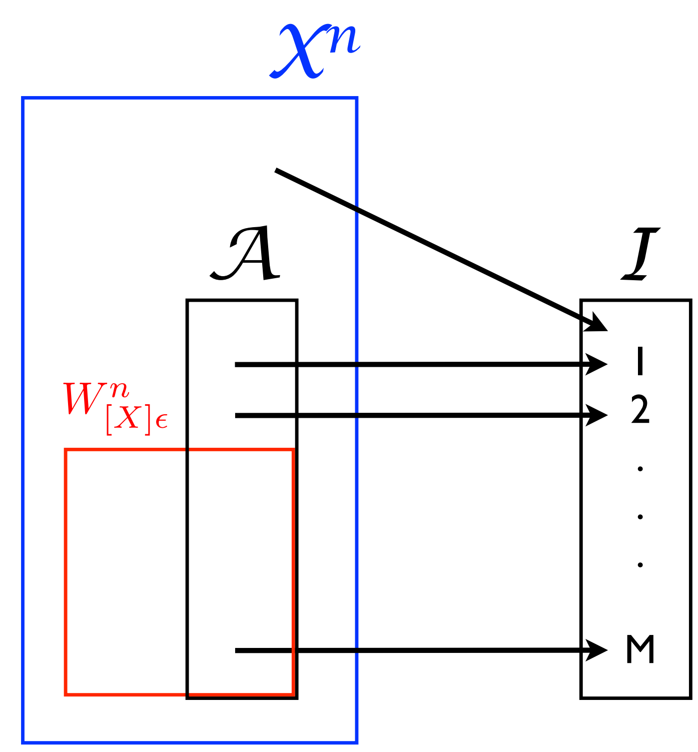

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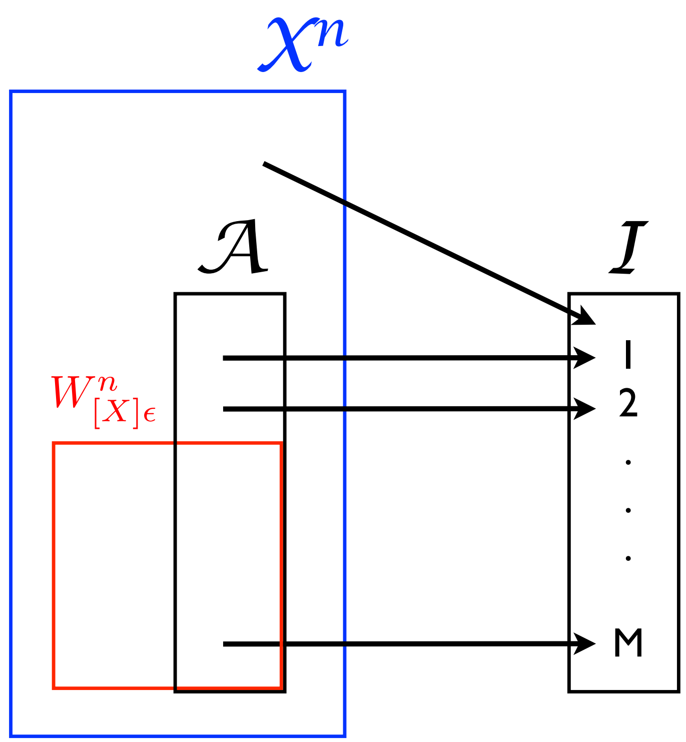

