香港中文大學
The Chinese University of Hong Kong

Chapter 3
The *I*-Measure

The Chinese University of Hong Kong

In this chapter

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- Set-theoretic structure of Shannon's information measures

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- Why Shannon's information measures can be represented by a form of Venn diagram – information diagram?

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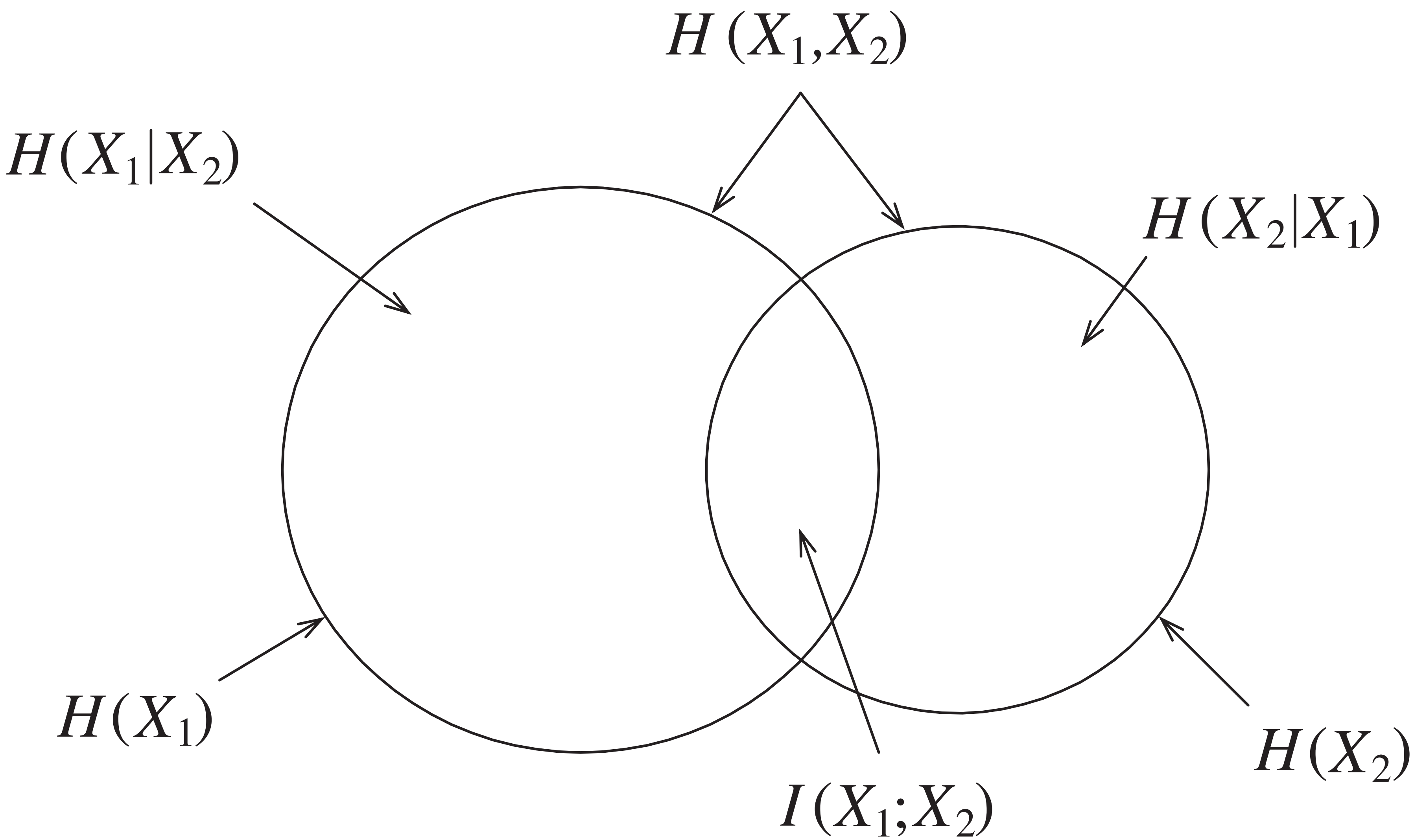
- Why Shannon's information measures can be represented by a form of Venn diagram – information diagram?
- How to use information diagrams to obtain information identities and inequalities
- Problem-solving examples

An Example



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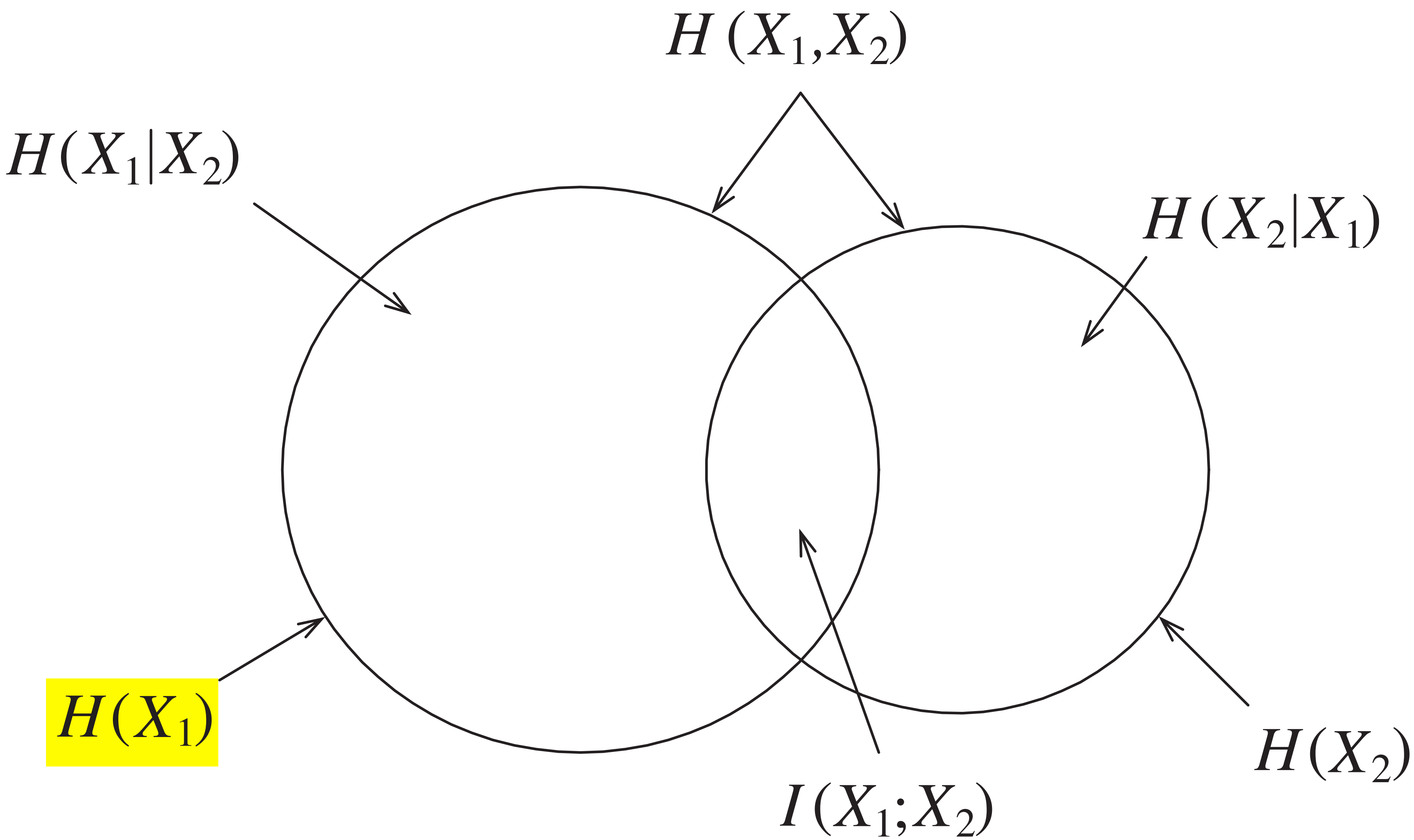

