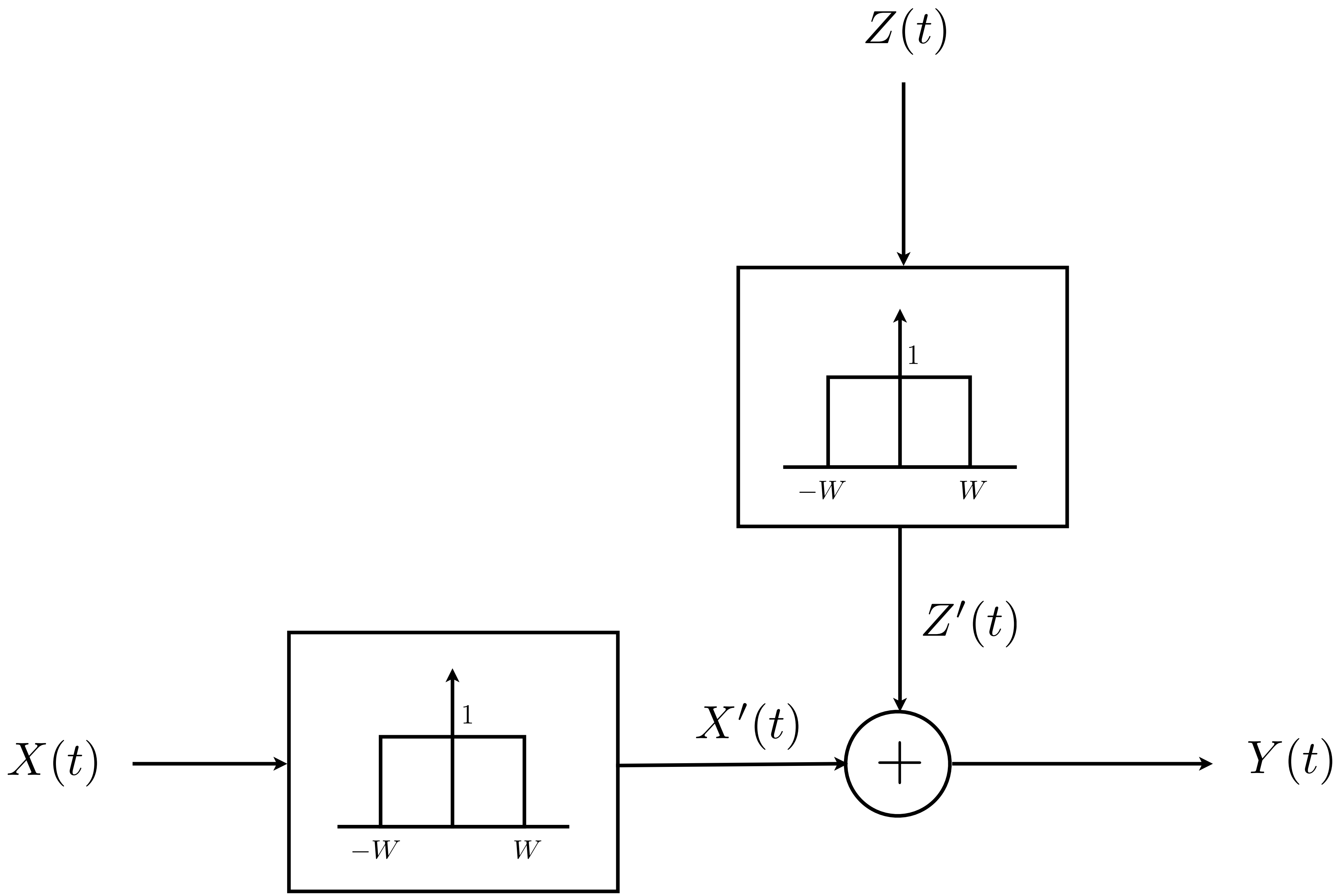
香港中文大學
The Chinese University of Hong Kong

11.8 The Bandlimited Colored Gaussian Channel

The Channel Model

The Channel Model



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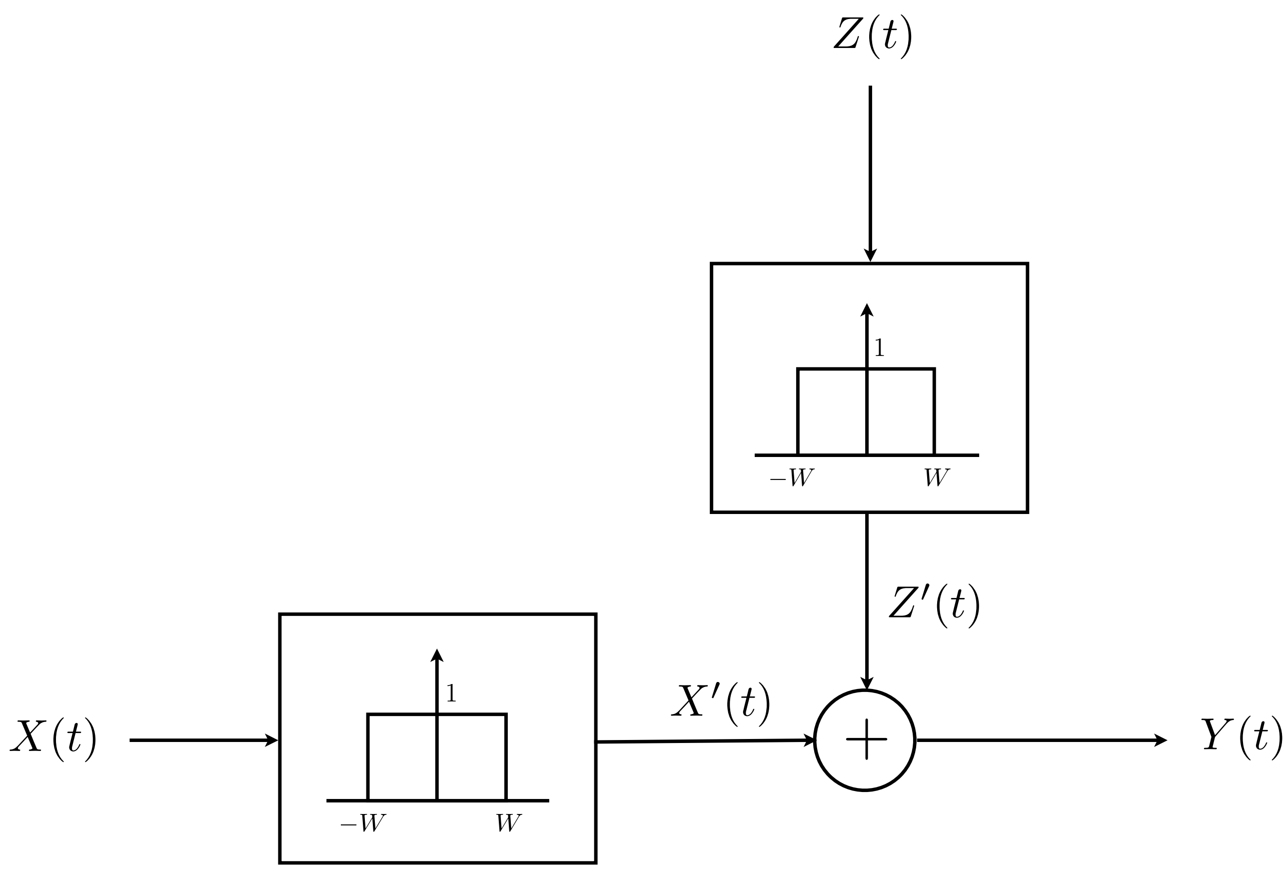

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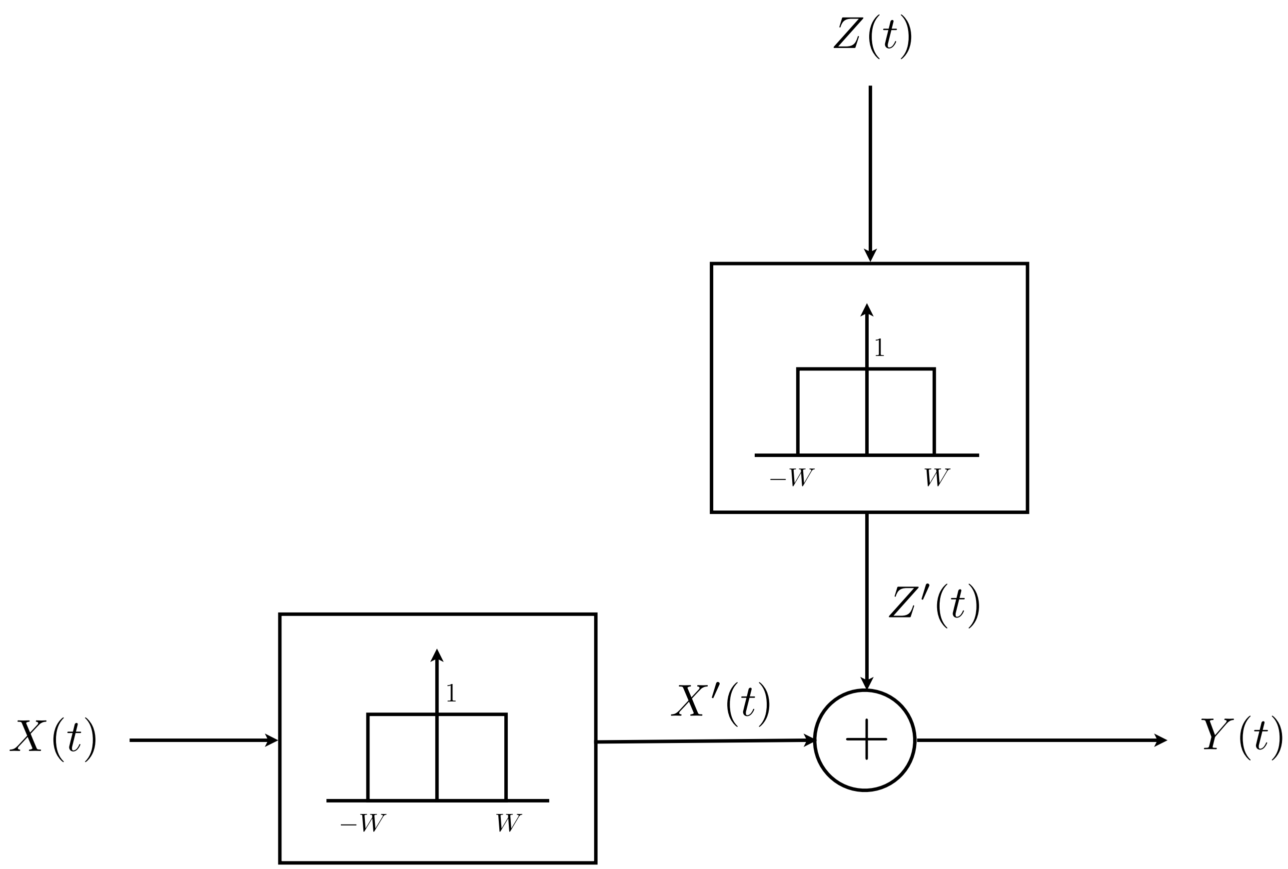

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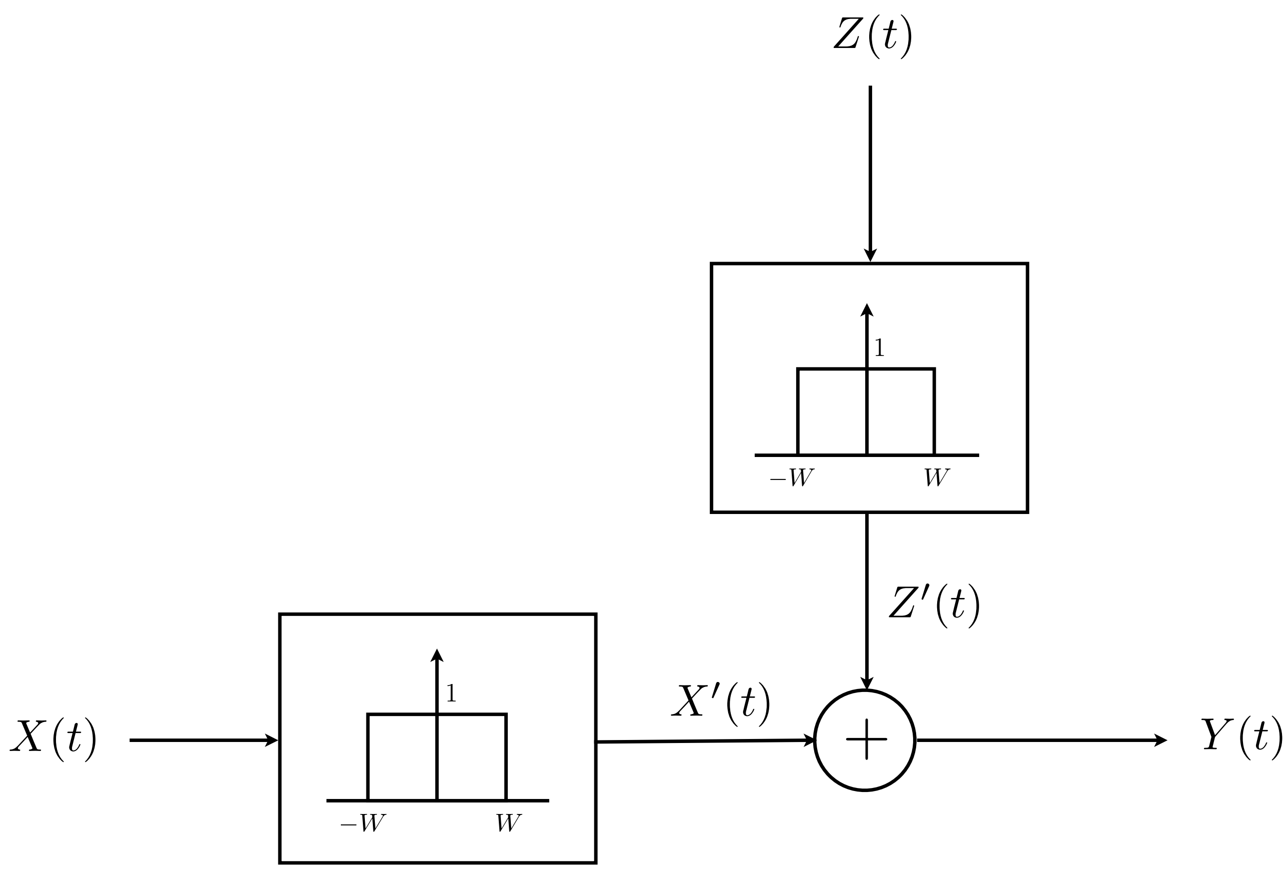

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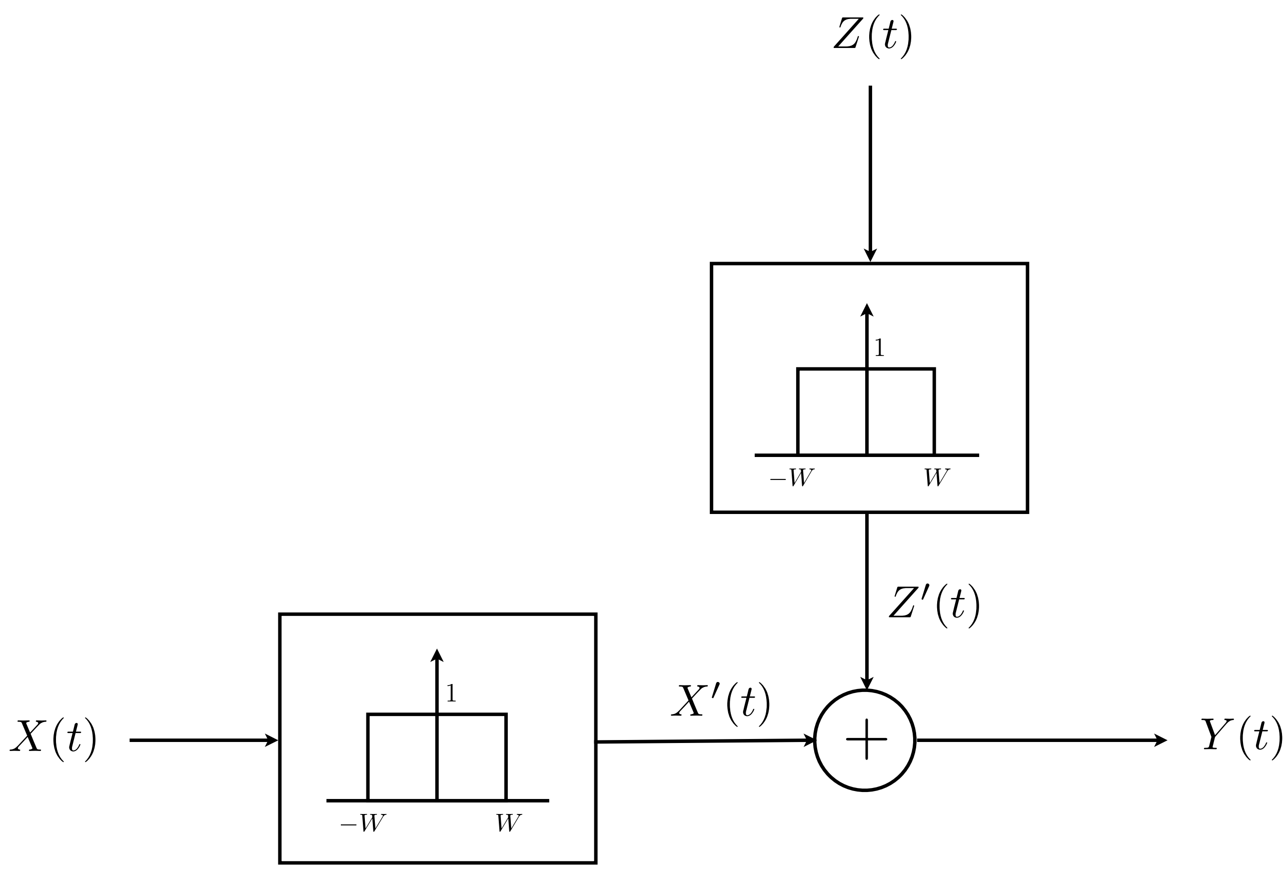