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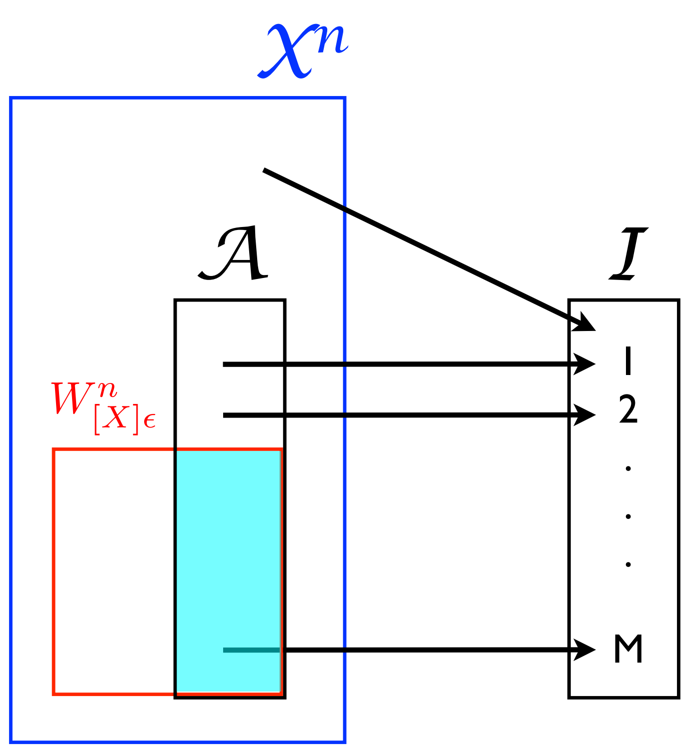

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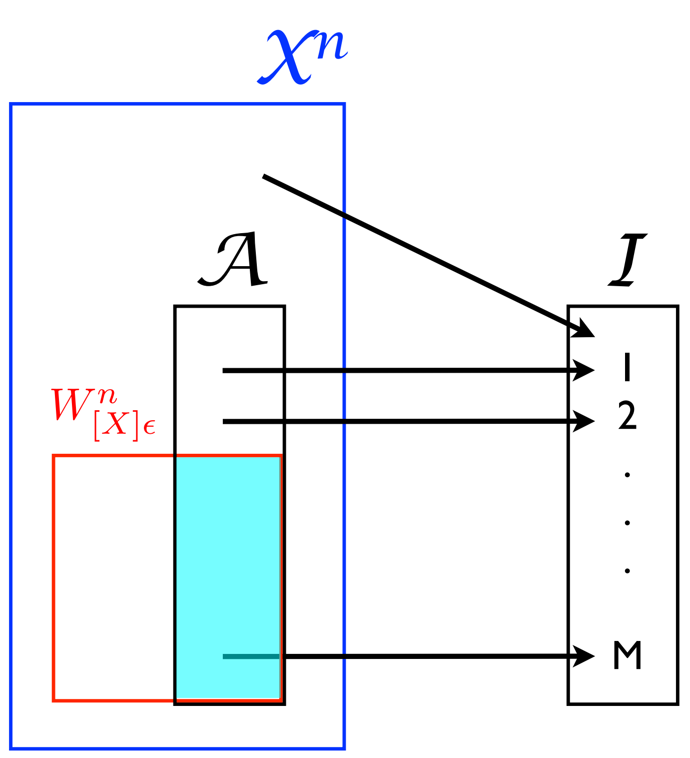

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1) If $\mathbf{x} \in W^n_{[X]\epsilon}$, then

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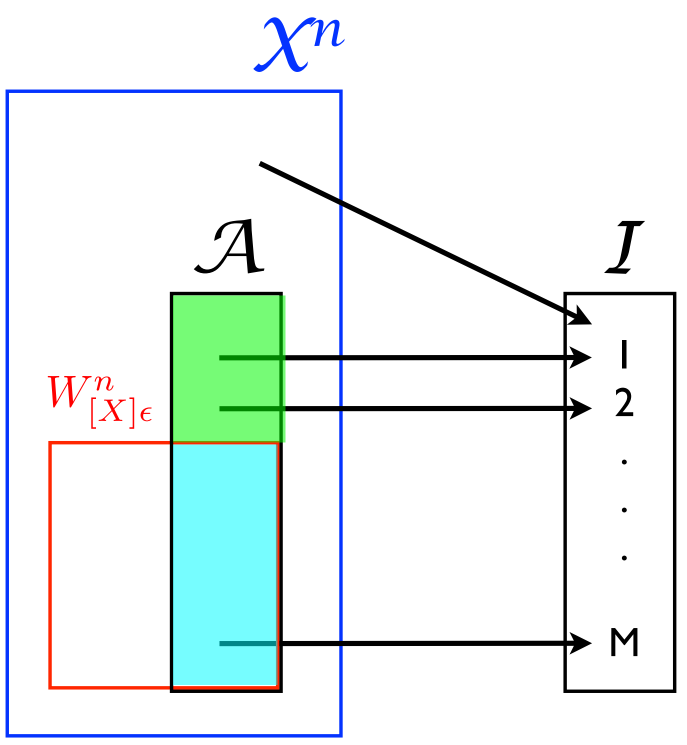