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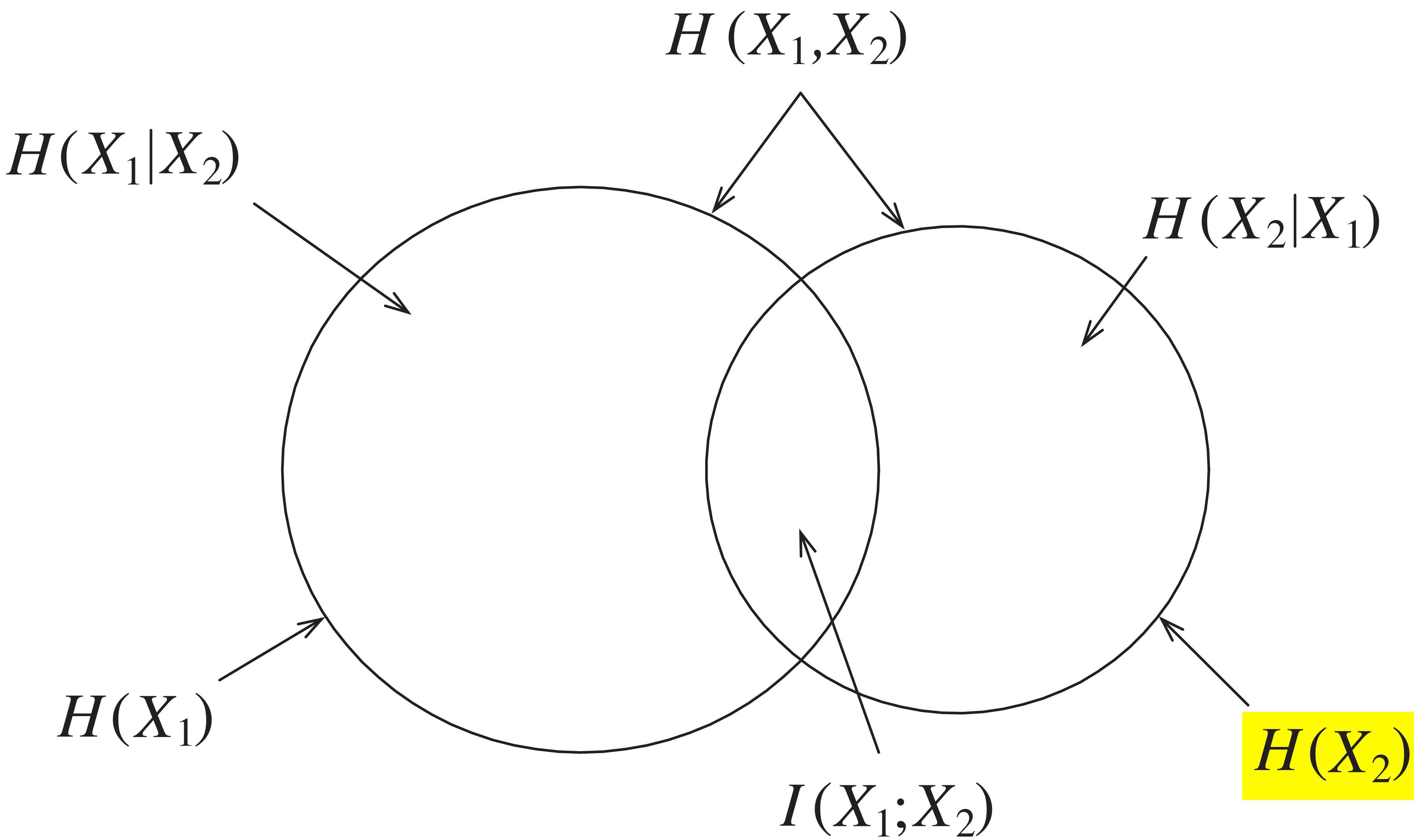

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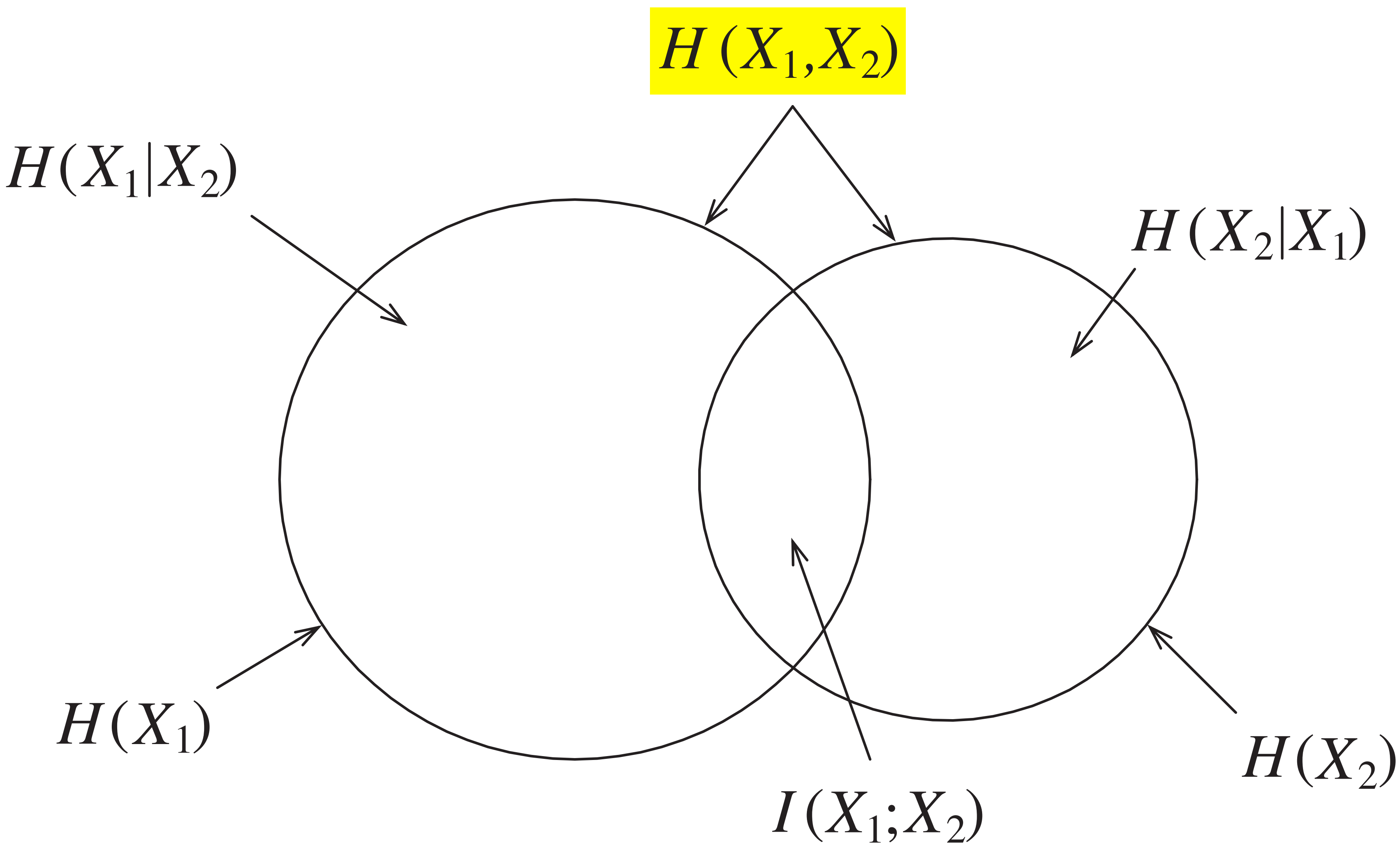

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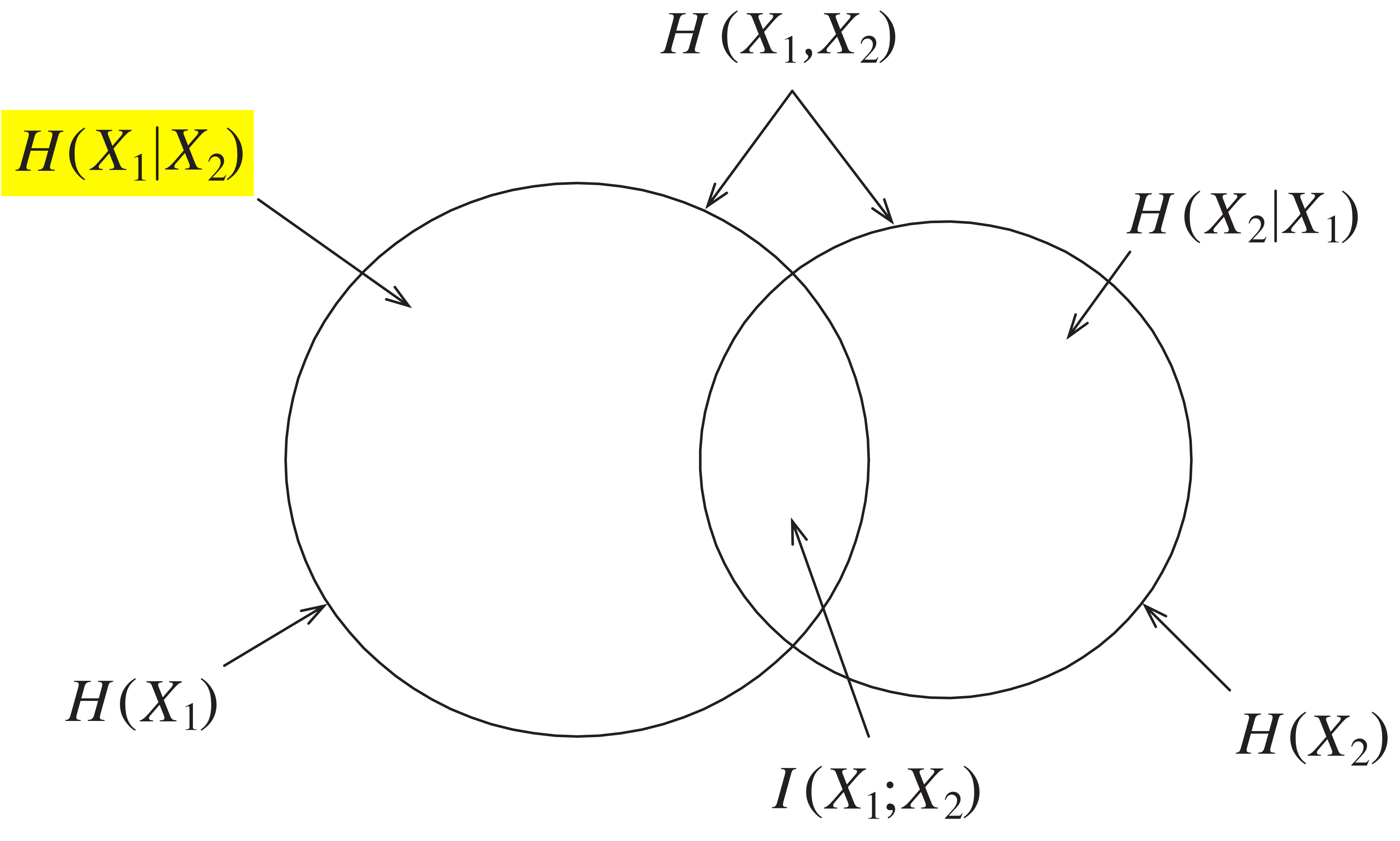