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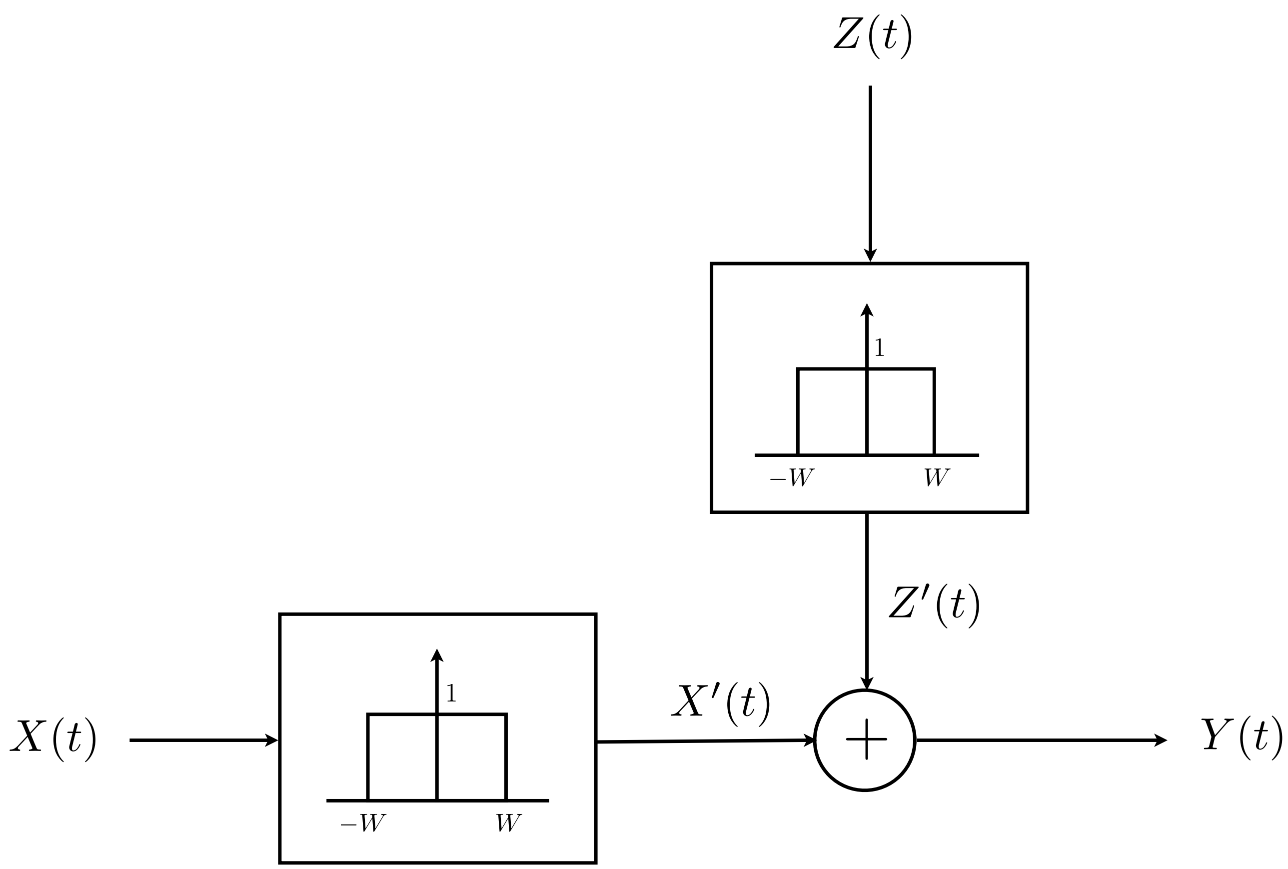

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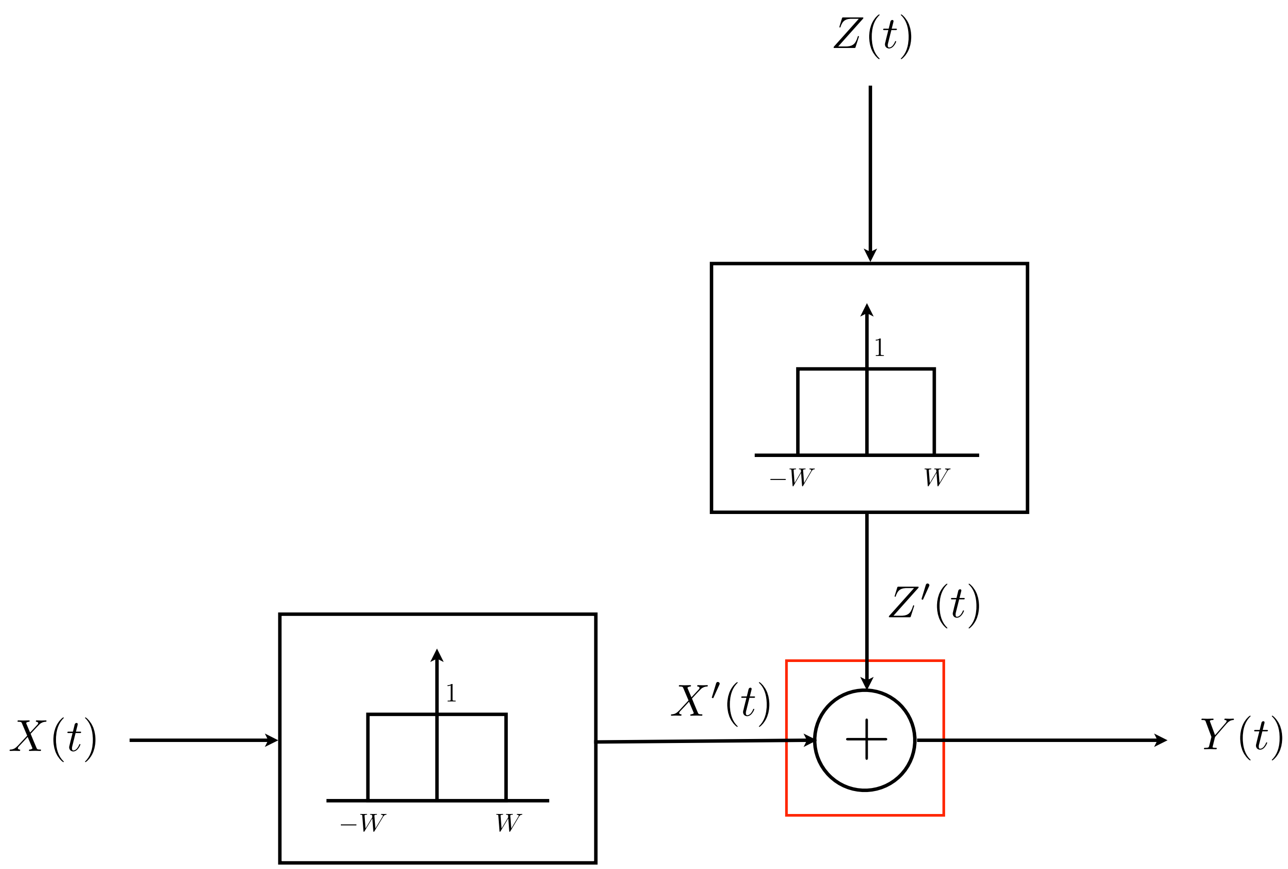

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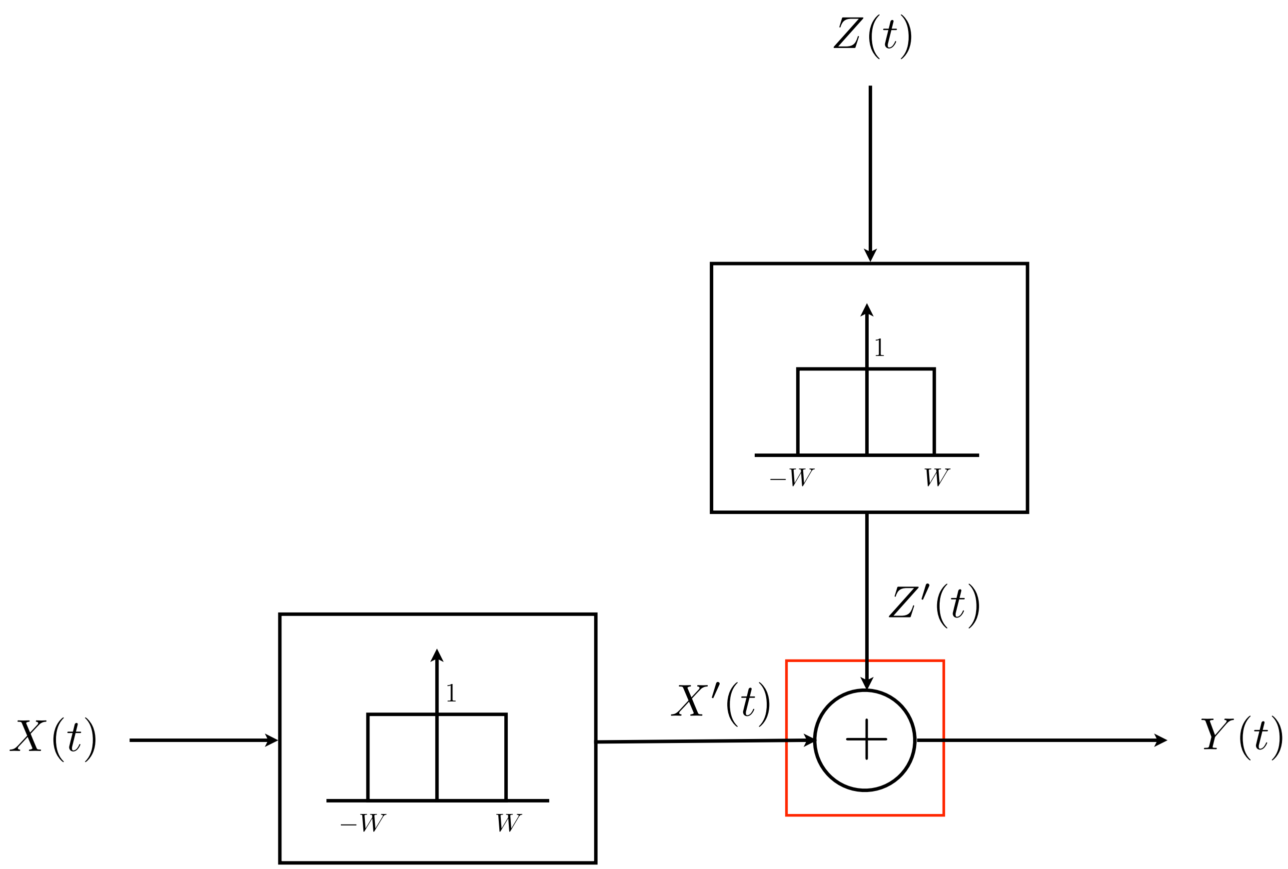

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- Impose a power constraint P on $X'(t)$.