Source Coding Theorem (Converse)
For any block code with block length n and coding rate less than $H(X) - \zeta$, where $\zeta > 0$ does not change with n, then $P_e \to 1$ as $n \to \infty$.

Proof

1. Consider any block code whose rate is less than $H(X) - \zeta$, i.e.,

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where $\zeta > 0$ does not change with n. Then total number of codewords

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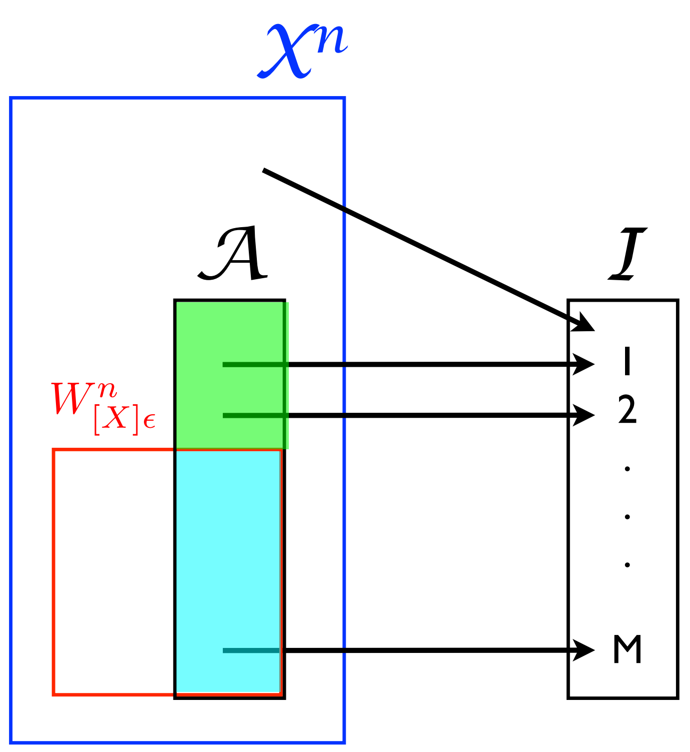

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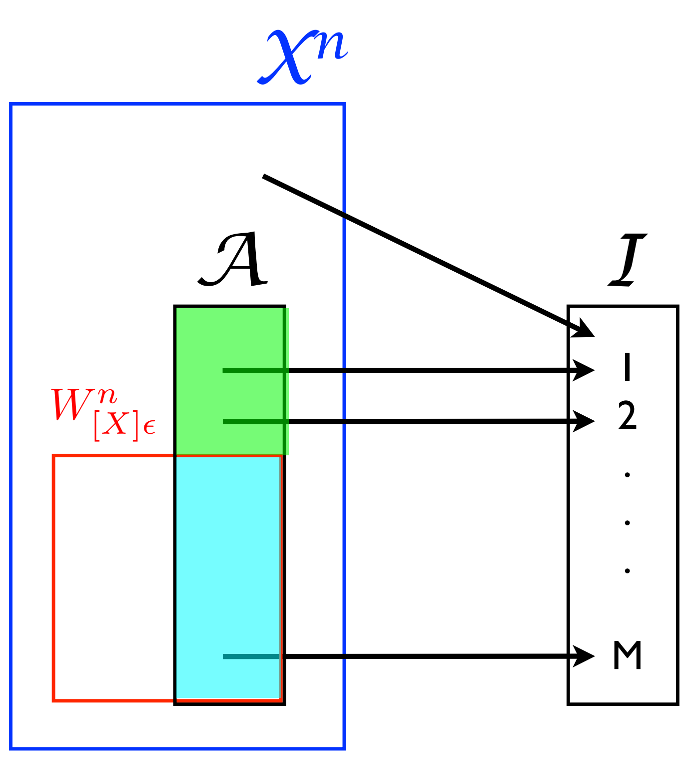