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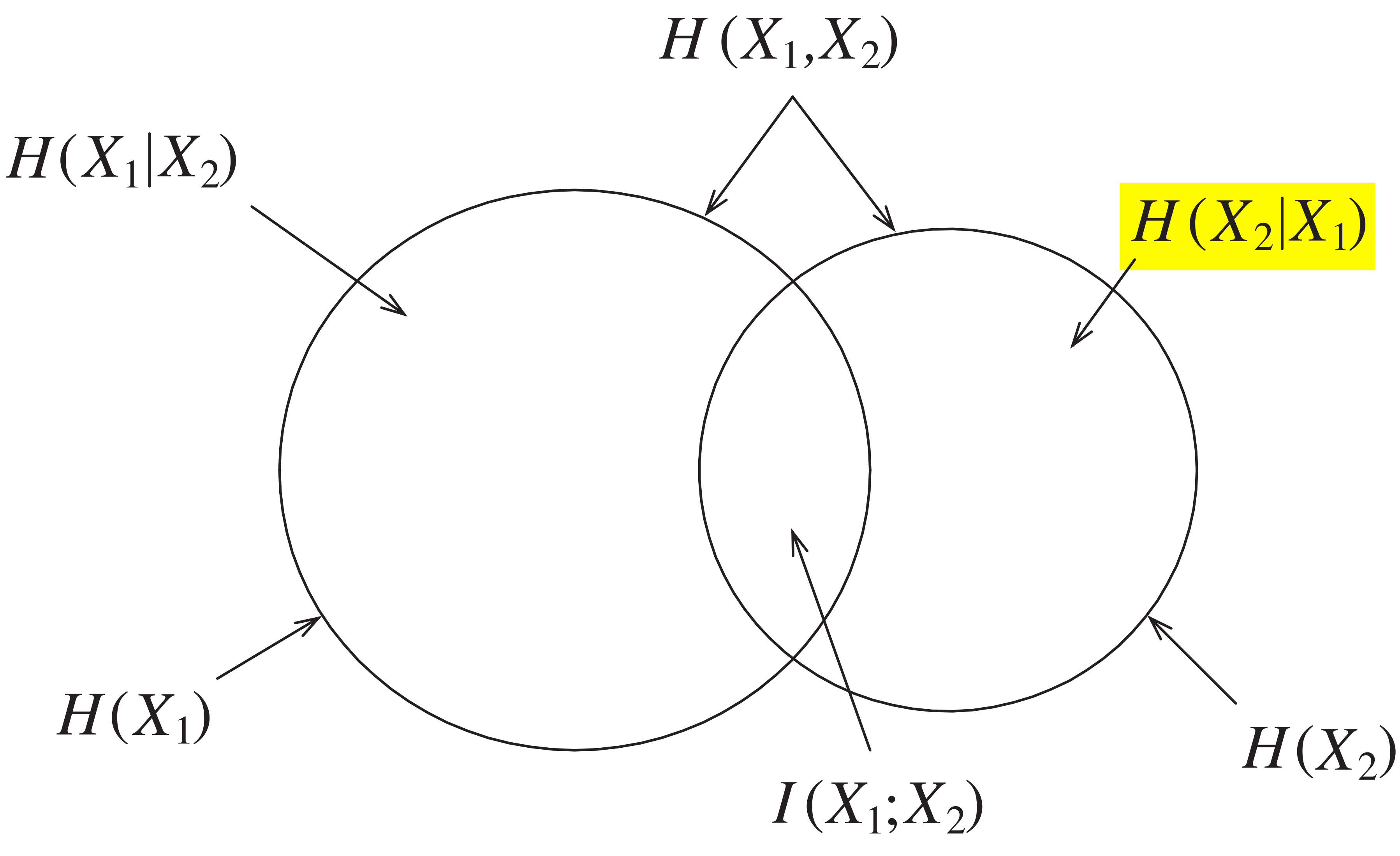

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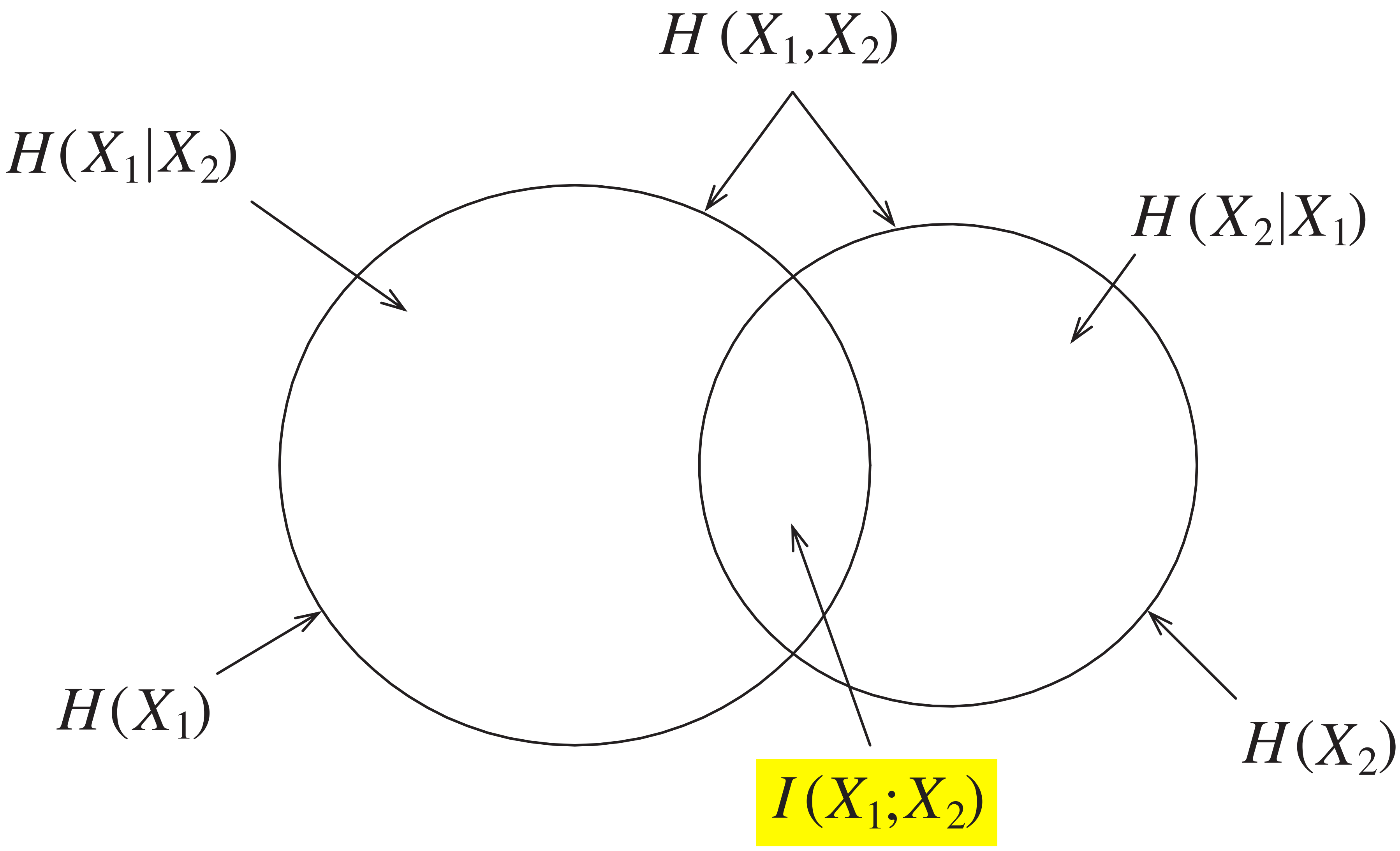

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$$\mu^*(\tilde{X}_1 \cup \tilde{X}_2) = \mu^*(\tilde{X}_1) + \mu^*(\tilde{X}_2) - \mu^*(\tilde{X}_1 \cap \tilde{X}_2)$$

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3.1 Preliminaries

Definition 3.1 The field $\mathcal{F}_n$ generated by sets $\tilde{X}_1, \tilde{X}_2, \cdots, \tilde{X}_n$ is the collection of sets which can be obtained by any sequence of usual set operations (union, intersection, complement, and difference) on $\tilde{X}_1, \tilde{X}_2, \cdots, \tilde{X}_n$.