Capacity of the Bandpass White Gaussian Channel

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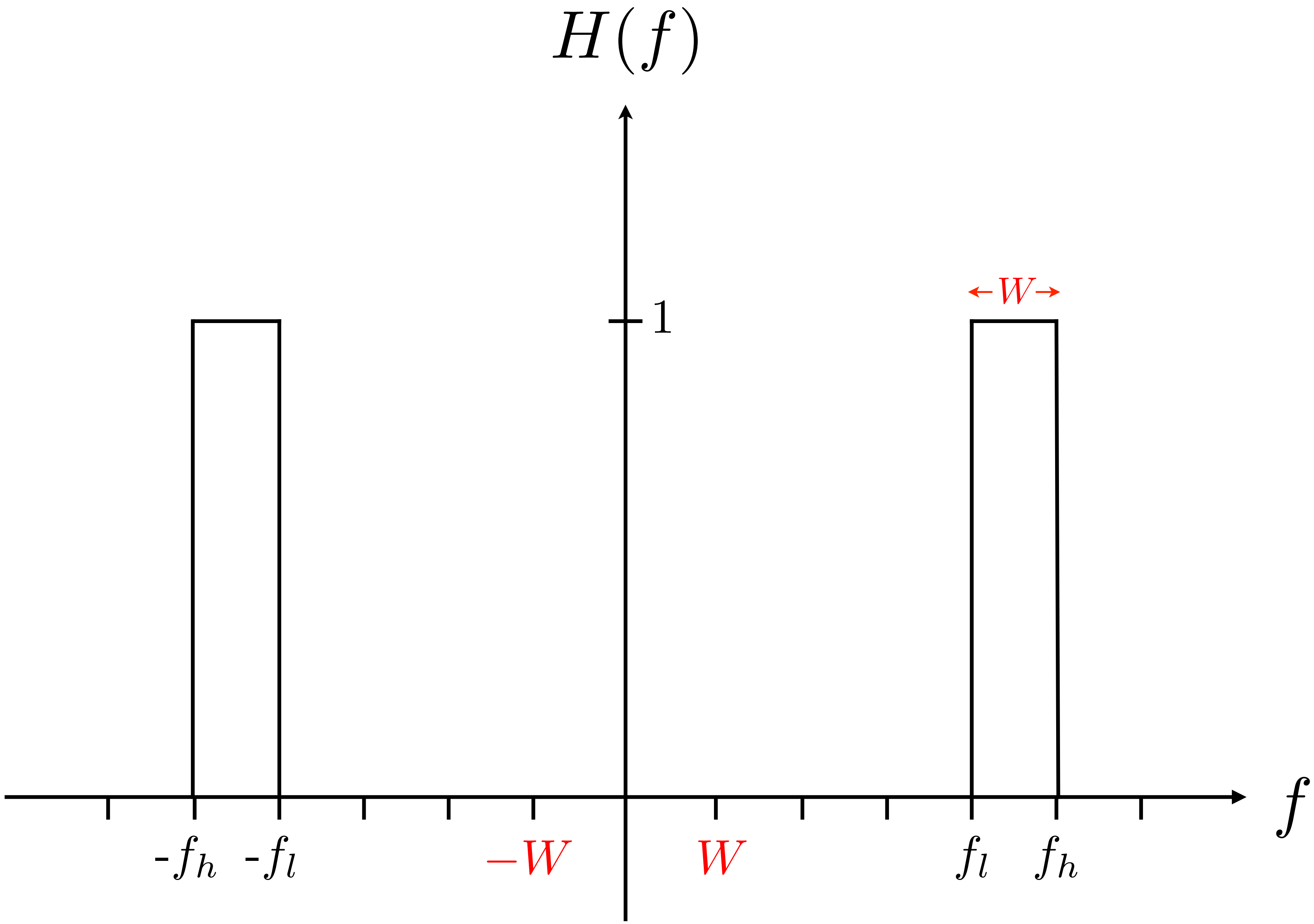
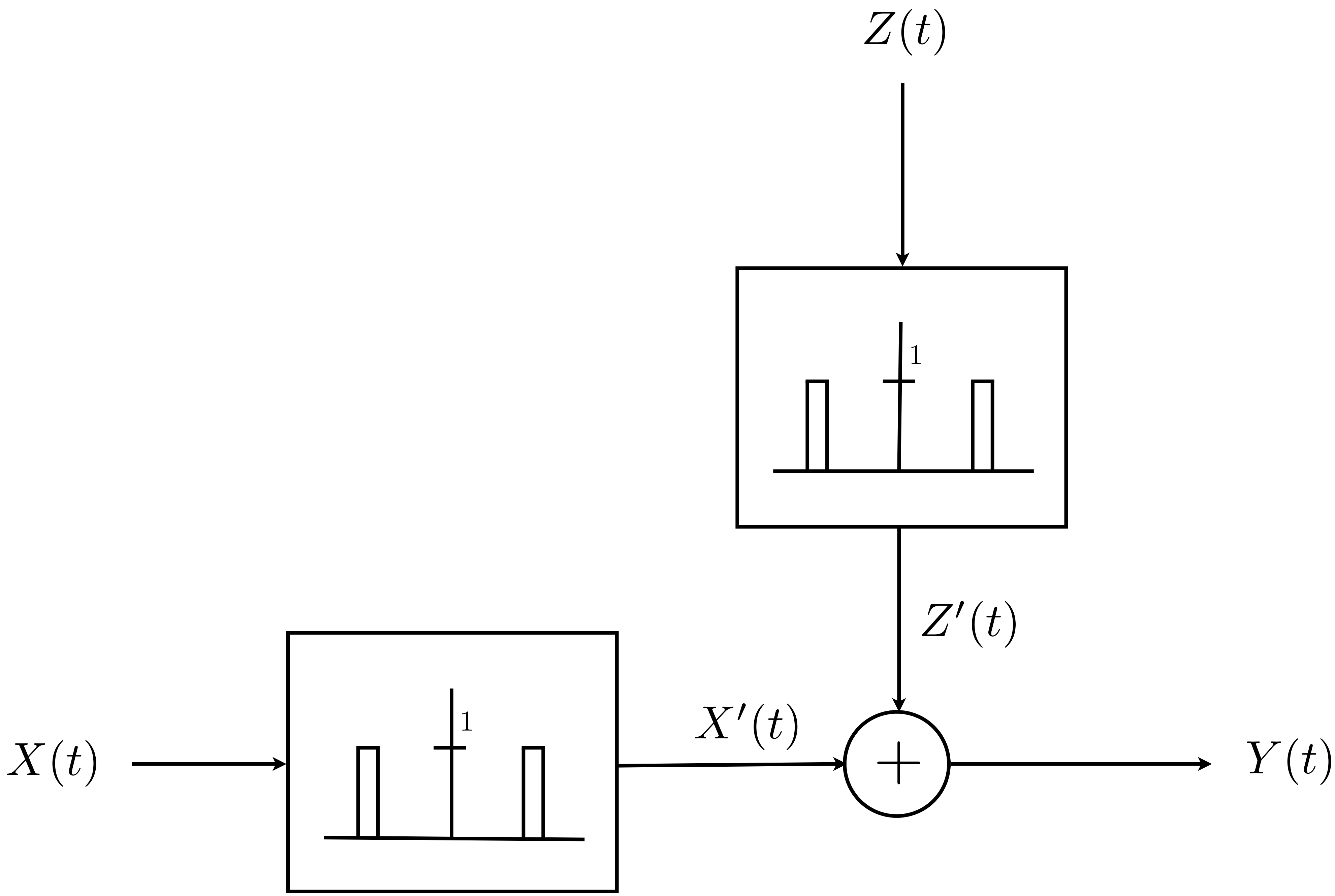
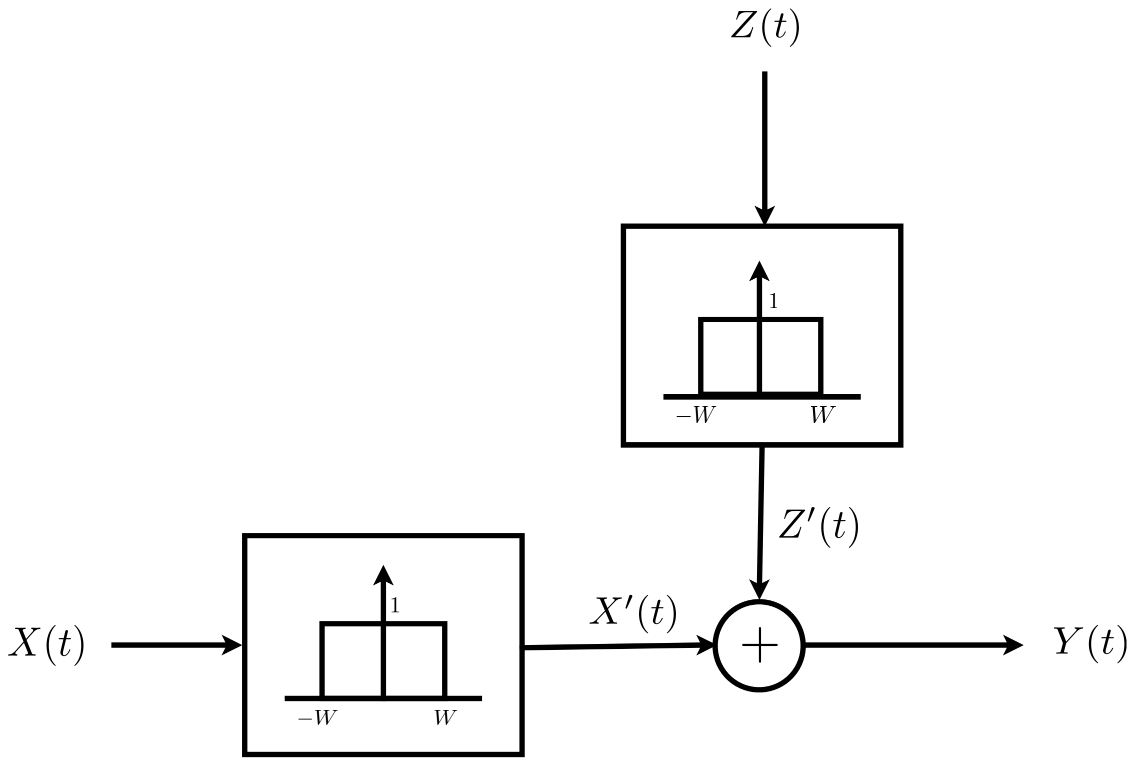
- When $f_l = 0$, the bandpass white Gaussian channel reduces to the bandlimited white Gaussian channel.



$H(f)$
1
W
f
-f_h
-f_l
$-W$
W
f_l
f_h


$Z(t)$
1
$Z'(t)$
$X'(t)$
1
$X(t)$
$Y(t)$

The Capacity of the Bandlimited Colored Gaussian Channel



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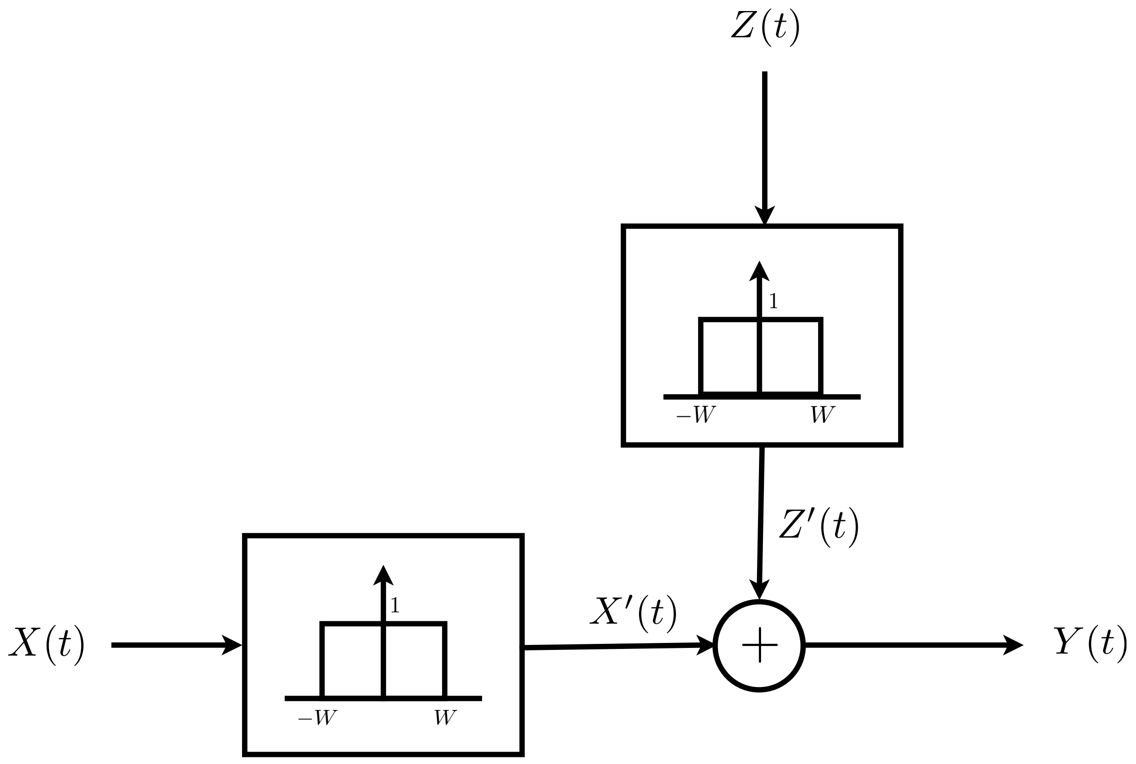

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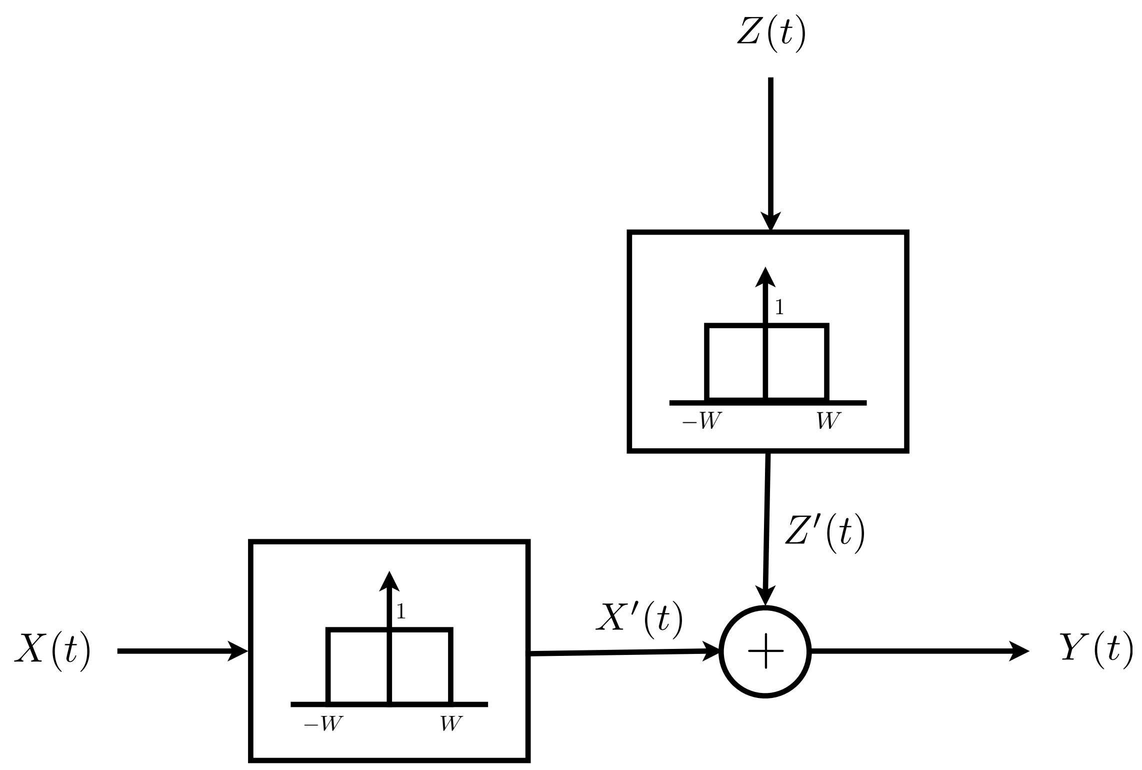

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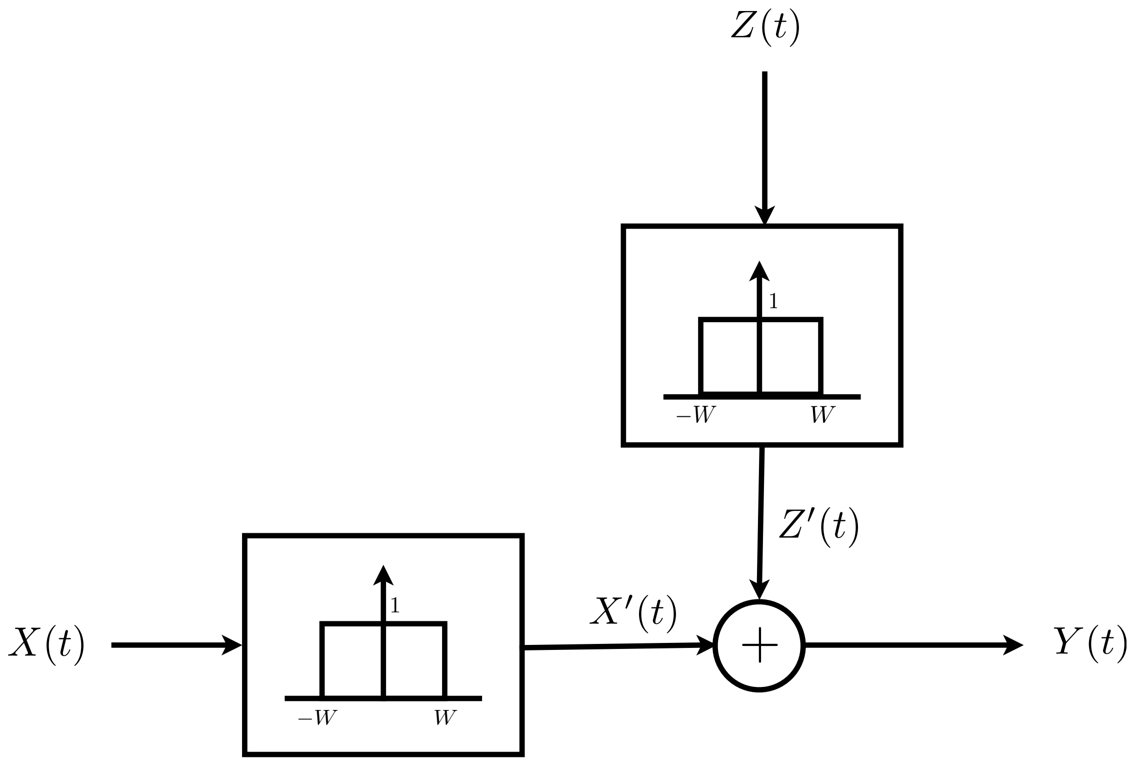