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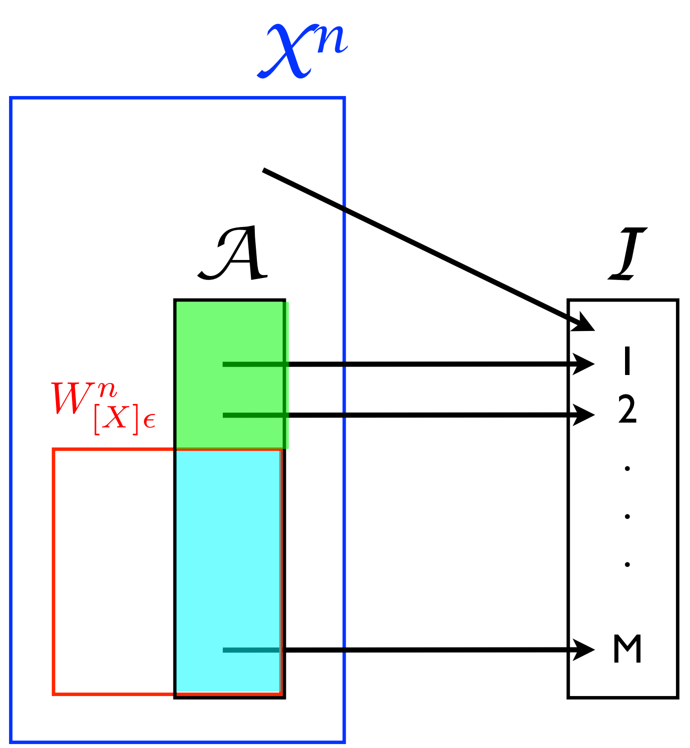

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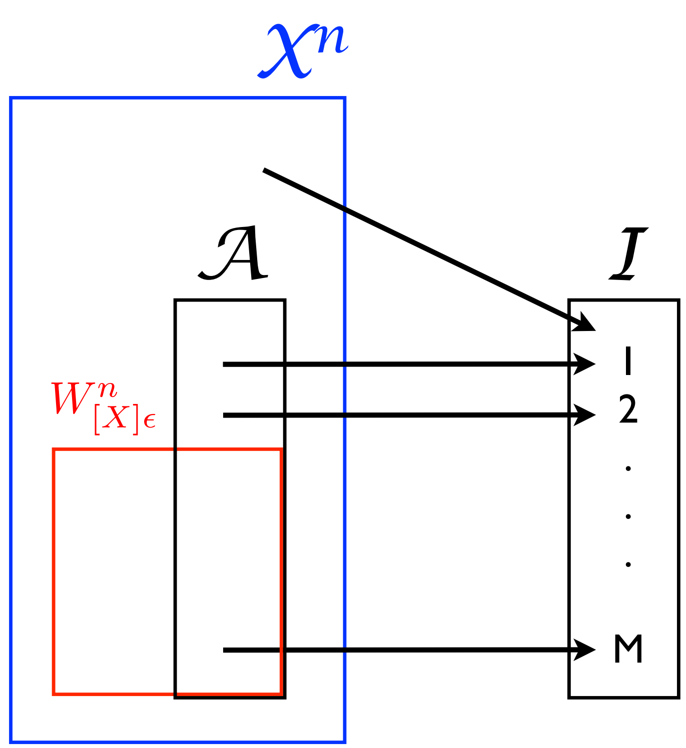