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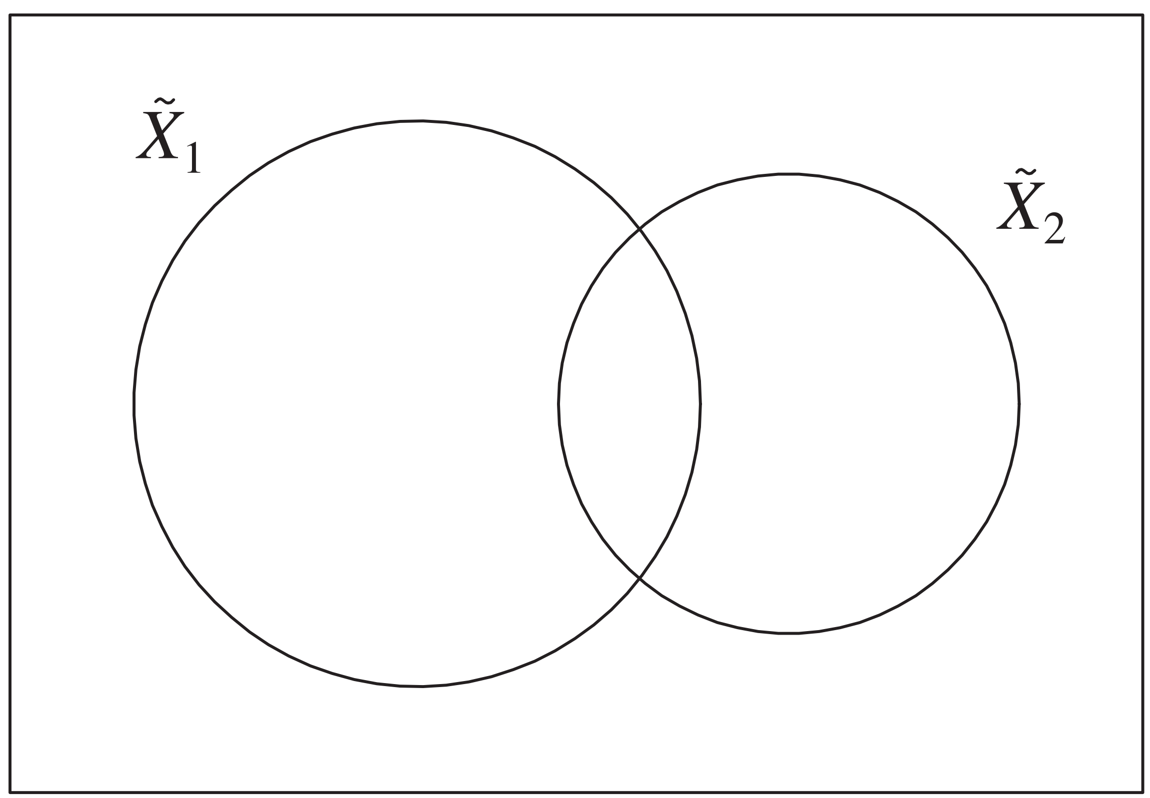
Definition 3.2 The atoms of $\mathcal{F}_n$ are sets of the form $\cap_{i=1}^n Y_i$, where Y_i is either $\tilde{X}_i$ or $\tilde{X}_i^c$, the complement of $\tilde{X}_i$.

Example 3.3

- The sets $\tilde{X}_1$ and $\tilde{X}_2$ generate the field $\mathcal{F}_2$.



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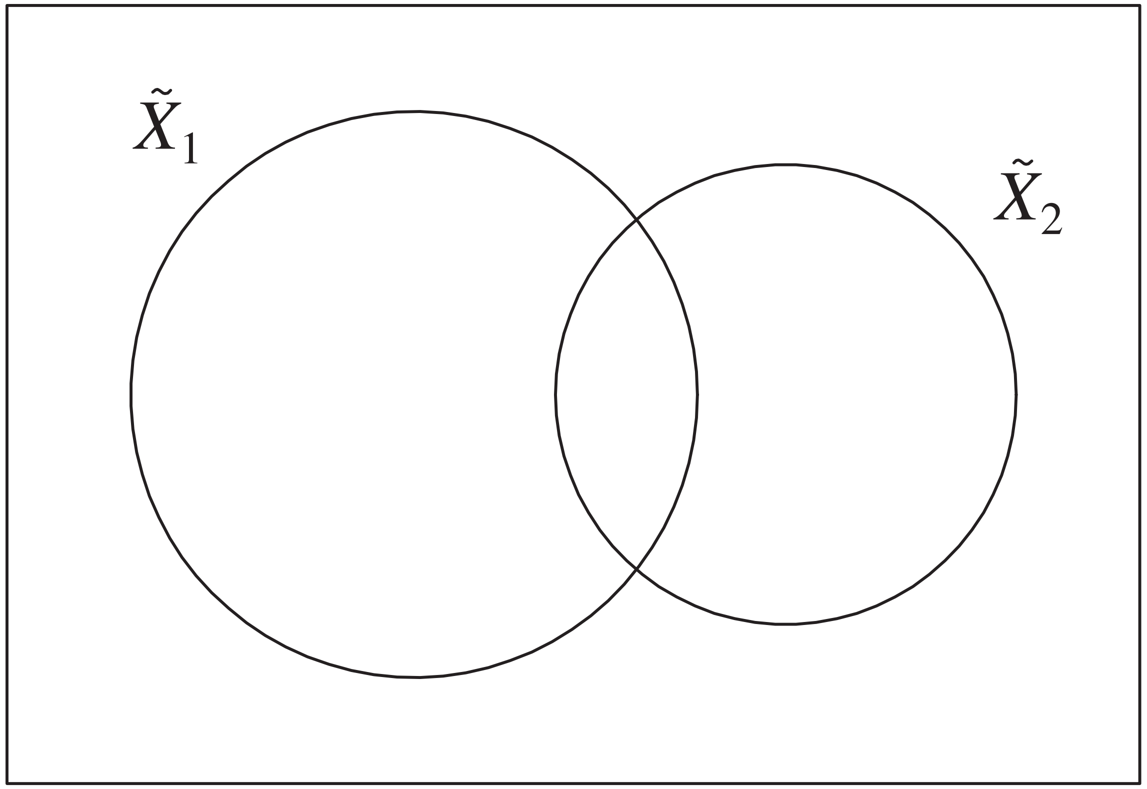