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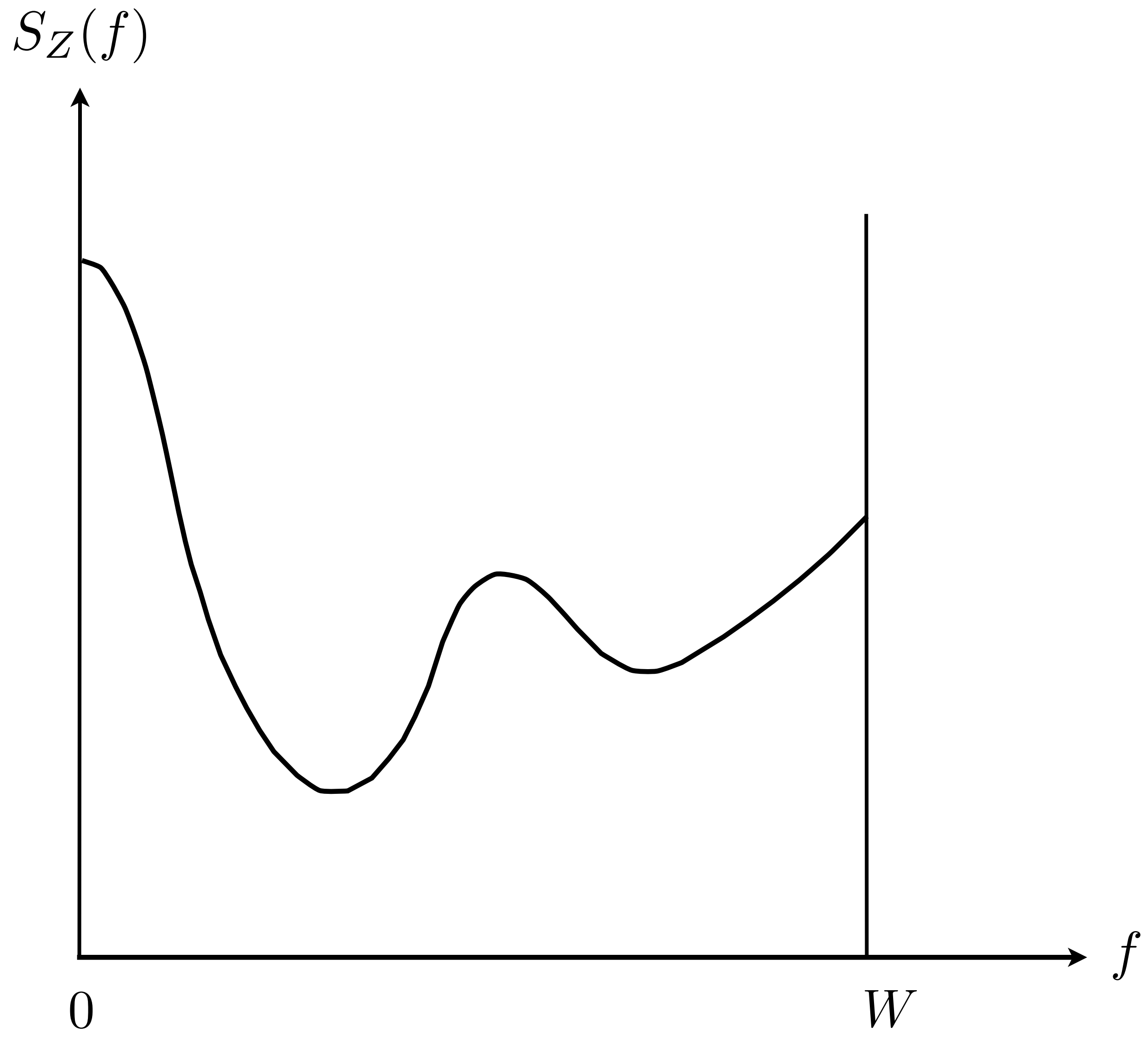

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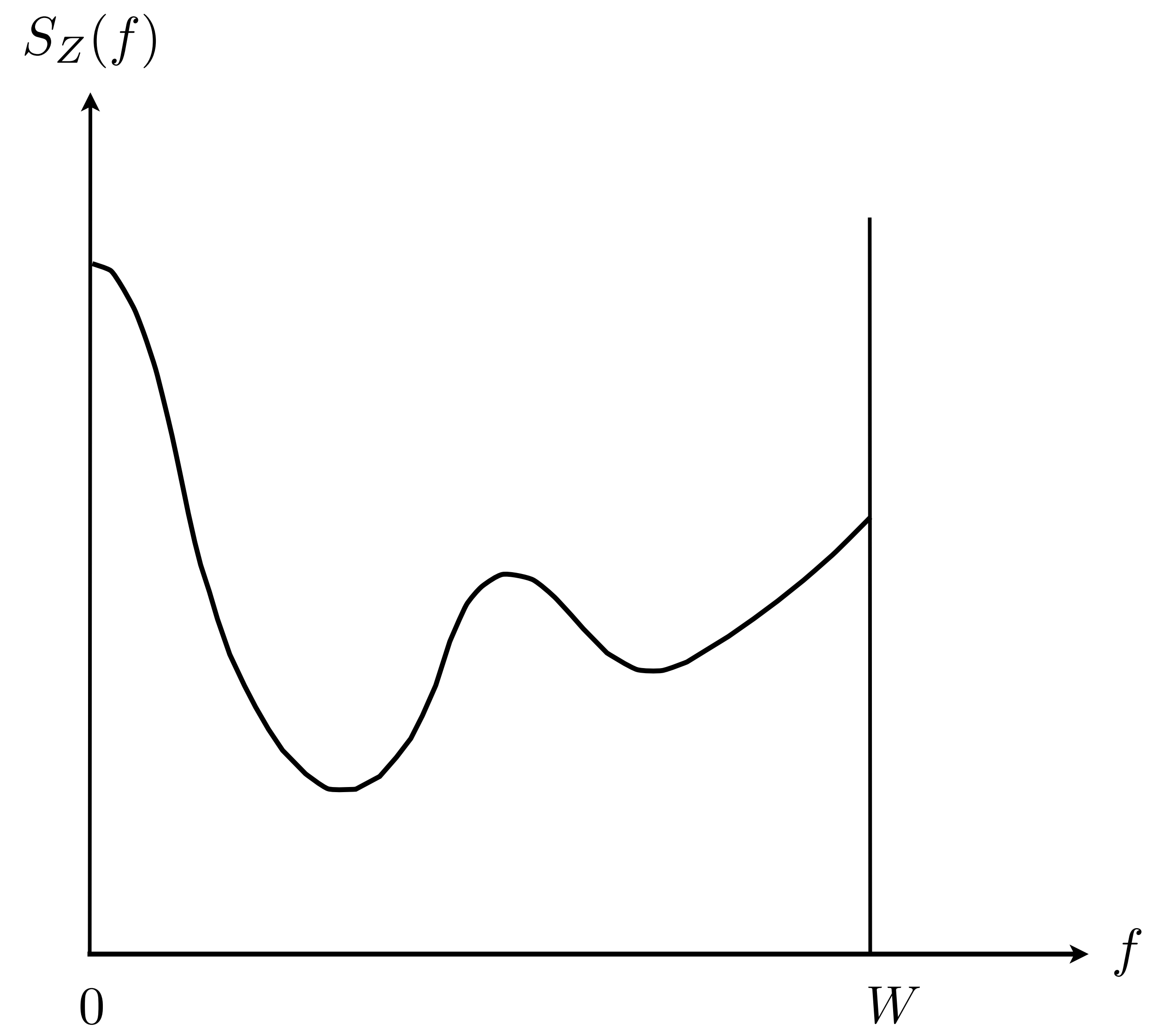

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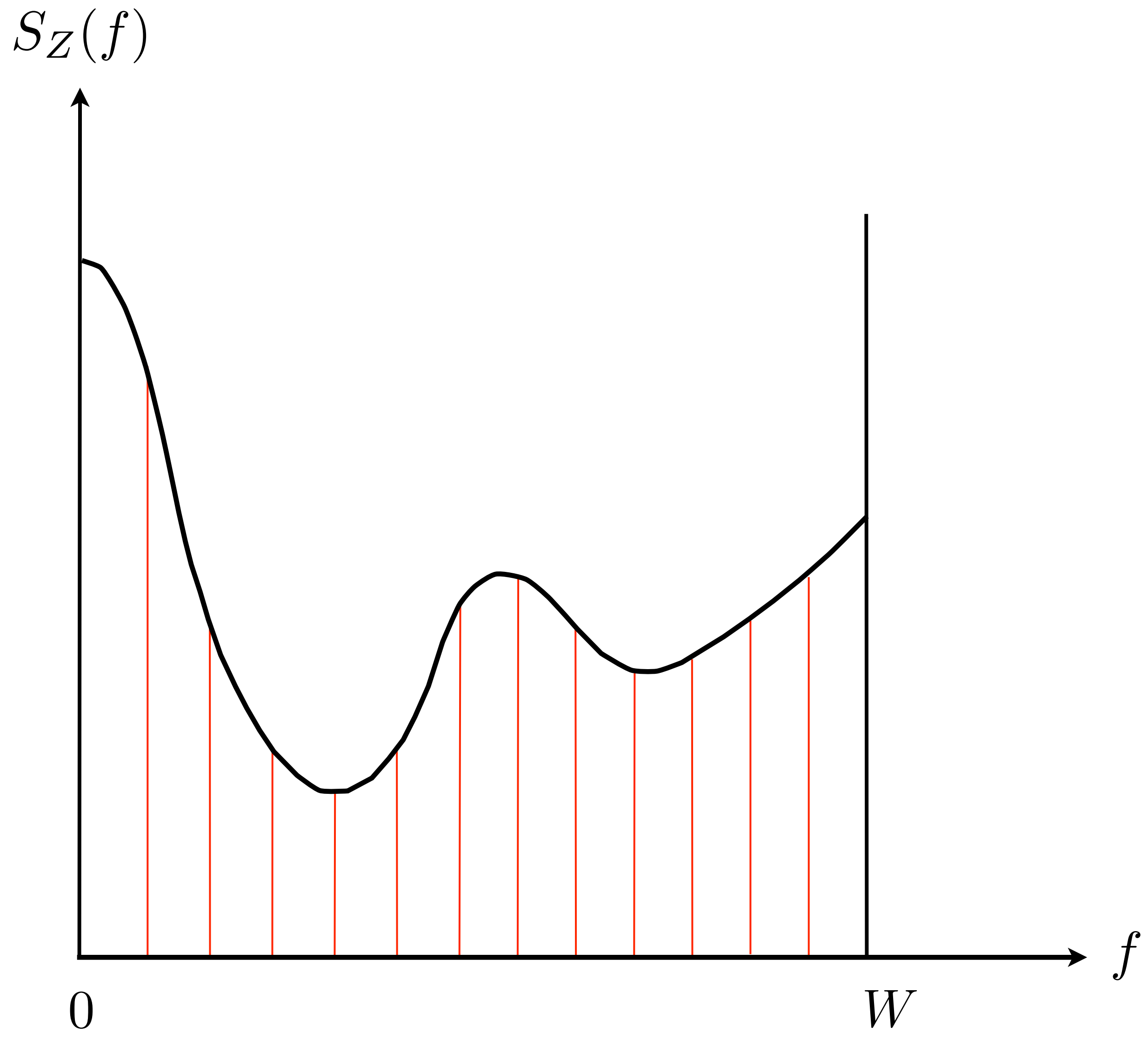

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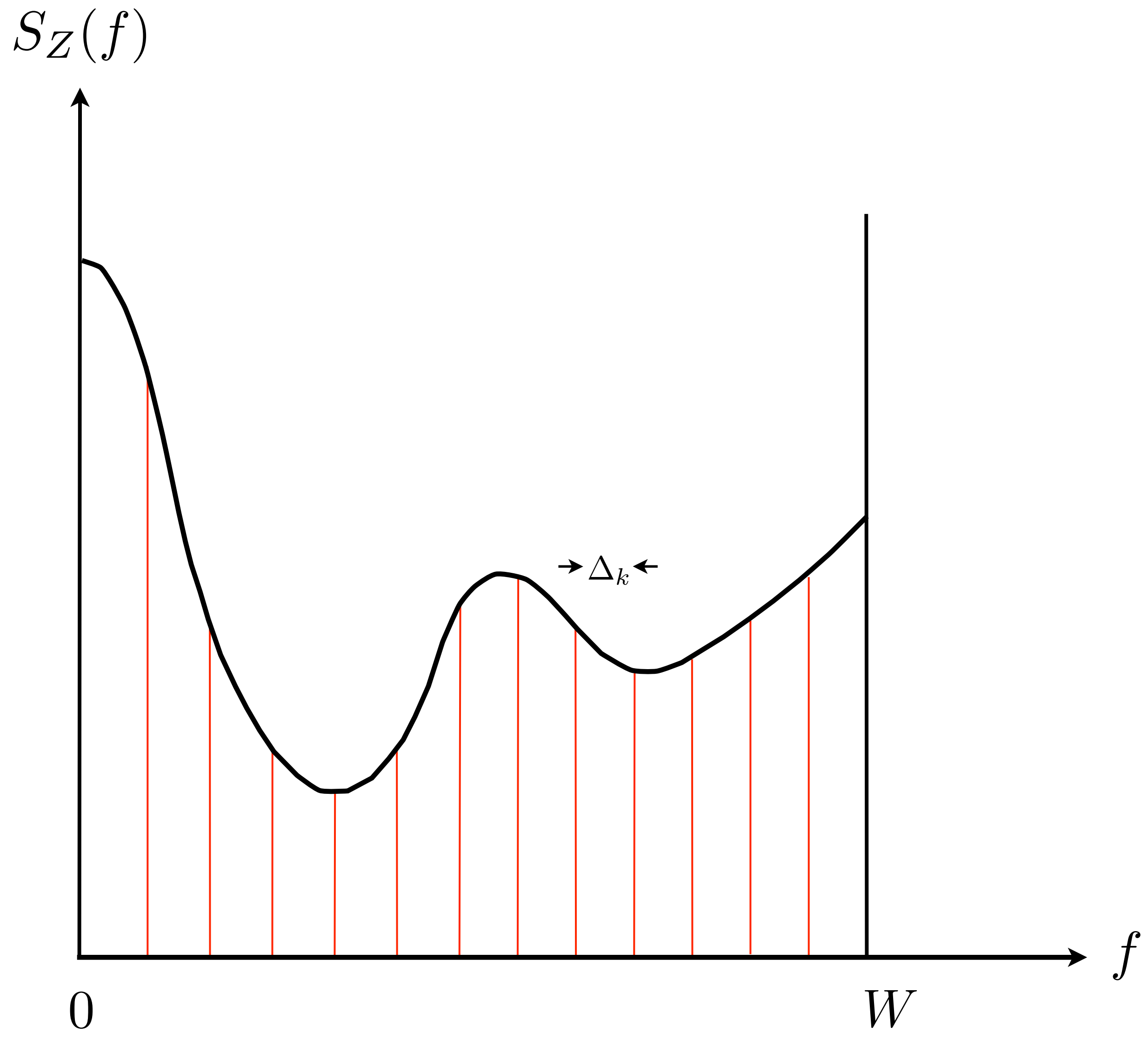

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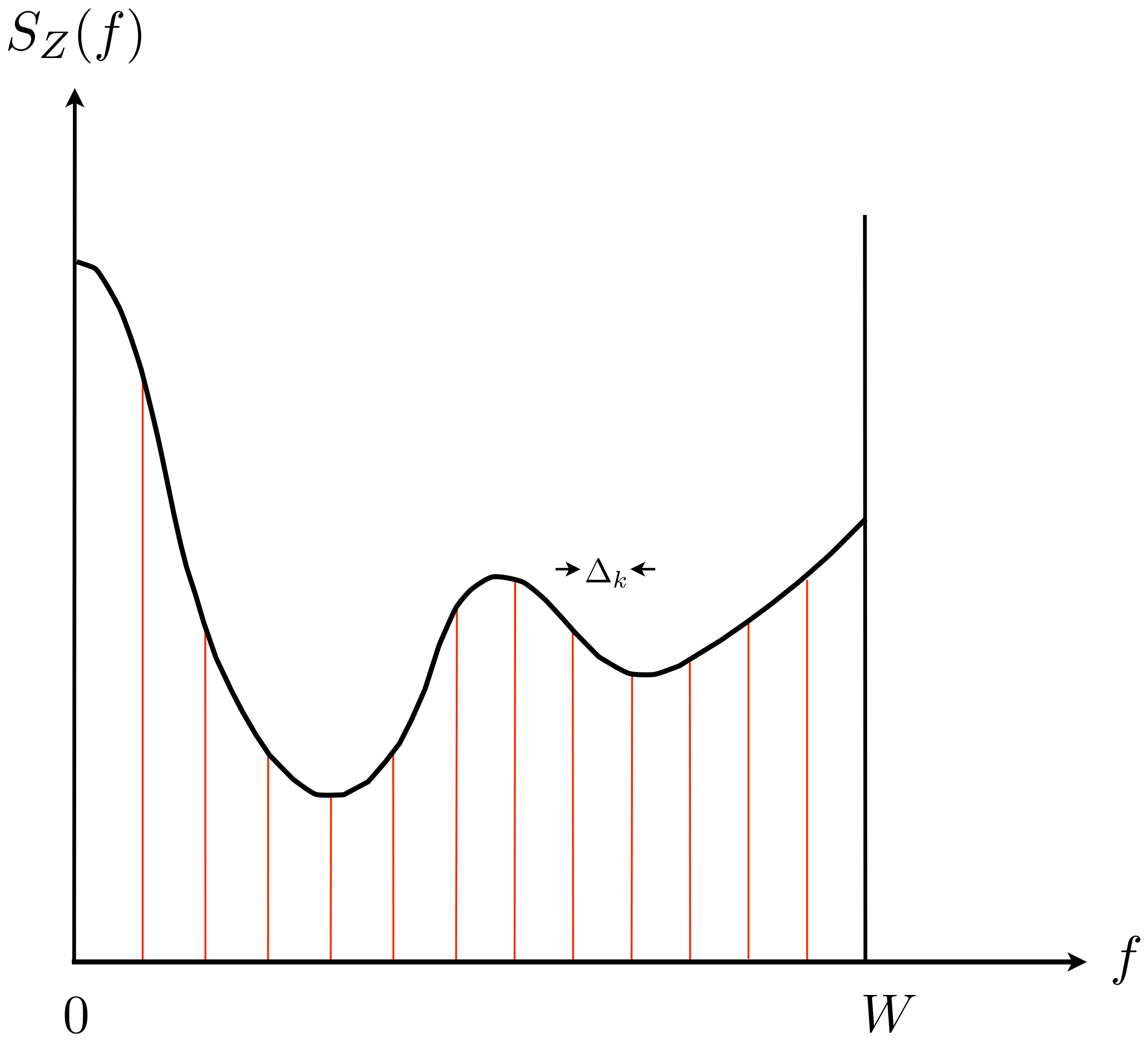