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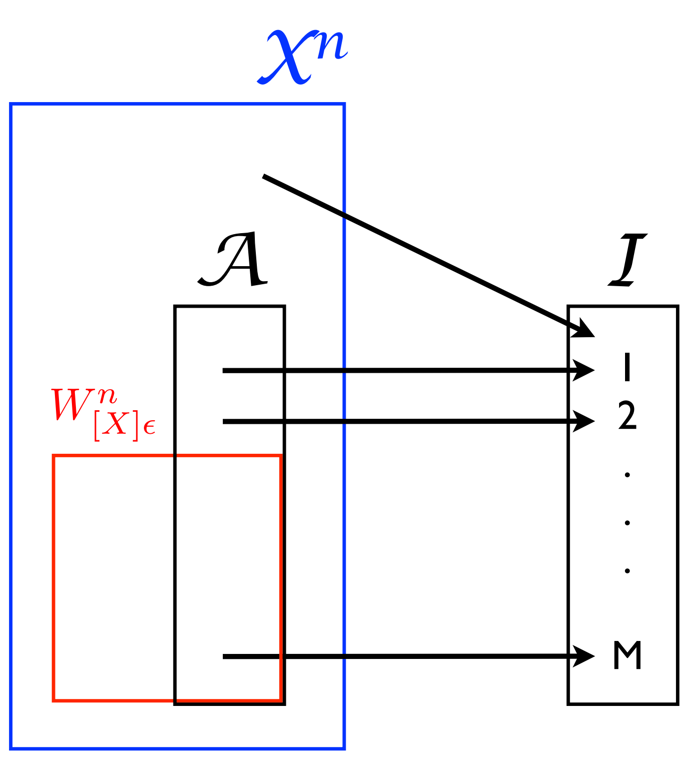

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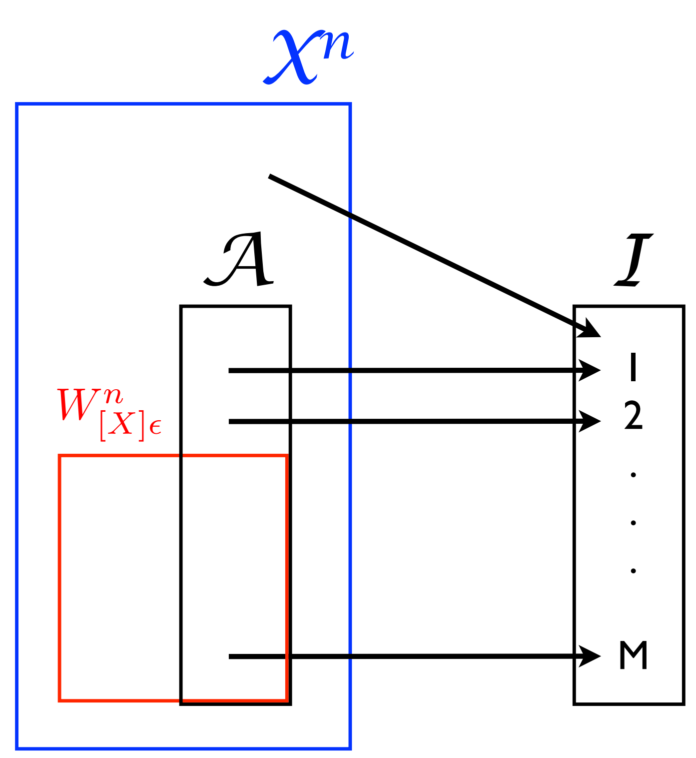