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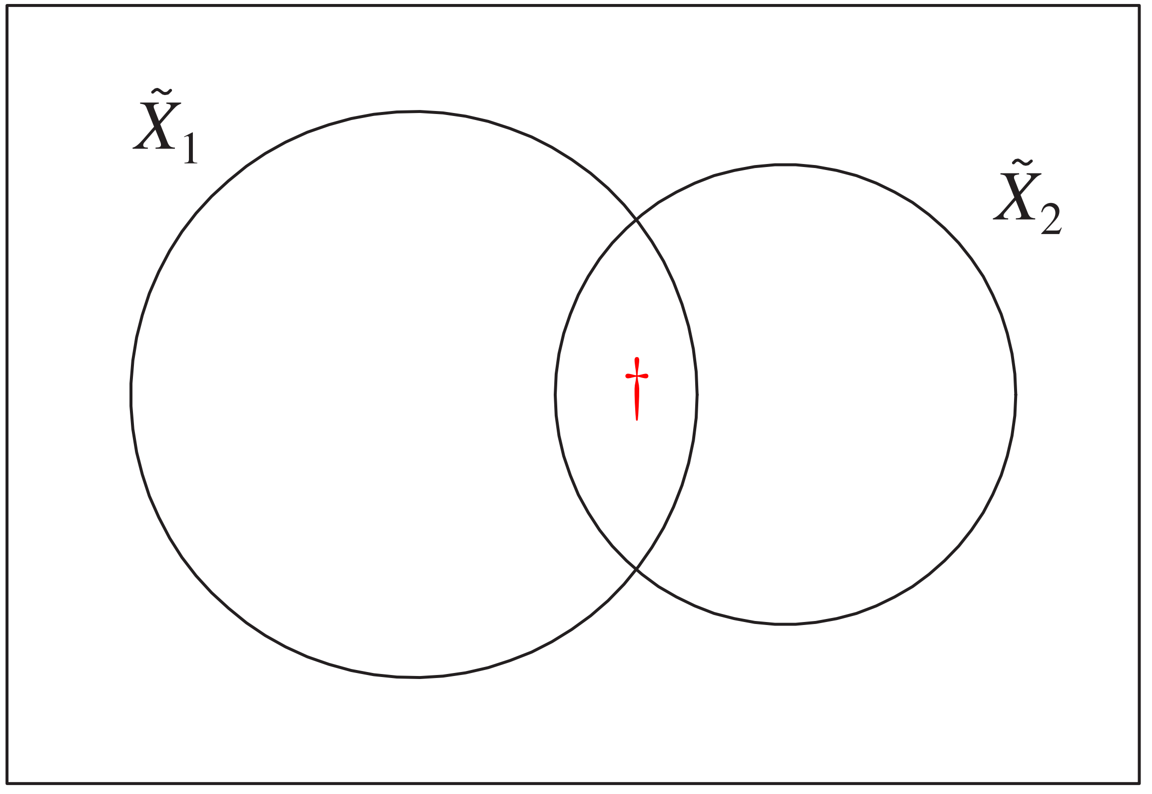

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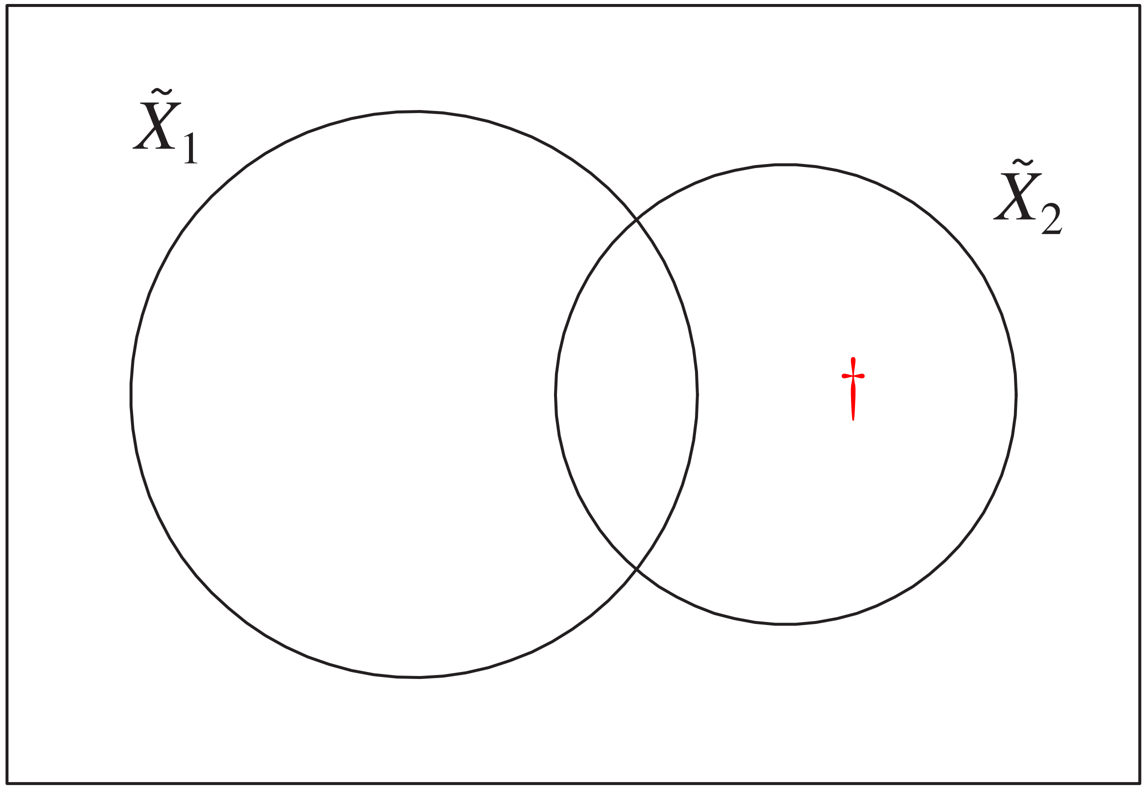

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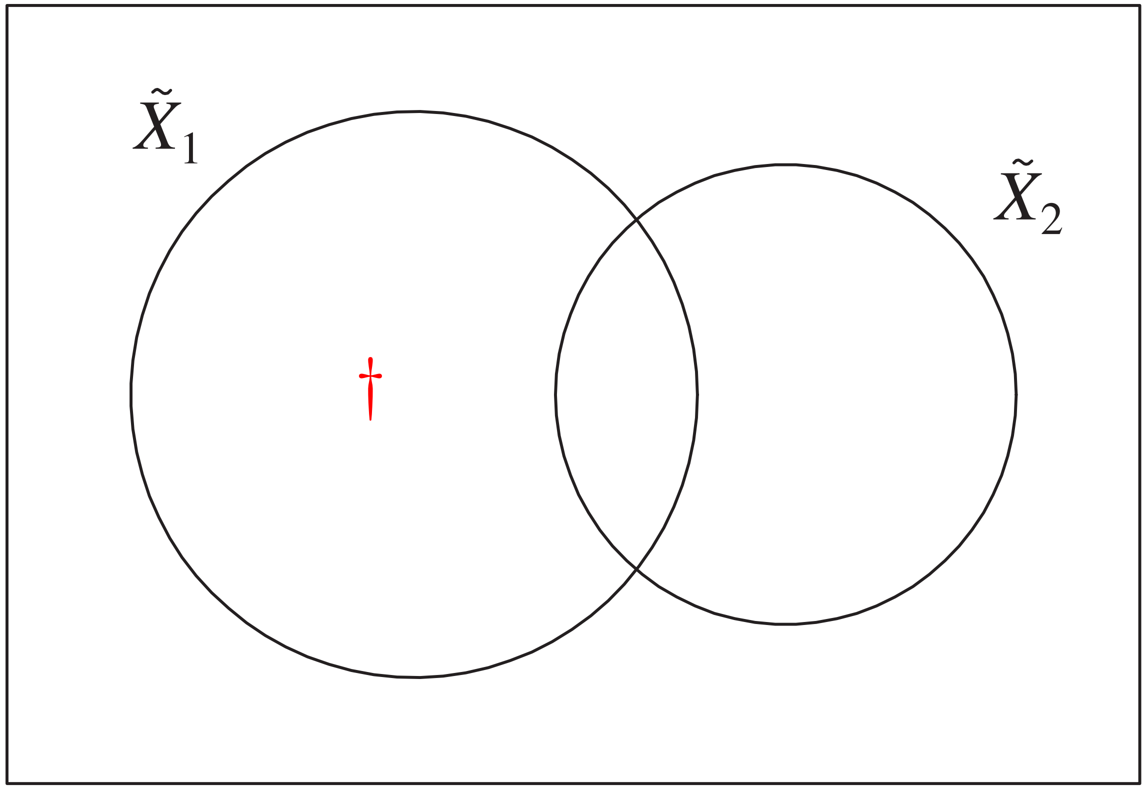