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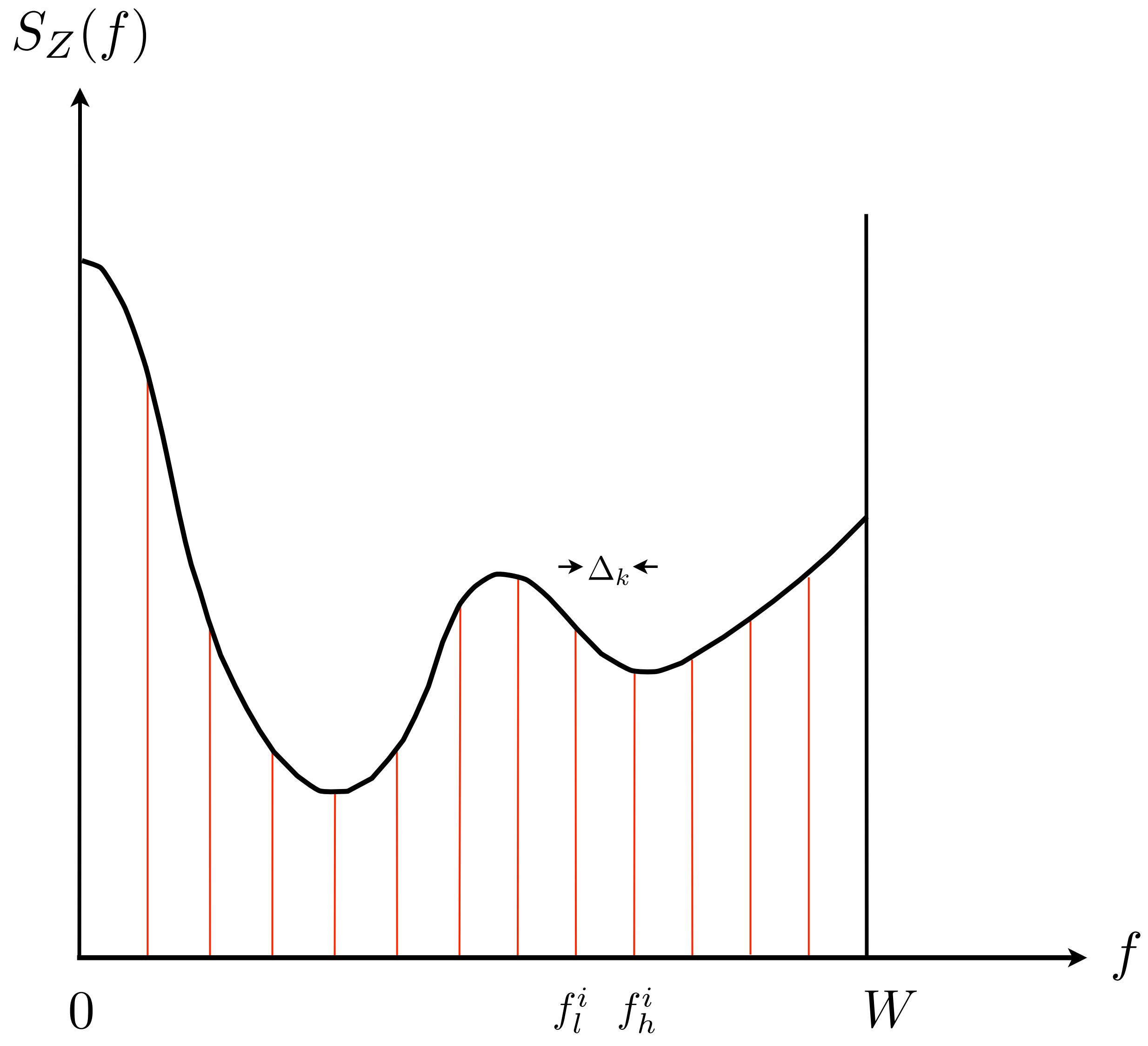

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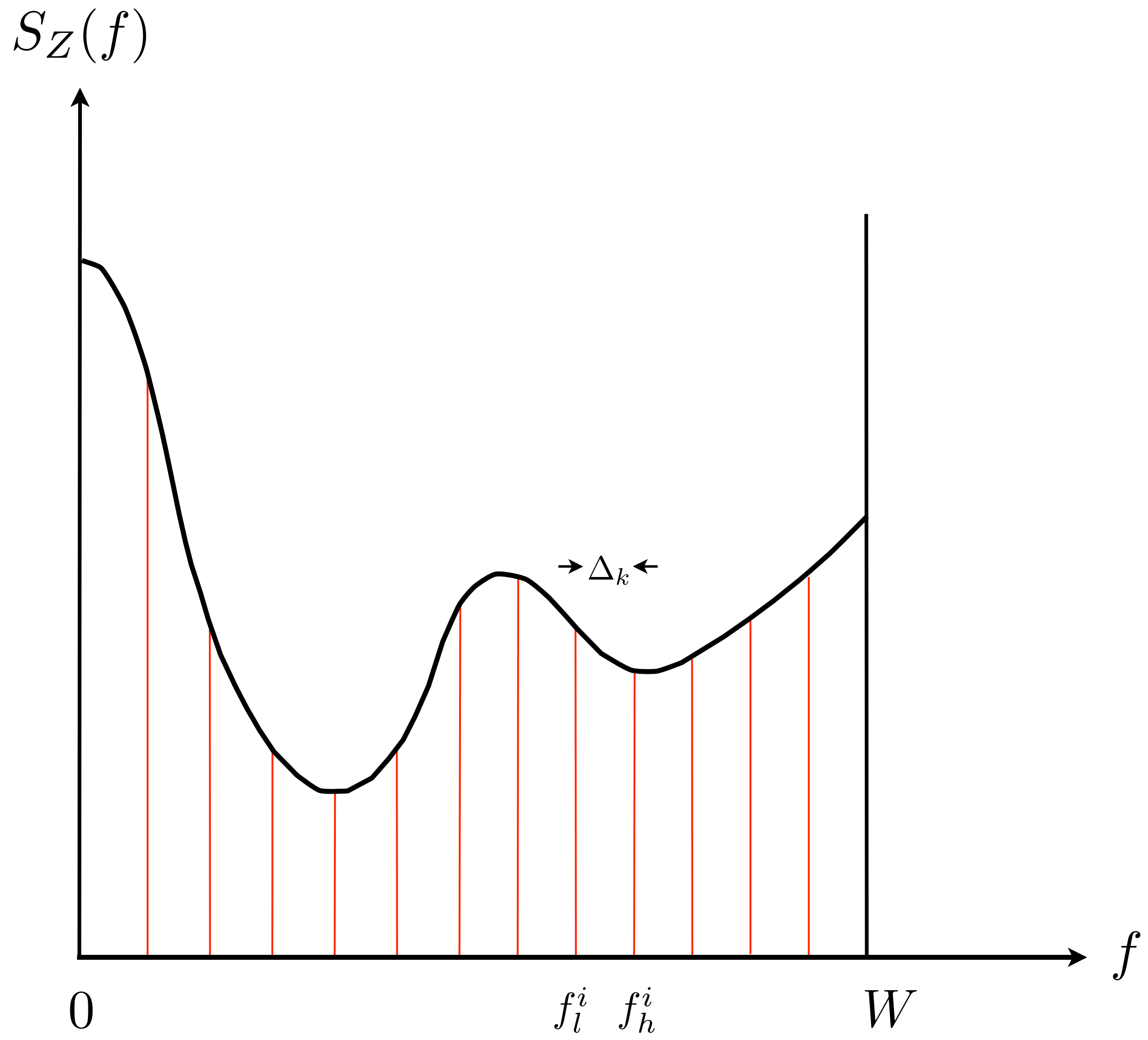

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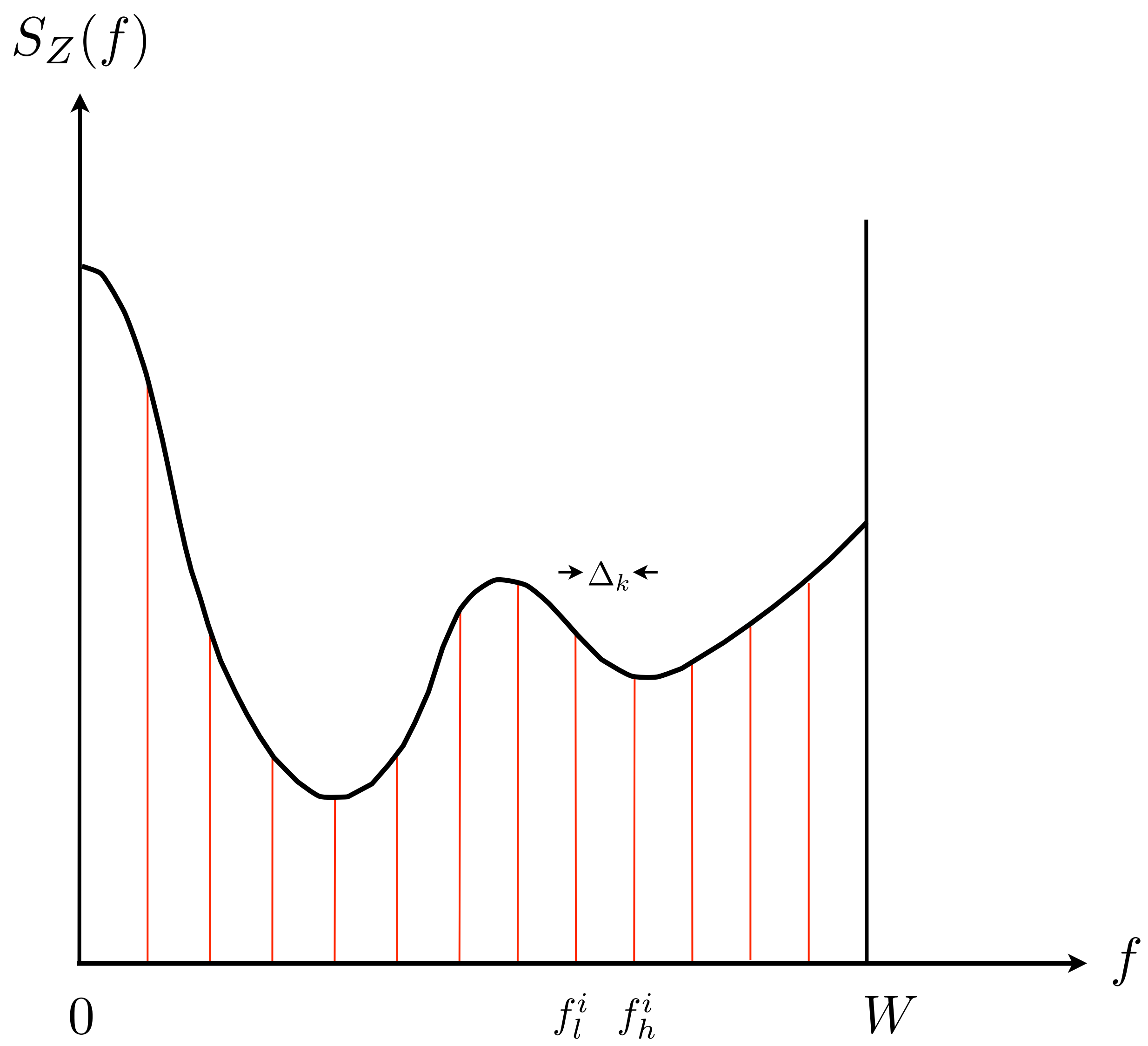

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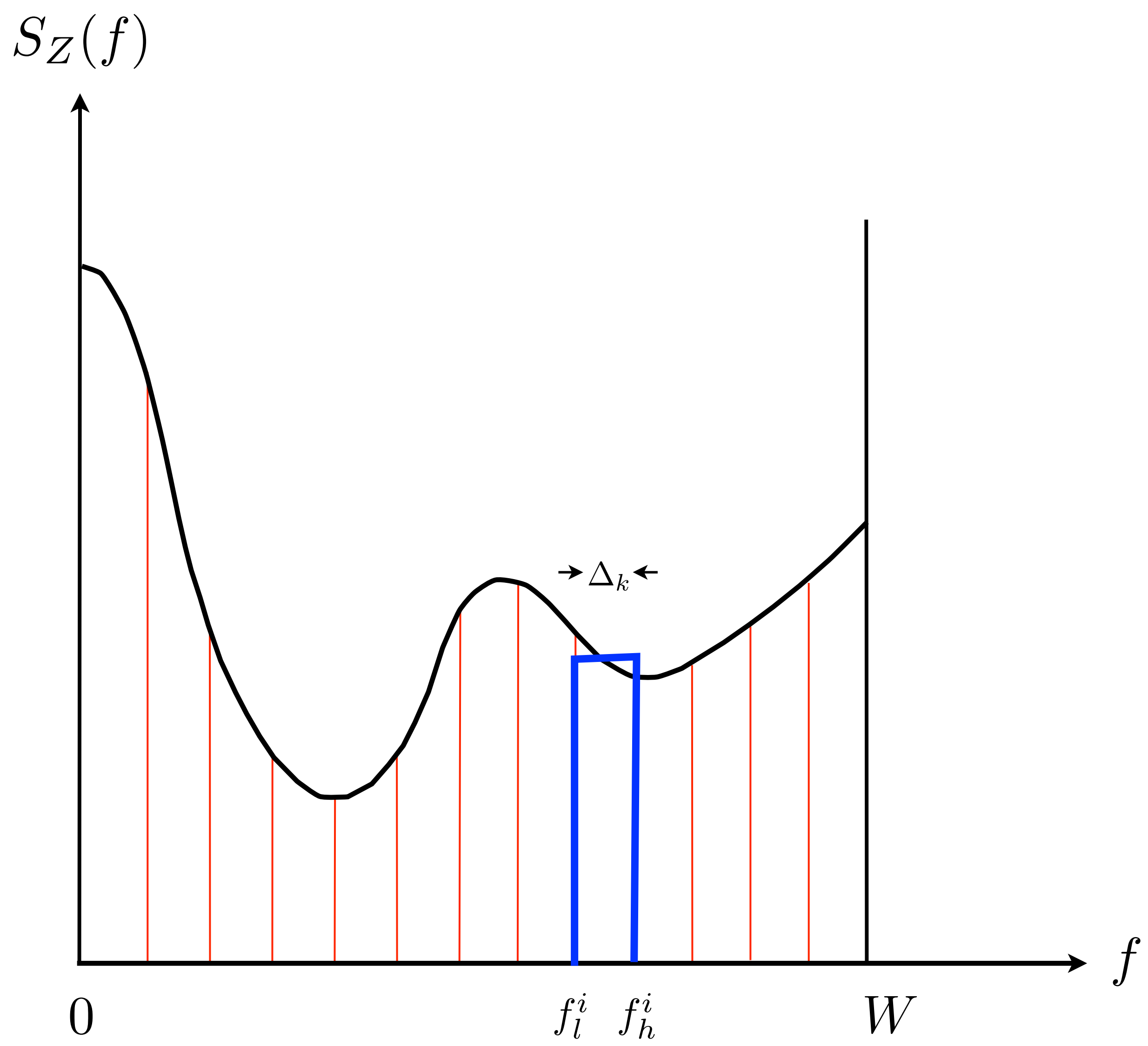