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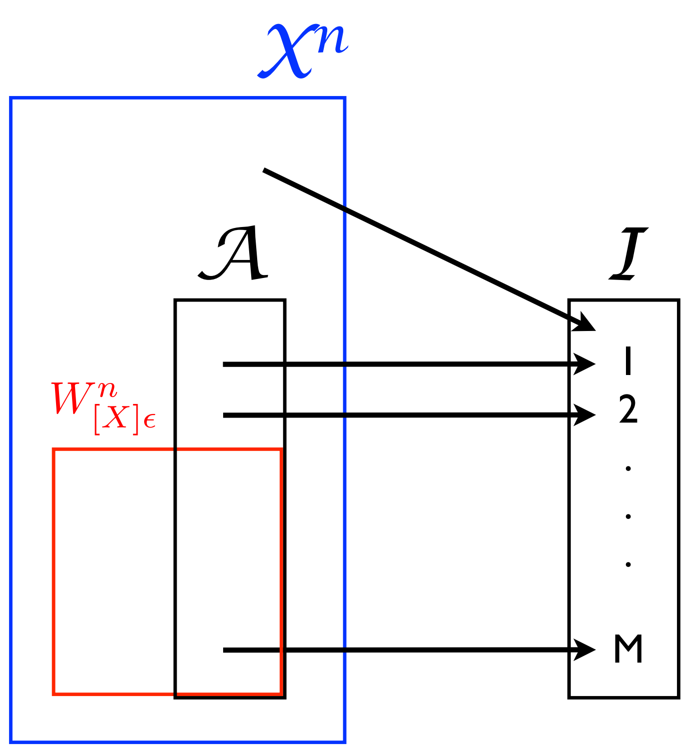

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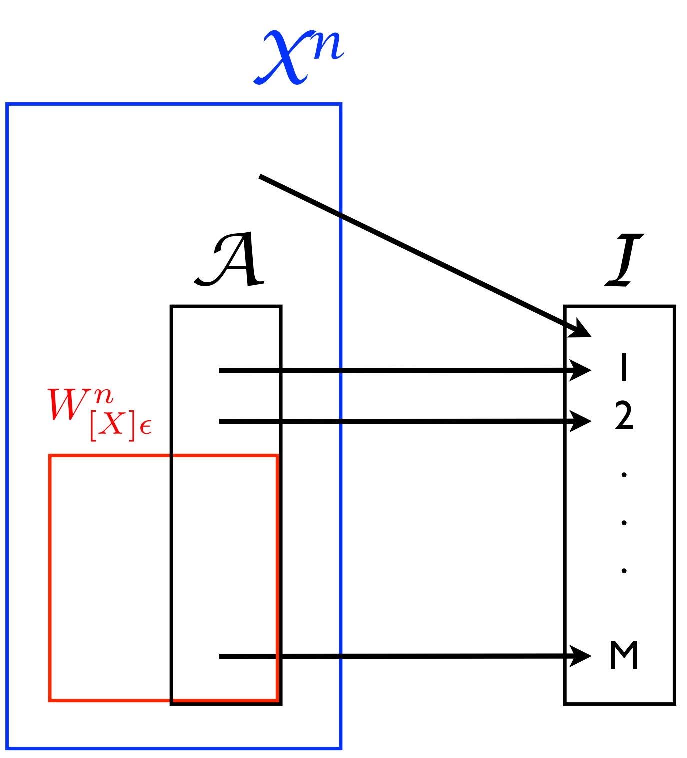

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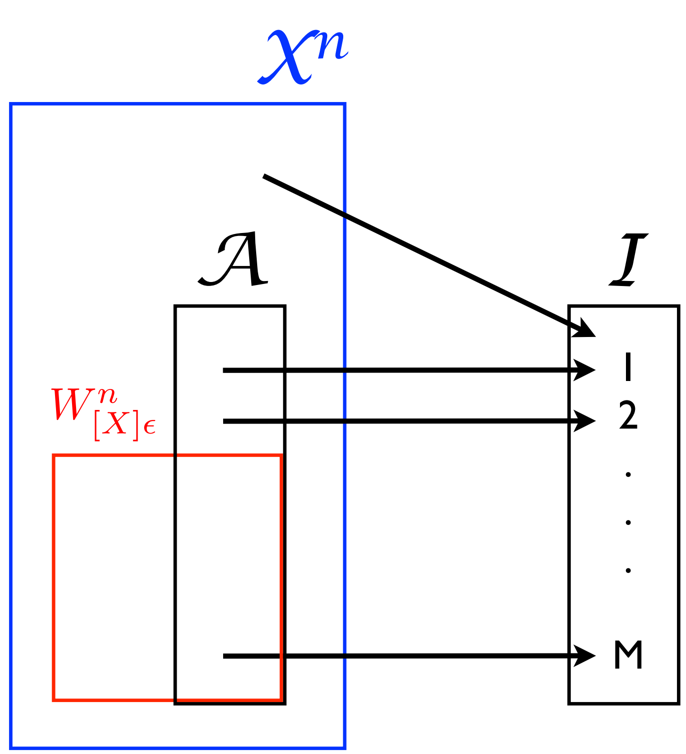