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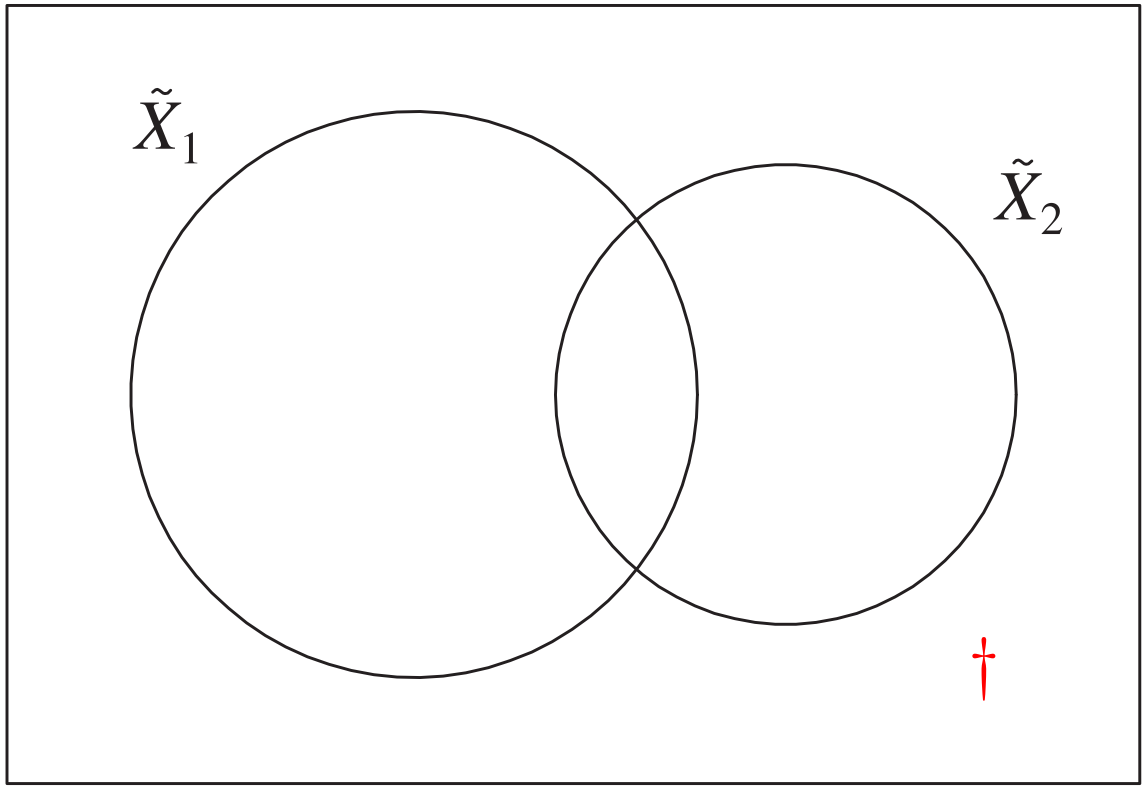

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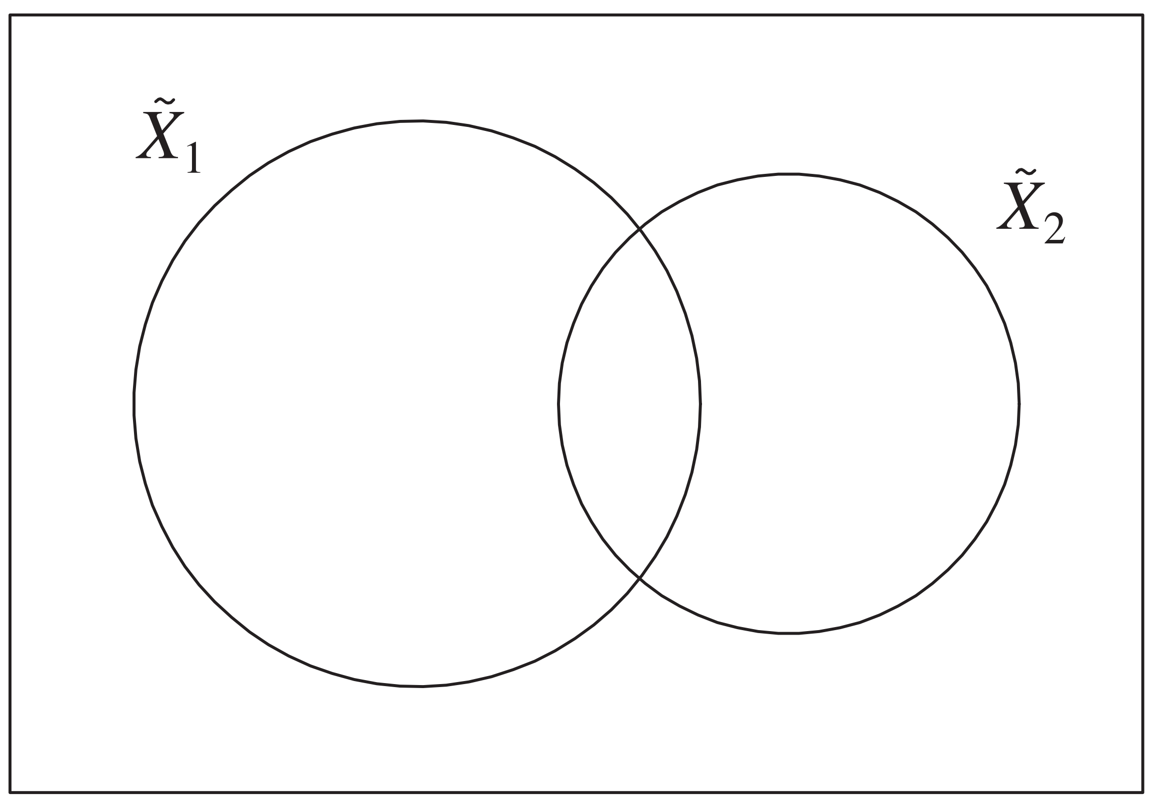

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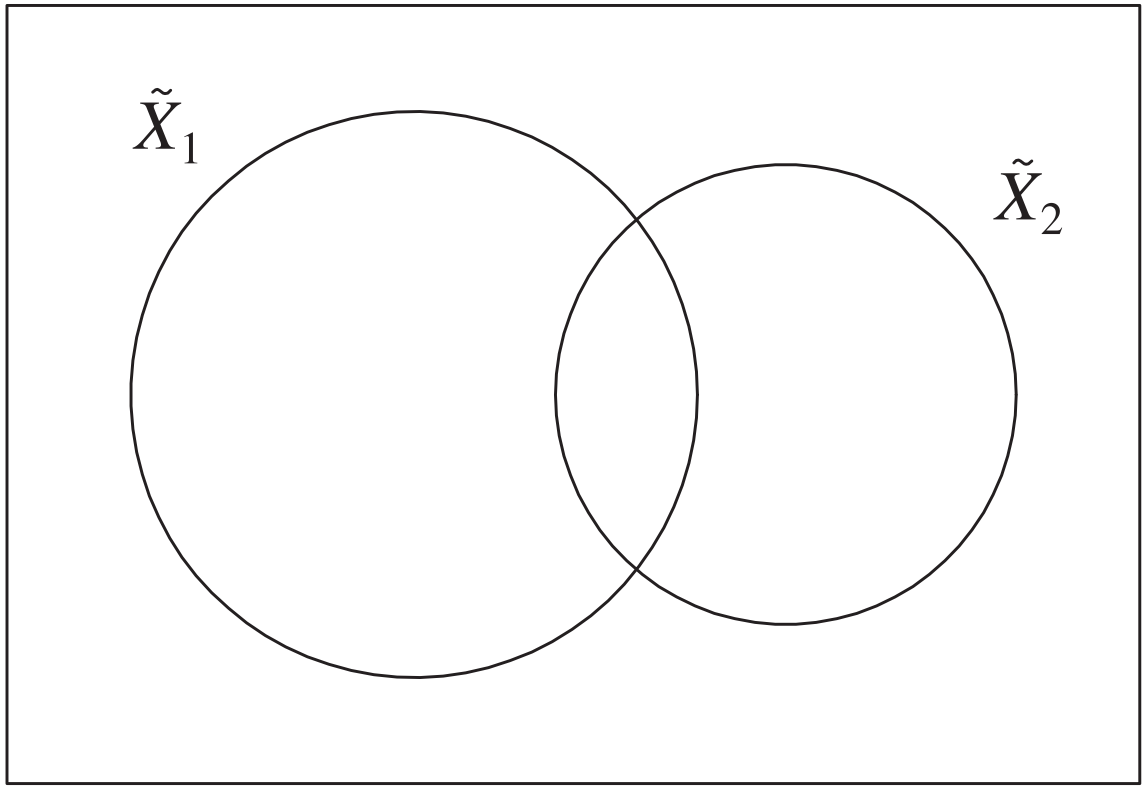

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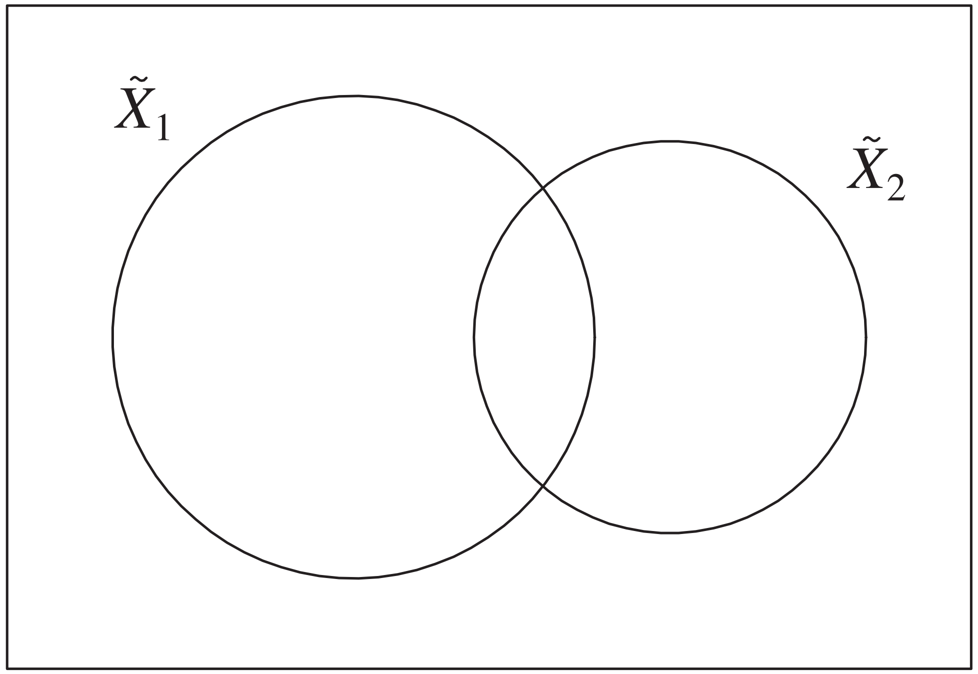
Remark A signed measure can take positive or negative values. If a signed measure takes only positive values, it is simply called a measure.