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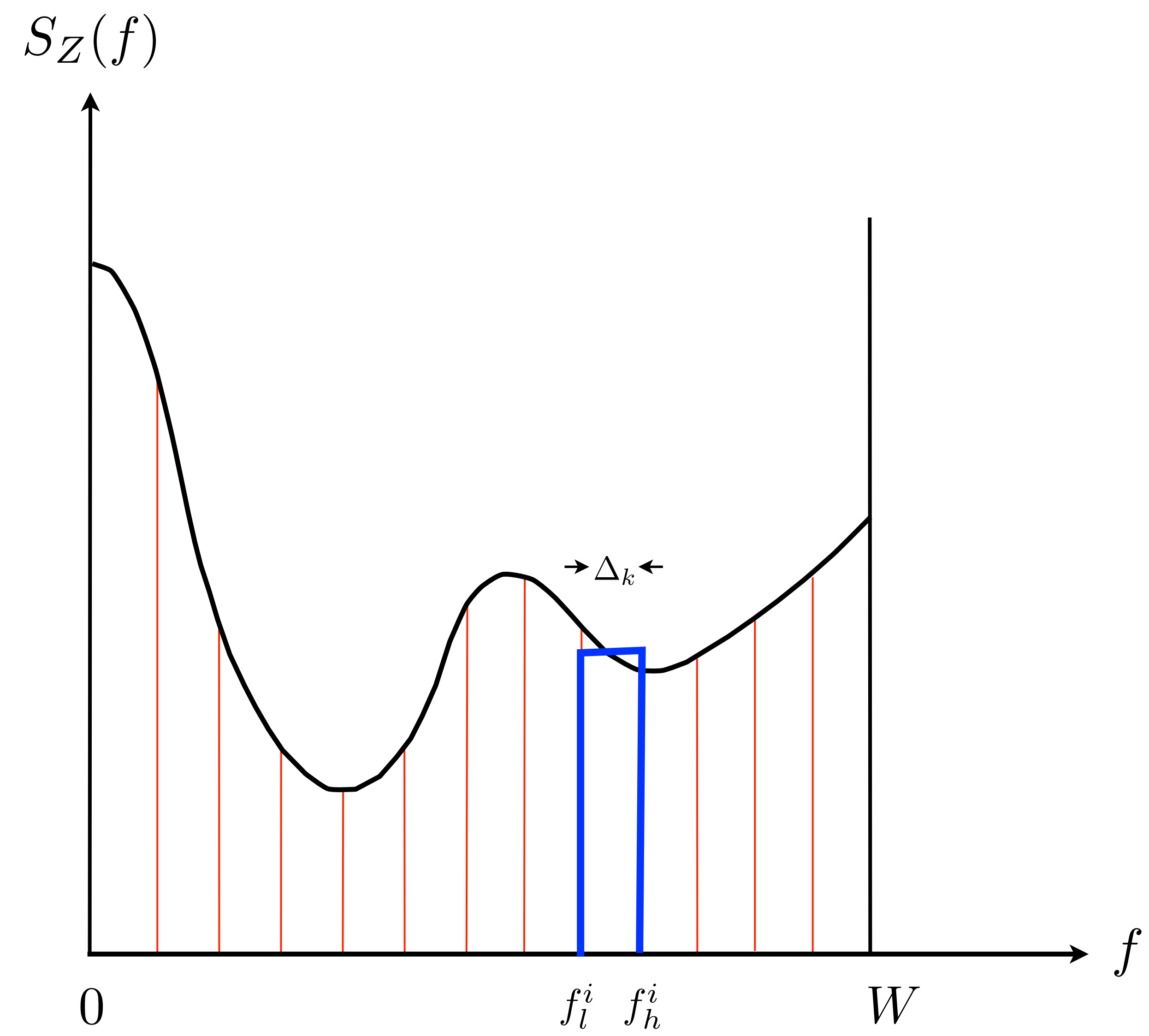

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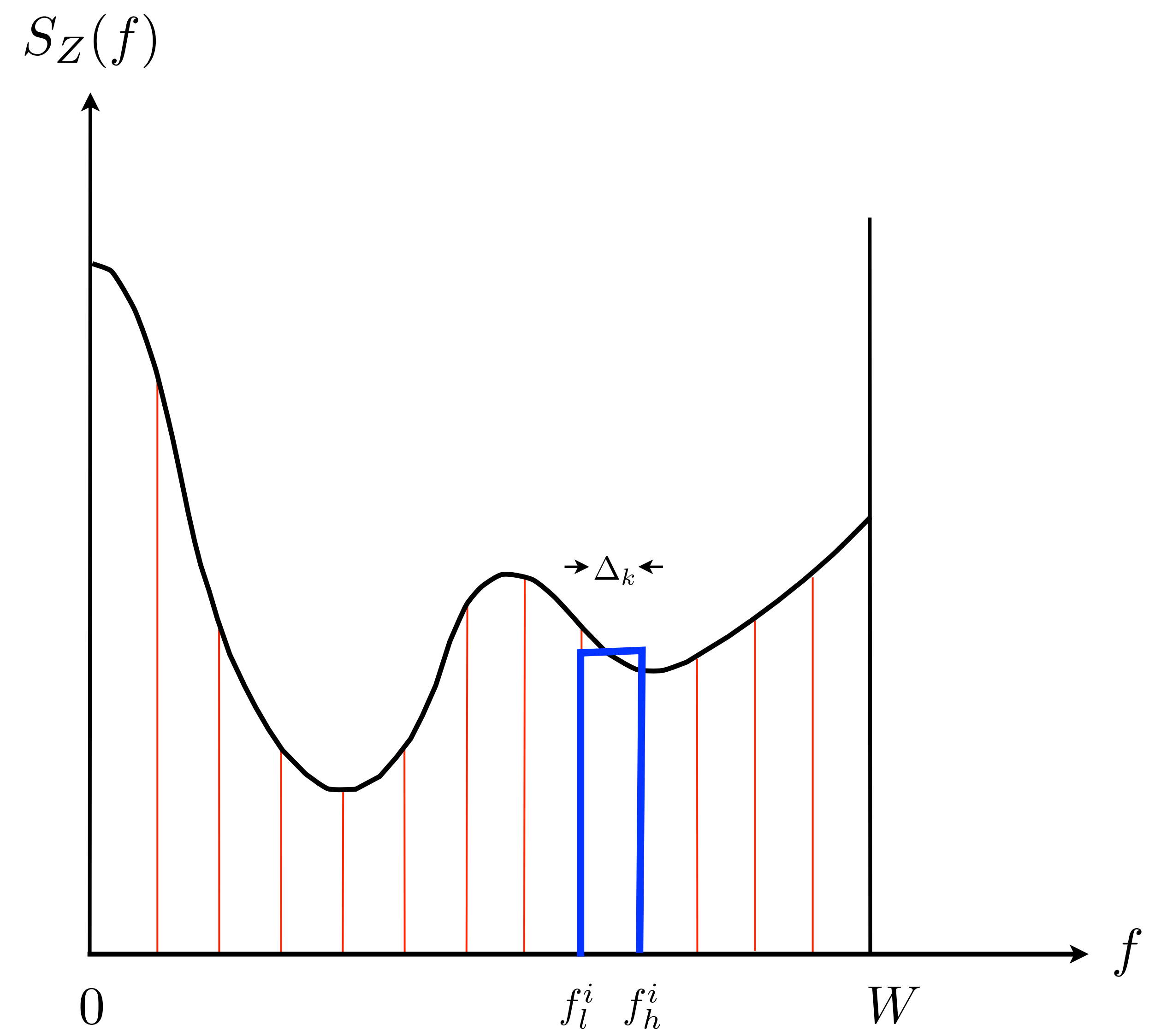

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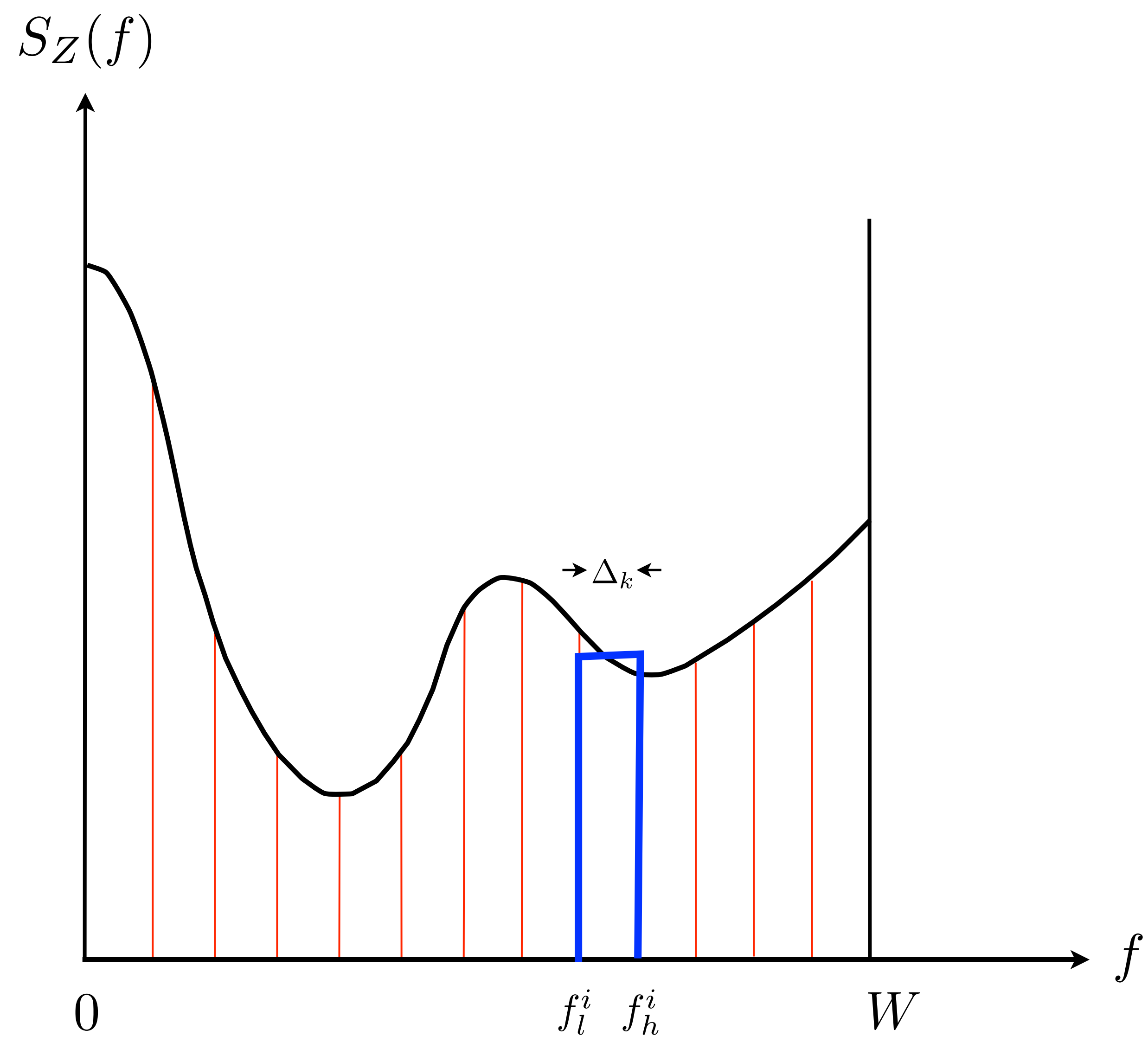

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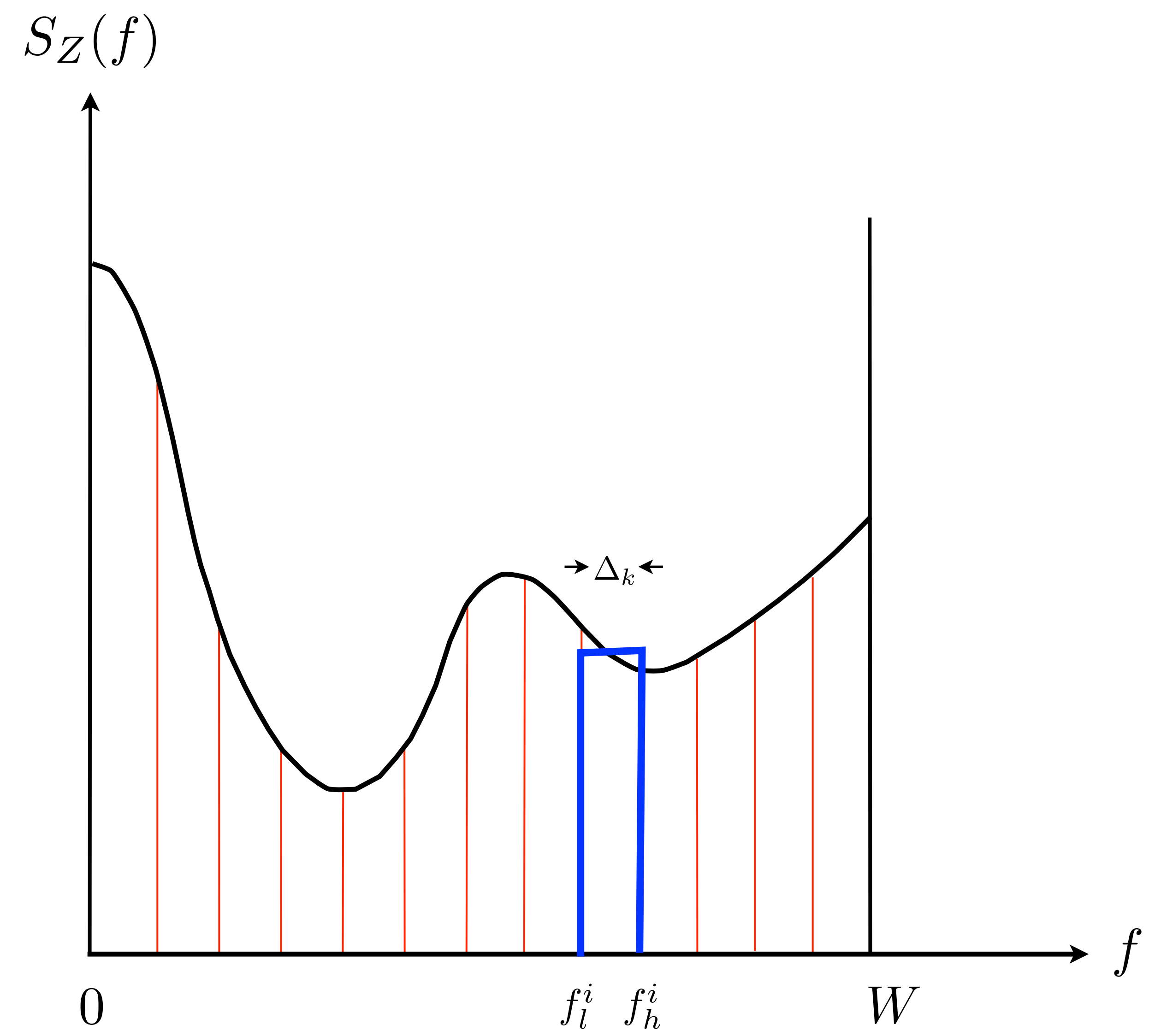

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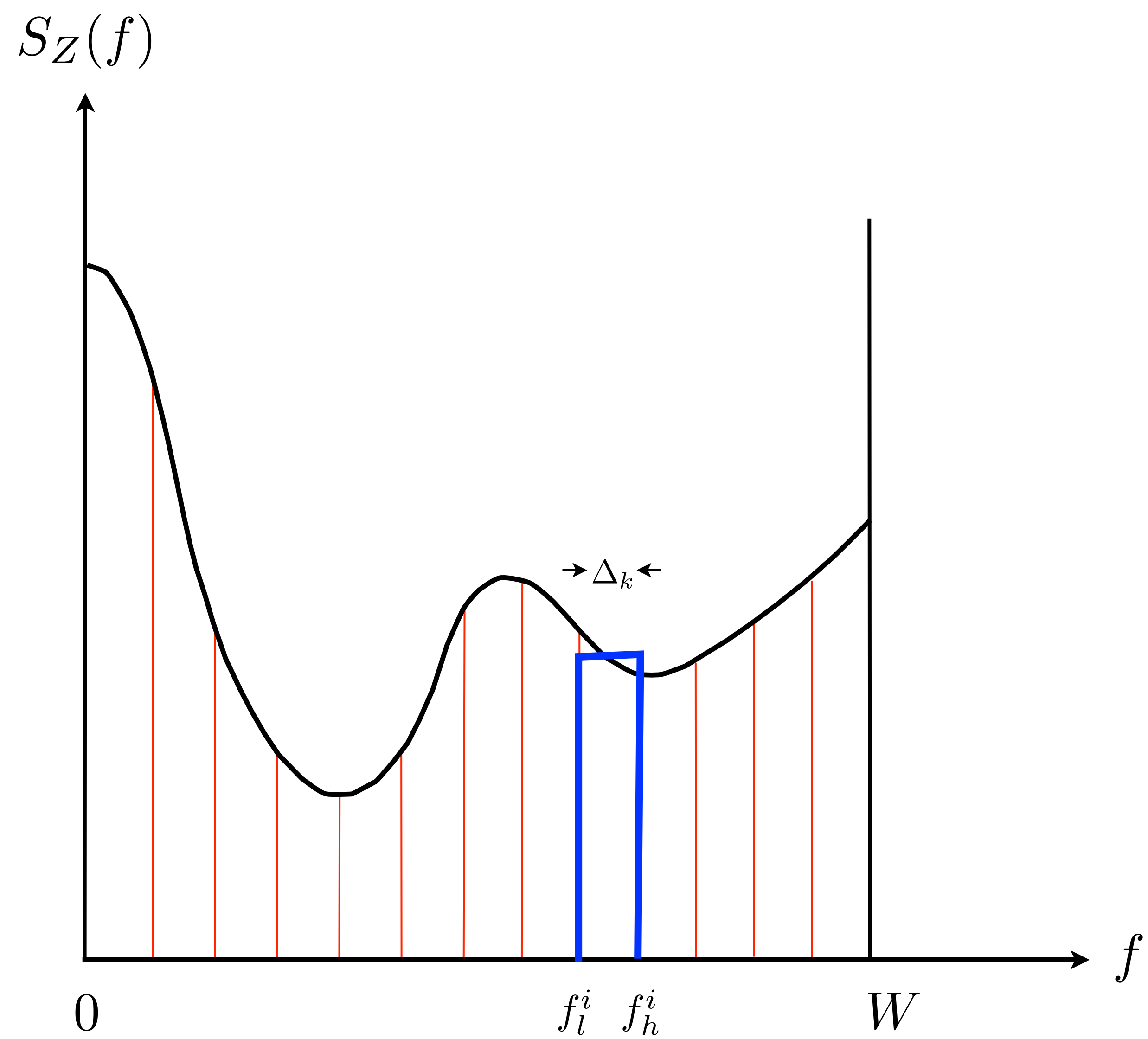