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$< \epsilon$

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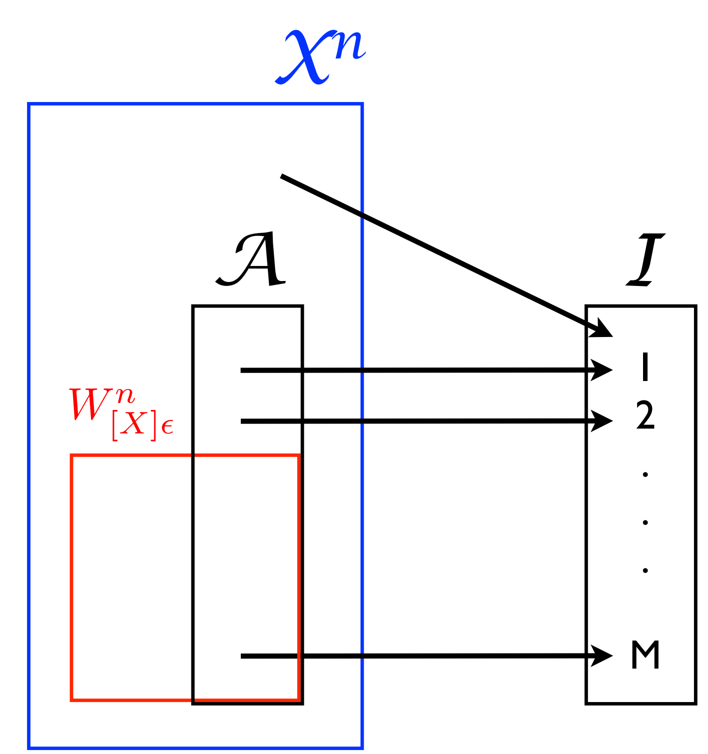

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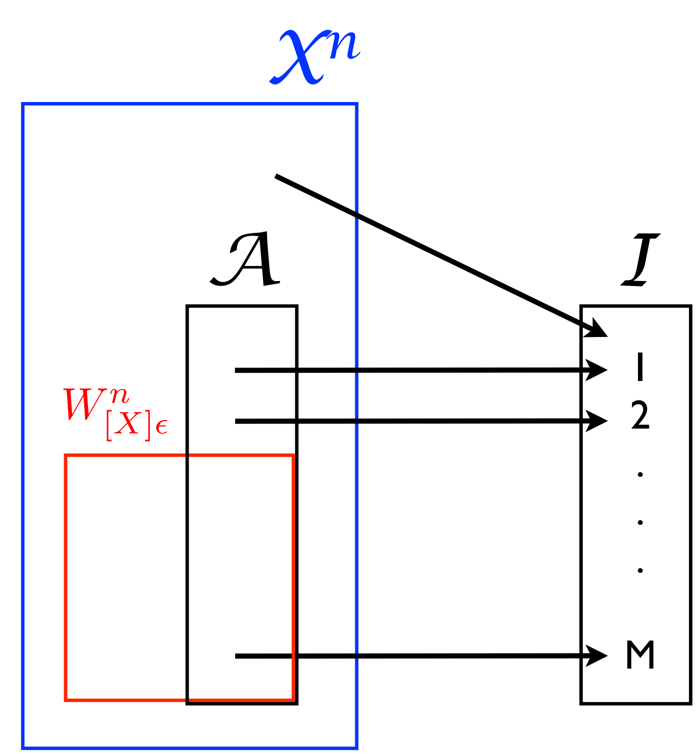

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7. Finally, let $\epsilon \to 0$.