Example 3.5

- A signed measure μ on $\mathcal{F}_2$ is completely specified by the values on the atoms

$$\mu(\tilde{X}_1 \cap \tilde{X}_2),\ \mu(\tilde{X}_1^c \cap \tilde{X}_2),\ \mu(\tilde{X}_1 \cap \tilde{X}_2^c),\ \mu(\tilde{X}_1^c \cap \tilde{X}_2^c)$$



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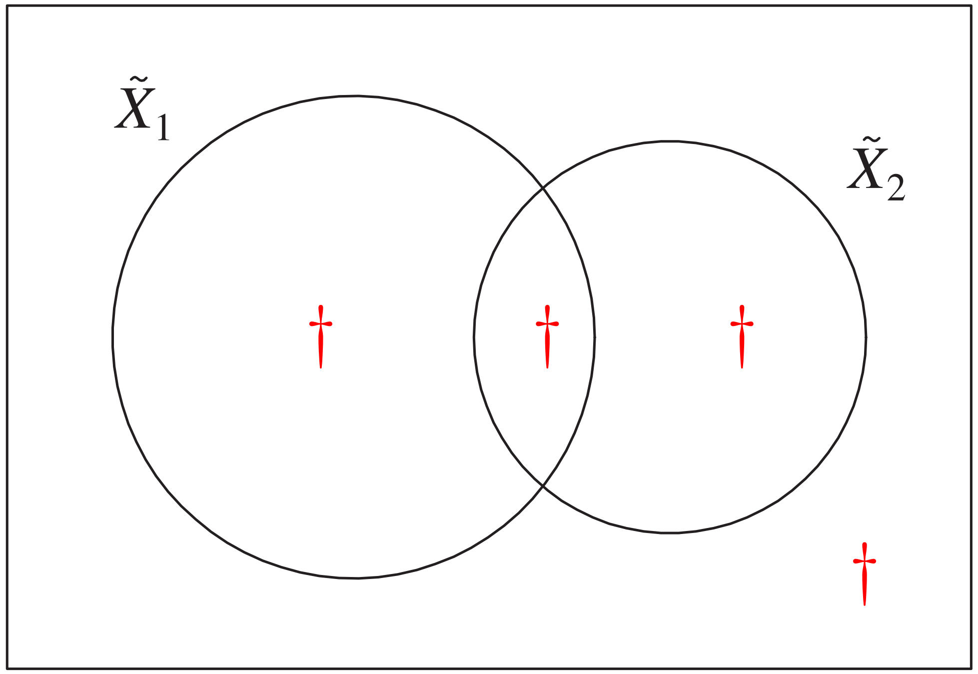

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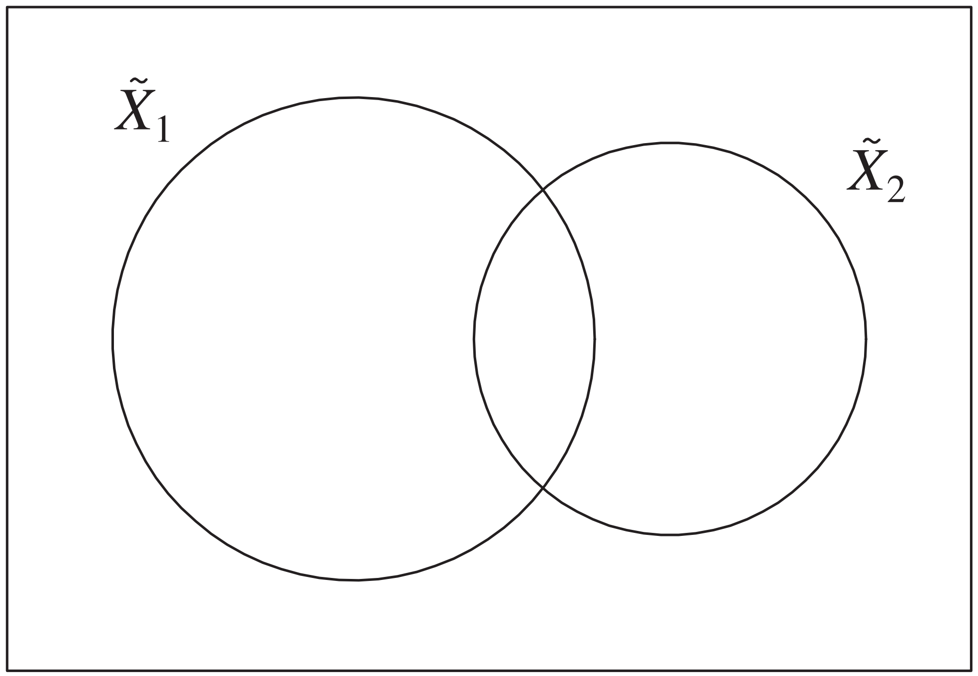
- The value of μ on other sets in $\mathcal{F}_2$ are obtained by set-additivity. For example,

$$\begin{aligned} \mu(\tilde{X}_1) &= \mu((\tilde{X}_1 \cap \tilde{X}_2) \cup (\tilde{X}_1 \cap \tilde{X}_2^c)) \\ &= \mu(\tilde{X}_1 \cap \tilde{X}_2) + \mu(\tilde{X}_1 \cap \tilde{X}_2^c) \end{aligned}$$



Example 3.5

- A signed measure μ on $\mathcal{F}_2$ is completely specified by the values on the atoms
$$\mu(\tilde{X}_1 \cap \tilde{X}_2),\ \mu(\tilde{X}_1^c \cap \tilde{X}_2),\ \mu(\tilde{X}_1 \cap \tilde{X}_2^c),\ \mu(\tilde{X}_1^c \cap \tilde{X}_2^c)$$

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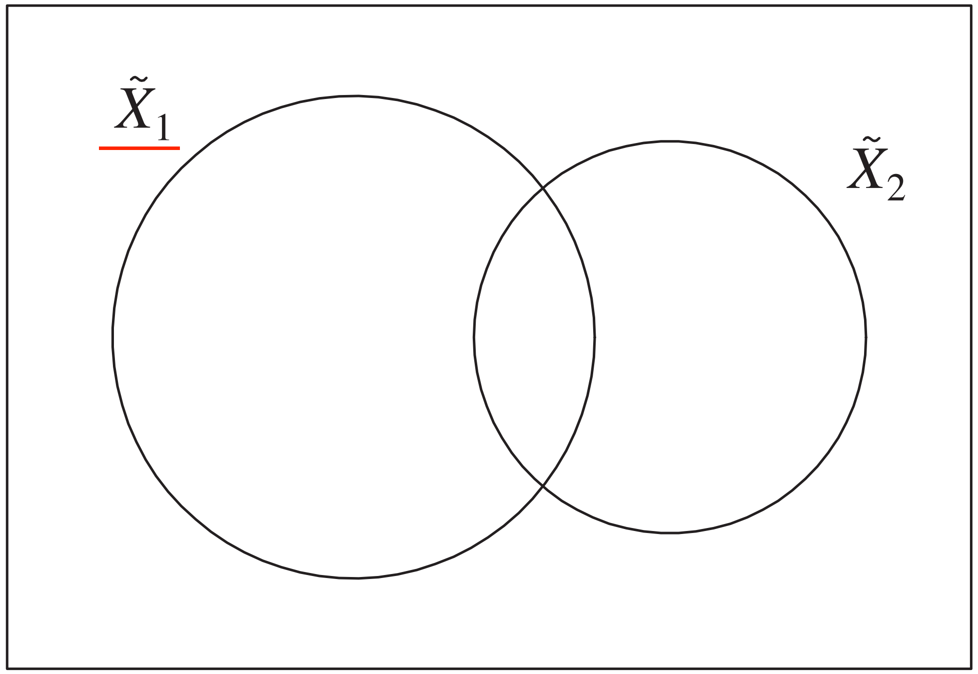