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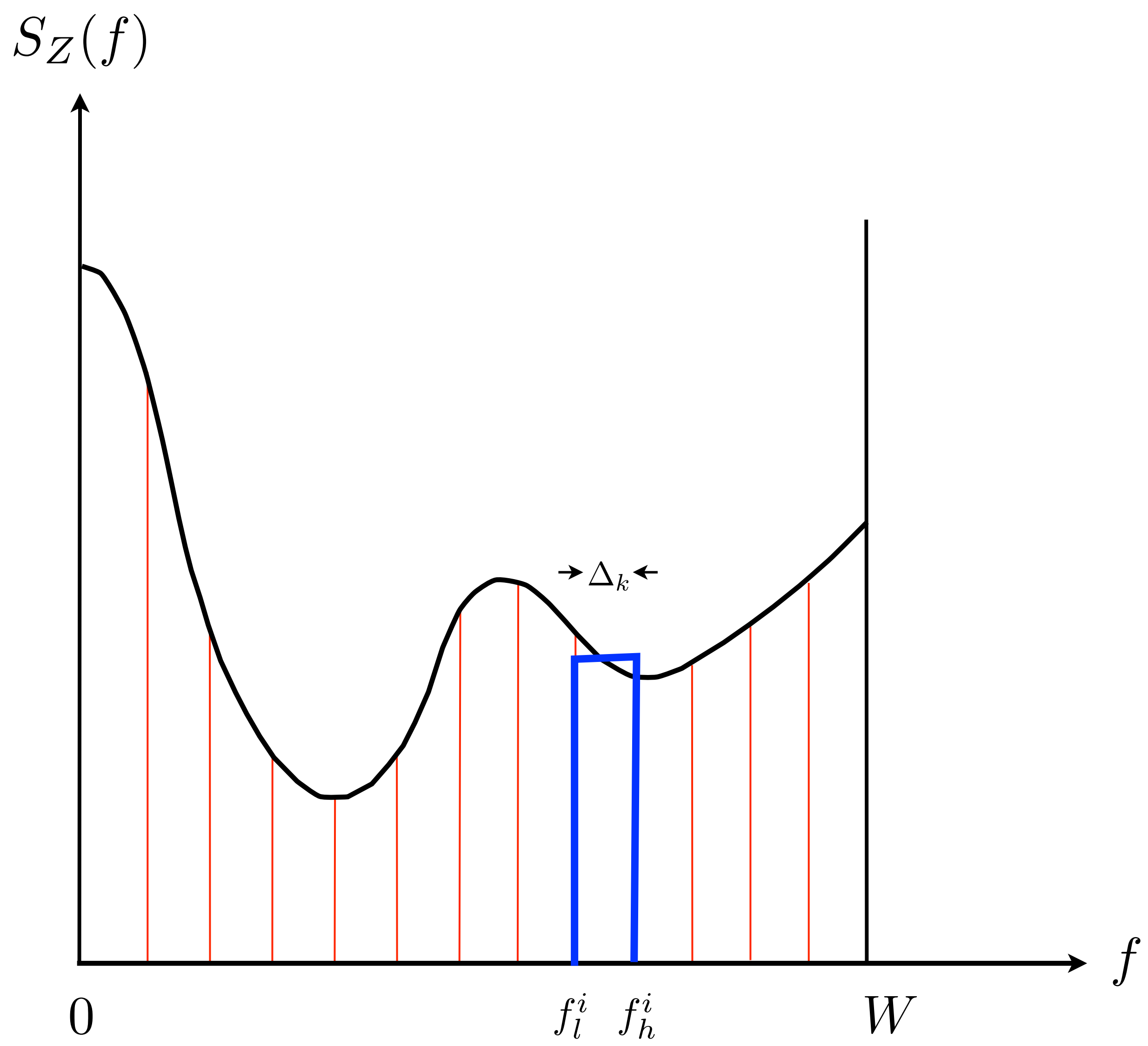

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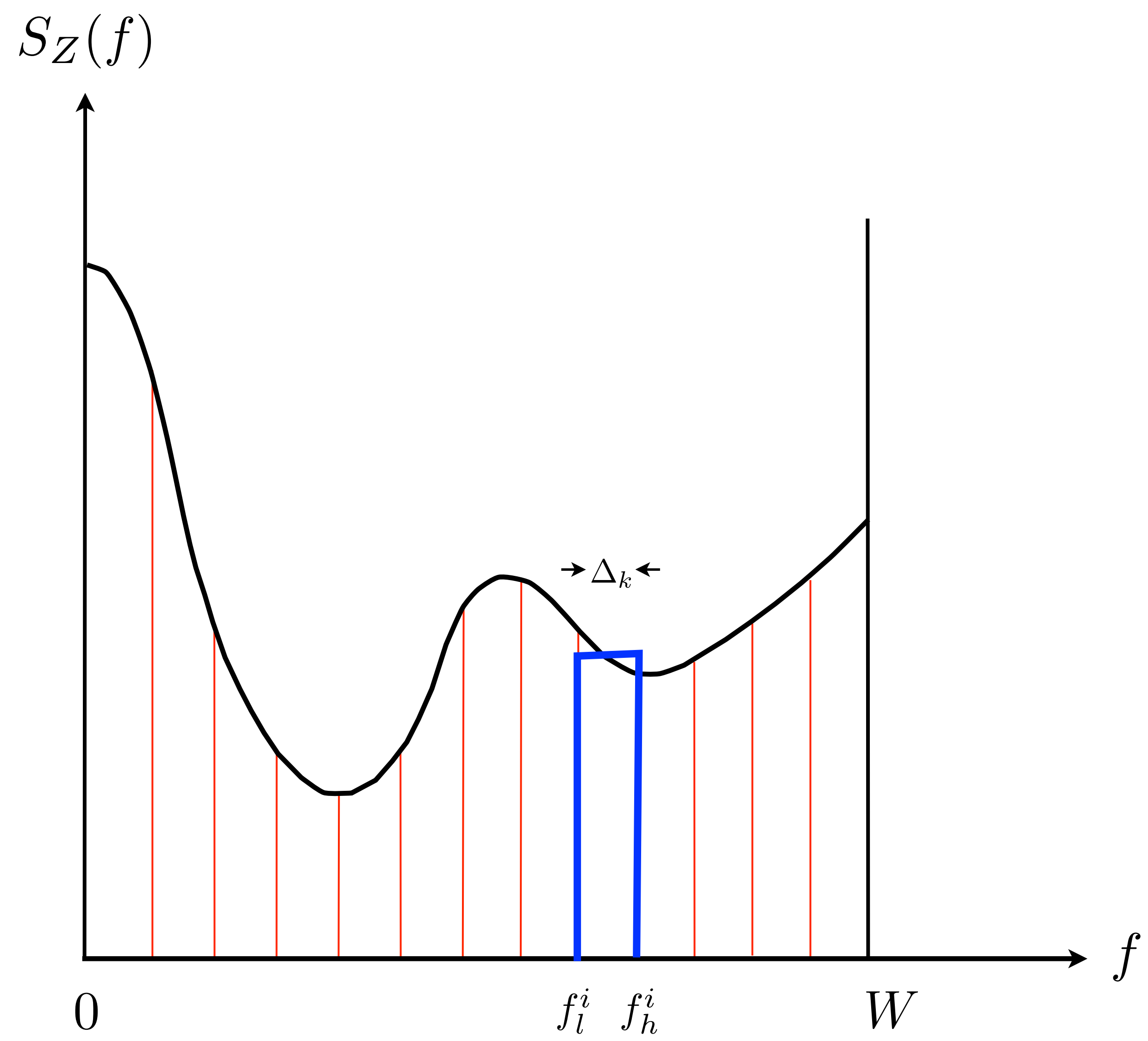

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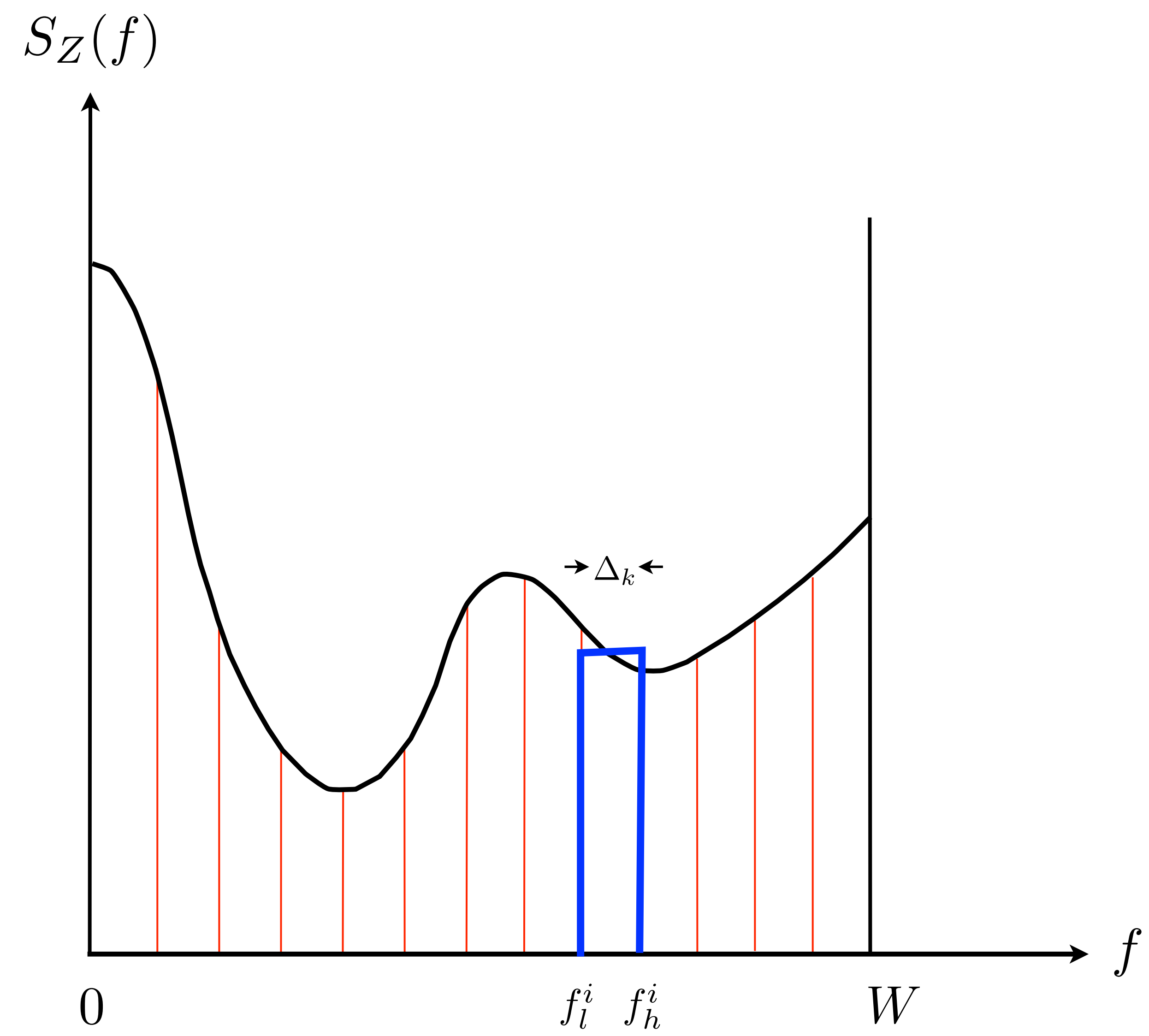

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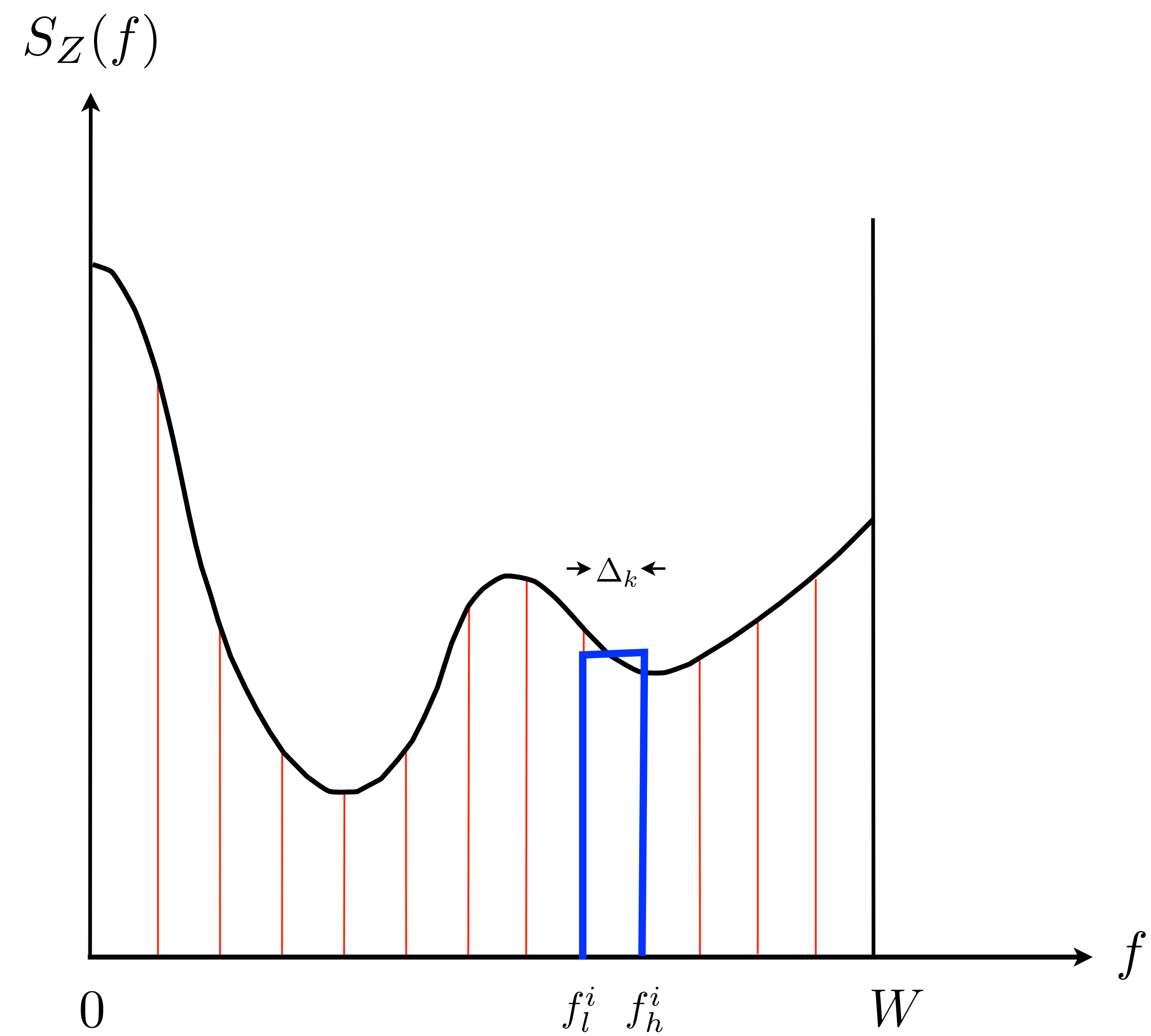

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2. Let the ith subinterval be $\left[f_l^i, f_h^i\right]$, $1 \le i \le k$.