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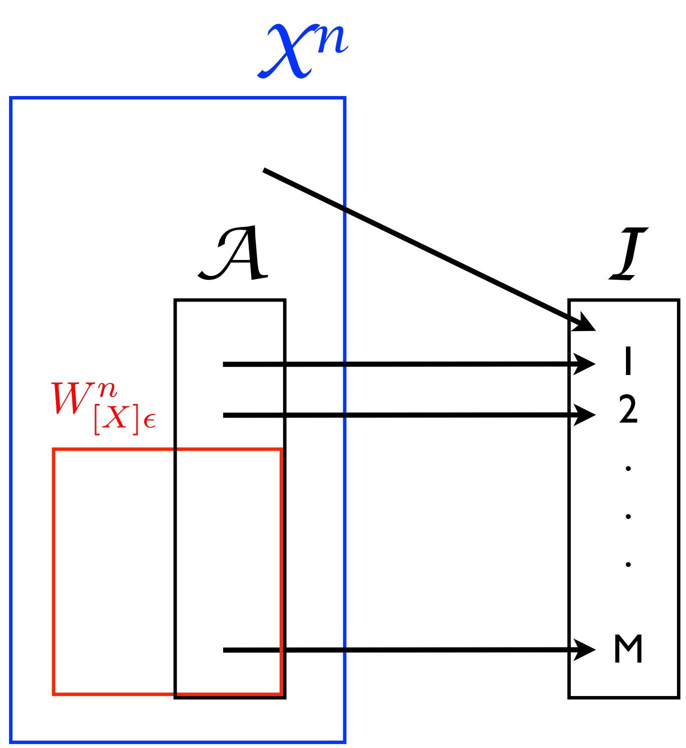

Source Coding Theorem (Converse)
For any block code with block length n and coding rate less than $H(X)-\zeta$, where $\zeta > 0$ does not change with n, then $P_e \to 1$ as $n \to \infty$.

Proof

1. Consider any block code whose rate is less than $H(X)-\zeta$, i.e.,

$$\frac{1}{n}\log M < H(X) - \zeta,$$

where $\zeta > 0$ does not change with n. Then total number of codewords

$$M \leq 2^{n(H(X)-\zeta)}.$$

2. In general, some of the indices in $\mathcal{I}$ cover $\mathbf{x} \in W^n_{[X]\epsilon}$, while the others cover $\mathbf{x} \notin W^n_{[X]\epsilon}$.

3. By WAEP, the total probability of typical sequences covered is upper bounded by

$$2^{n(H(X)-\zeta)}2^{-n(H(X)-\epsilon)} = 2^{-n(\zeta-\epsilon)}.$$

4. By WAEP, the total probability covered by $\mathcal{I}$, i.e., $\Pr\{\mathbf{X} \in \mathcal{A}\}$, is upper bounded by

$$2^{-n(\zeta-\epsilon)} + \Pr\{\mathbf{X} \notin W^n_{[X]\epsilon}\} < 2^{-n(\zeta-\epsilon)} + \epsilon.$$

5. Then $P_e = \Pr\{\mathbf{X} \notin \mathcal{A}\} > 1 - (2^{-n(\zeta-\epsilon)} + \epsilon)$ holds for any $\epsilon > 0$ and n sufficiently large.

6. Take $\epsilon < \zeta$. Then $\underline{P_e > 1 - 2\epsilon}$ for $n(\epsilon)$ sufficiently large.

7. Finally, let $\epsilon \to 0$.

Theorem 5.2 (Weak AEP II)

1) If $\mathbf{x} \in W^n_{[X]\epsilon}$, then

$$2^{-n(H(X)+\epsilon)} \leq p(\mathbf{x}) \leq 2^{-n(H(X)-\epsilon)}.$$

2) For n sufficiently large,

$$\Pr\{\mathbf{X} \in W^n_{[X]\epsilon}\} > 1 - \epsilon.$$

3) For n sufficiently large,

$$(1-\epsilon)2^{n(H(X)-\epsilon)} \leq |W^n_{[X]\epsilon}| \leq 2^{n(H(X)+\epsilon)}.$$