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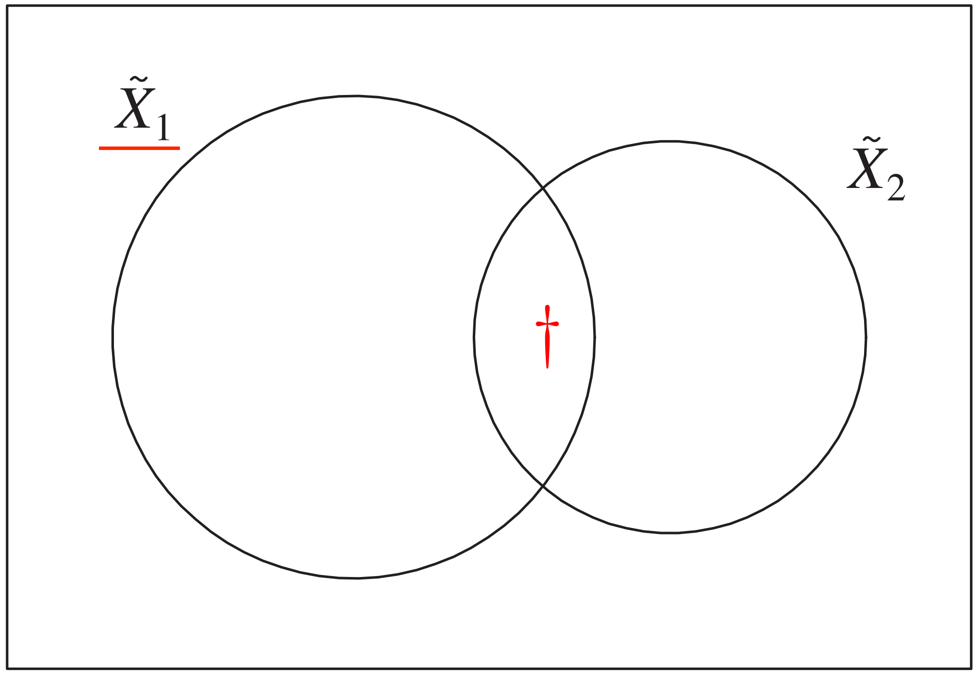

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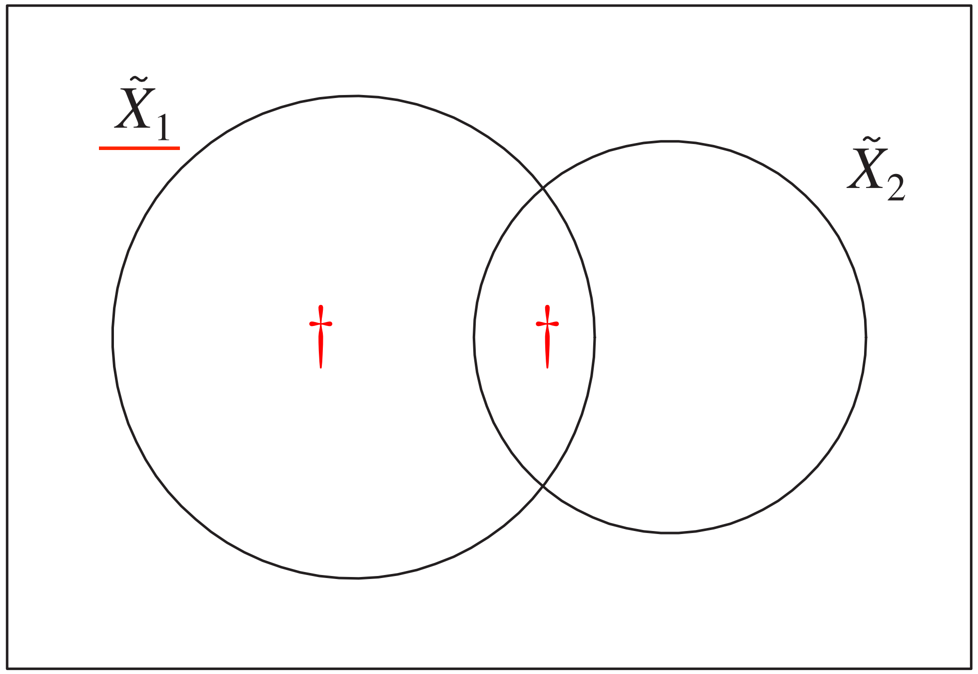

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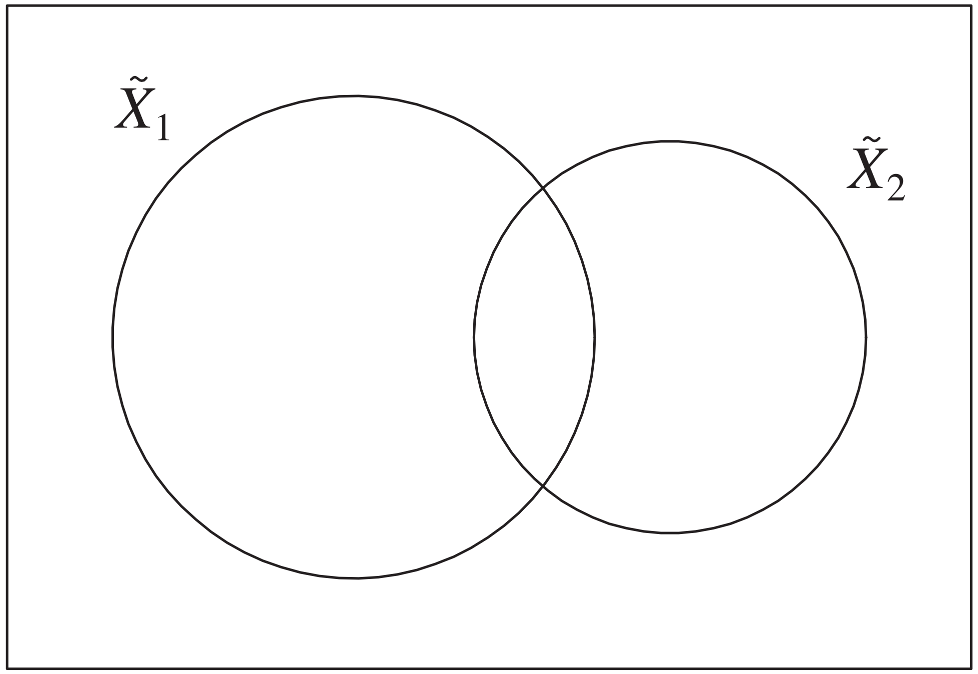

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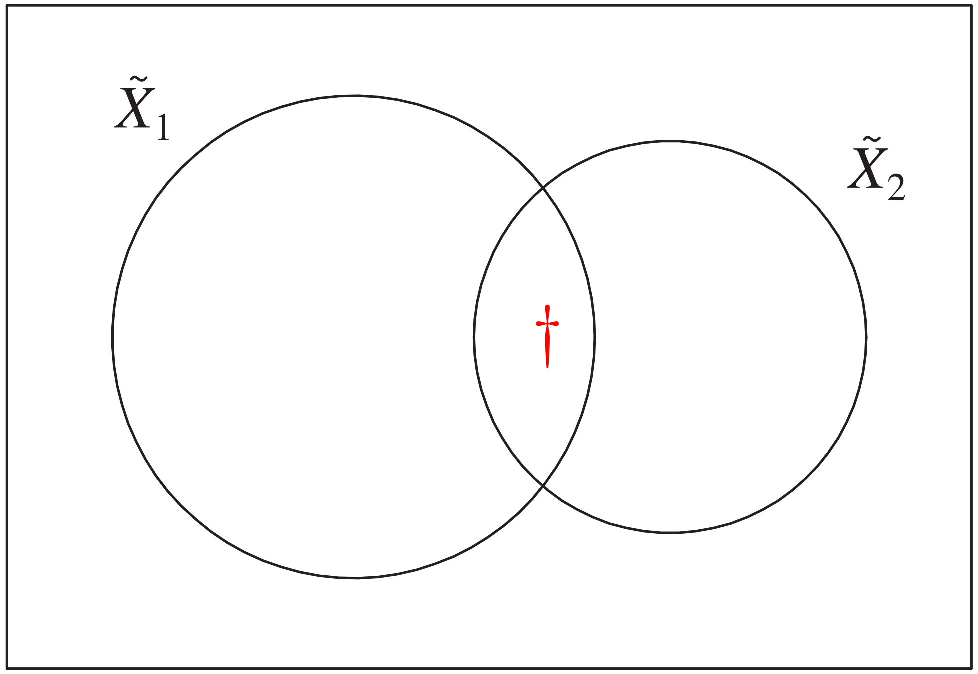

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