3. As an approximation, assume that the noise power over the ith subinterval is a constant $S_{Z,i}$.

4. Then the channel consists of k sub-channels, with the ith sub-channel being a bandpass white Gaussian channel occupying the frequency band $\left[f_l^i, f_h^i\right]$.

5. Let P_i be the power allocated to the ith sub-channel. Then the capacity of the ith sub-channel is

$$\Delta_k \log\left(1 + \frac{P_i}{2S_{Z,i}\Delta_k}\right).$$

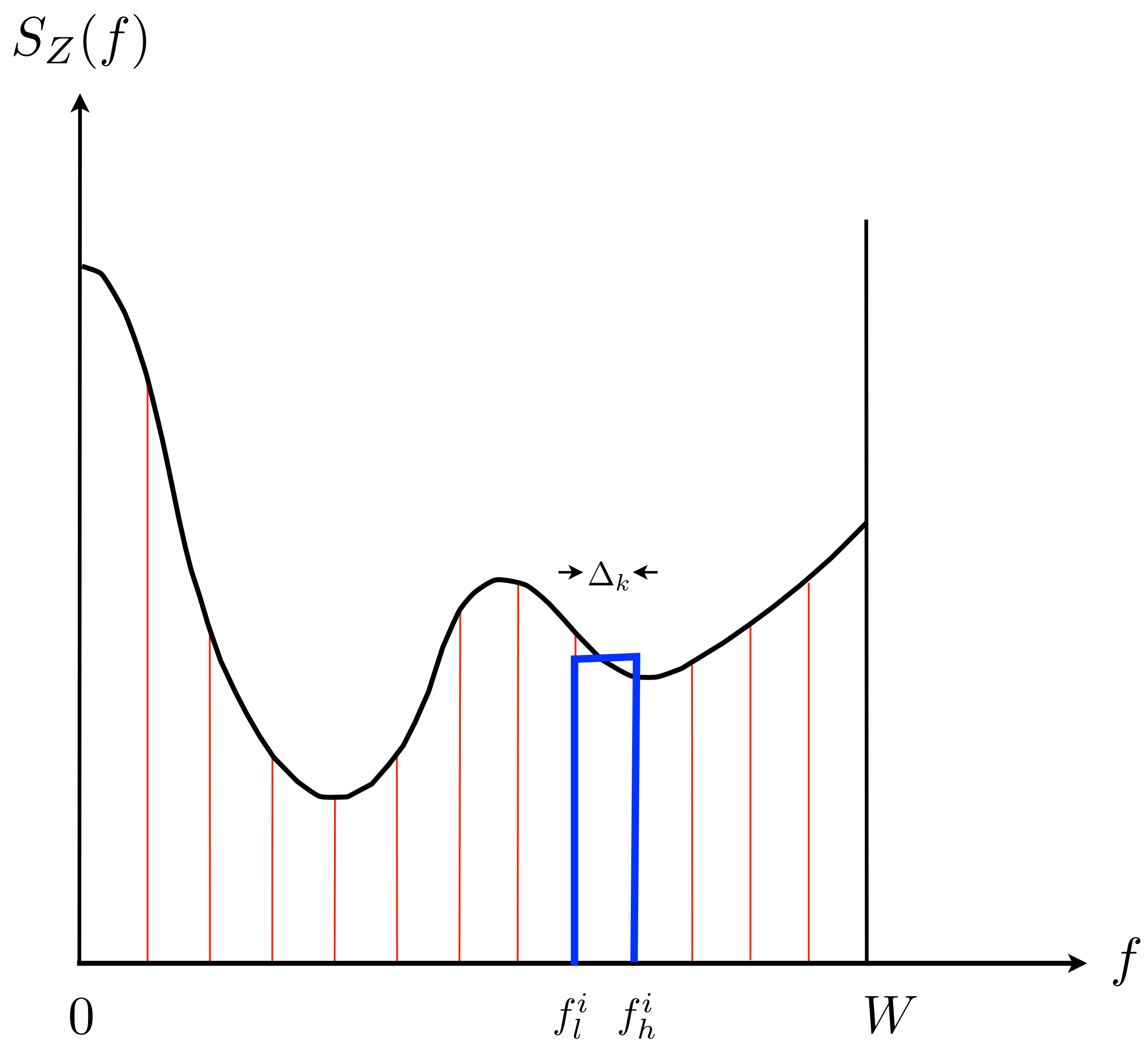
For a white Gaussian channel

- bandlimited to $[f_l, f_h]$, where f_l is a multiple of $W' = f_h - f_l$;
- noise level $= \frac{N_0}{2}$;
- power constraint $= P$;

the capacity is

$$W' \log\left(1 + \frac{P}{N_o W'}\right).$$

Here, $W' = \Delta_k$, $P = P_i$, and $\frac{N_0}{2} = S_{Z,i}$, or $N_0 = 2S_{Z,i}$.



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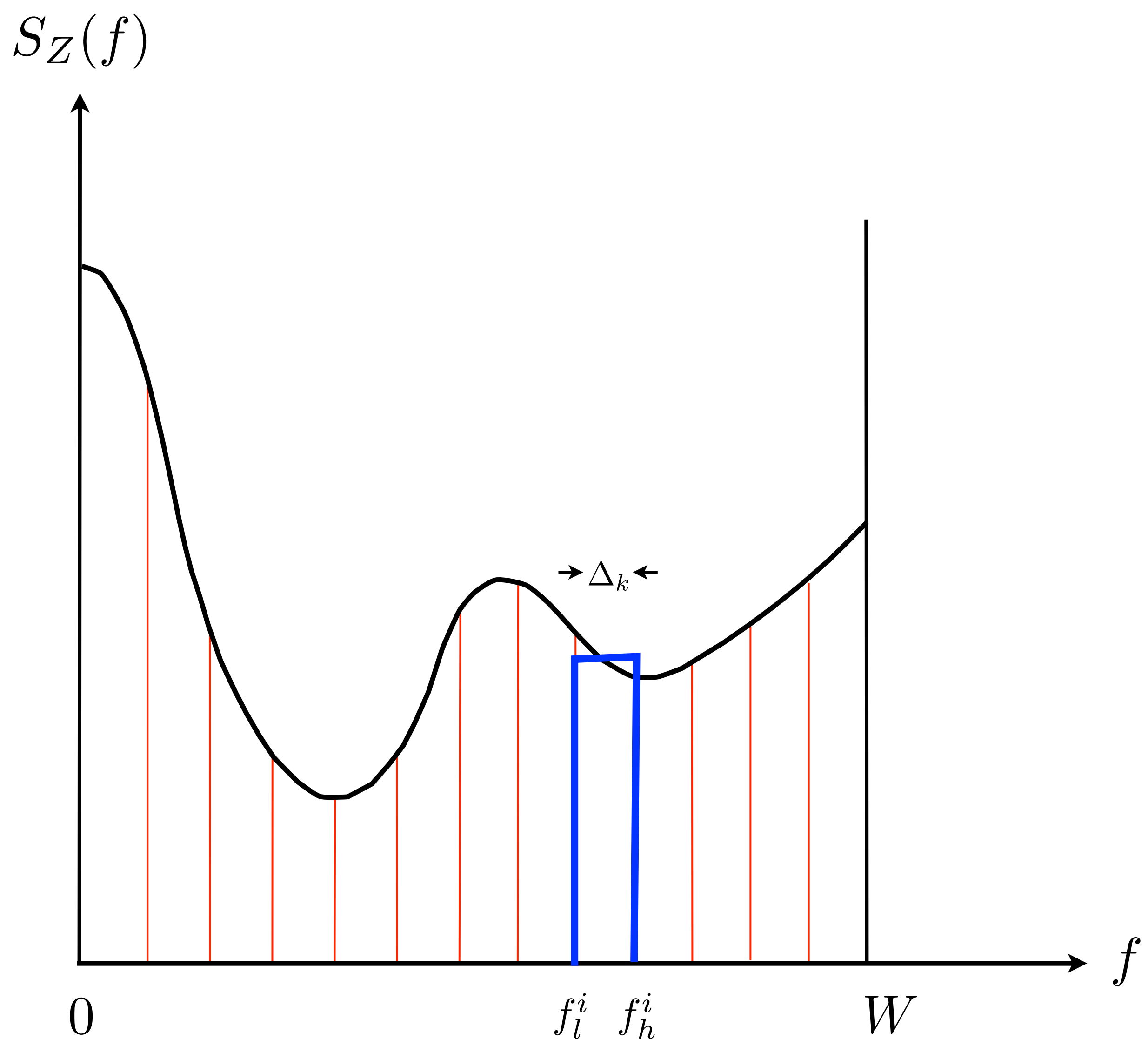

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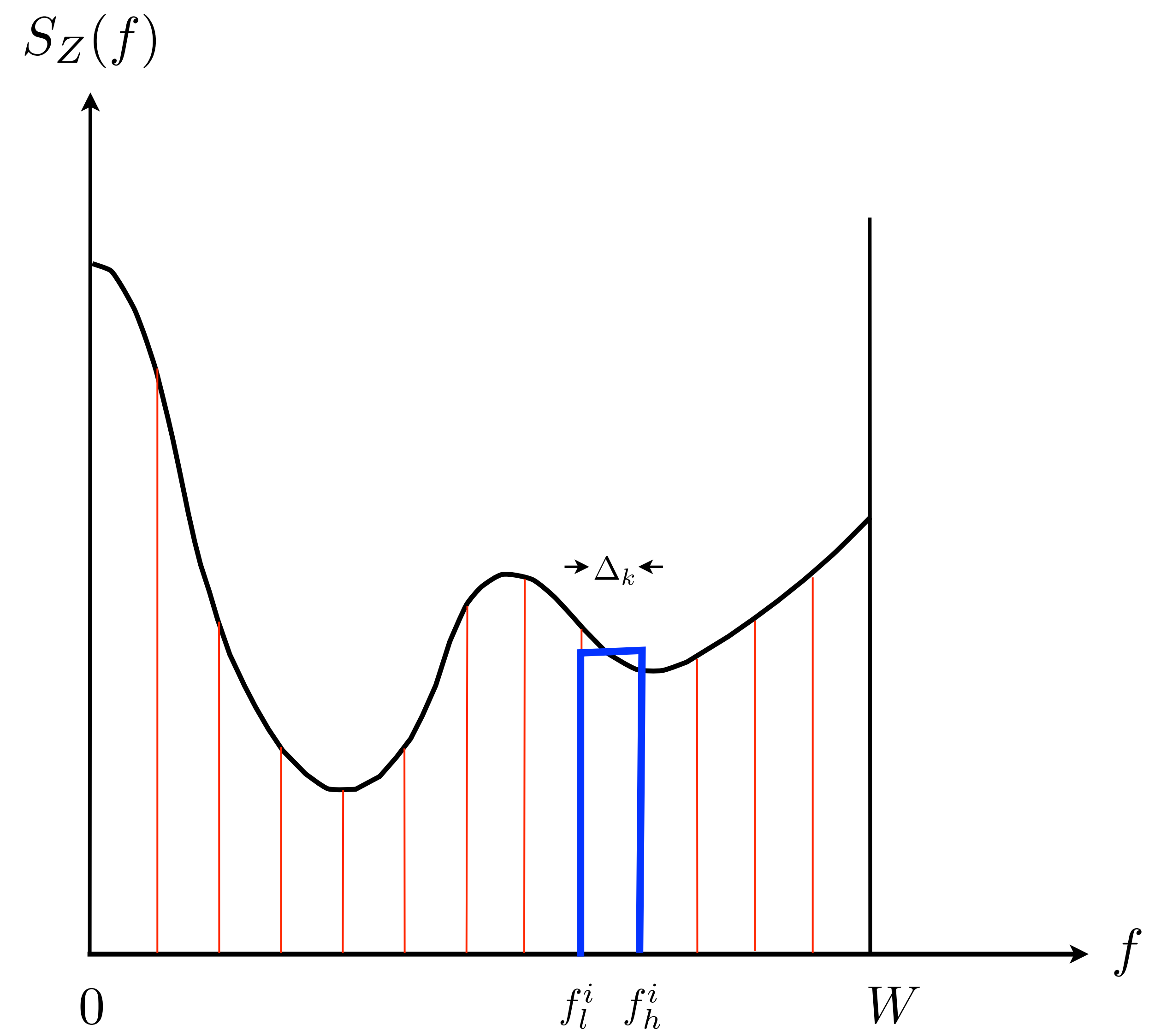

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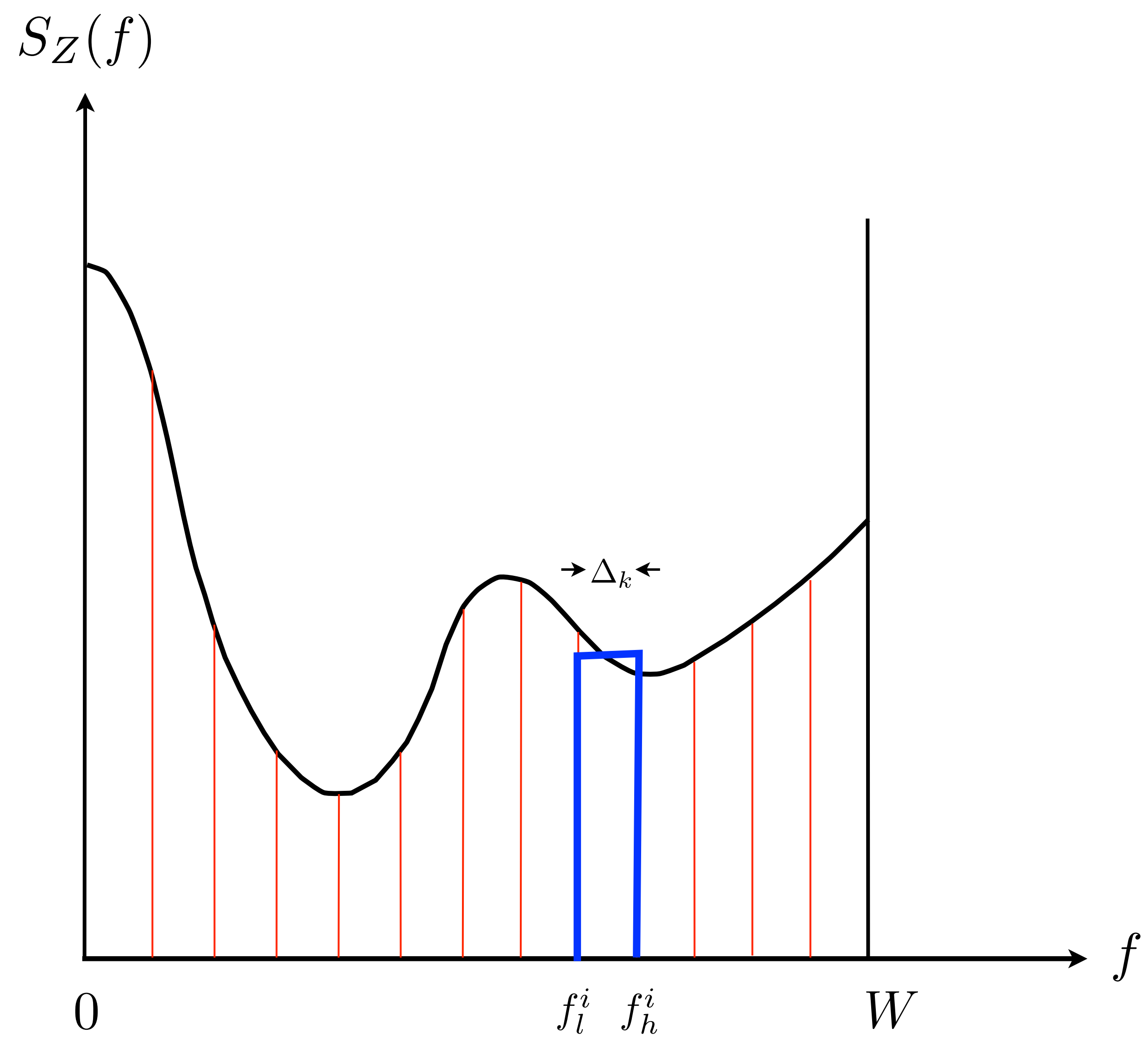

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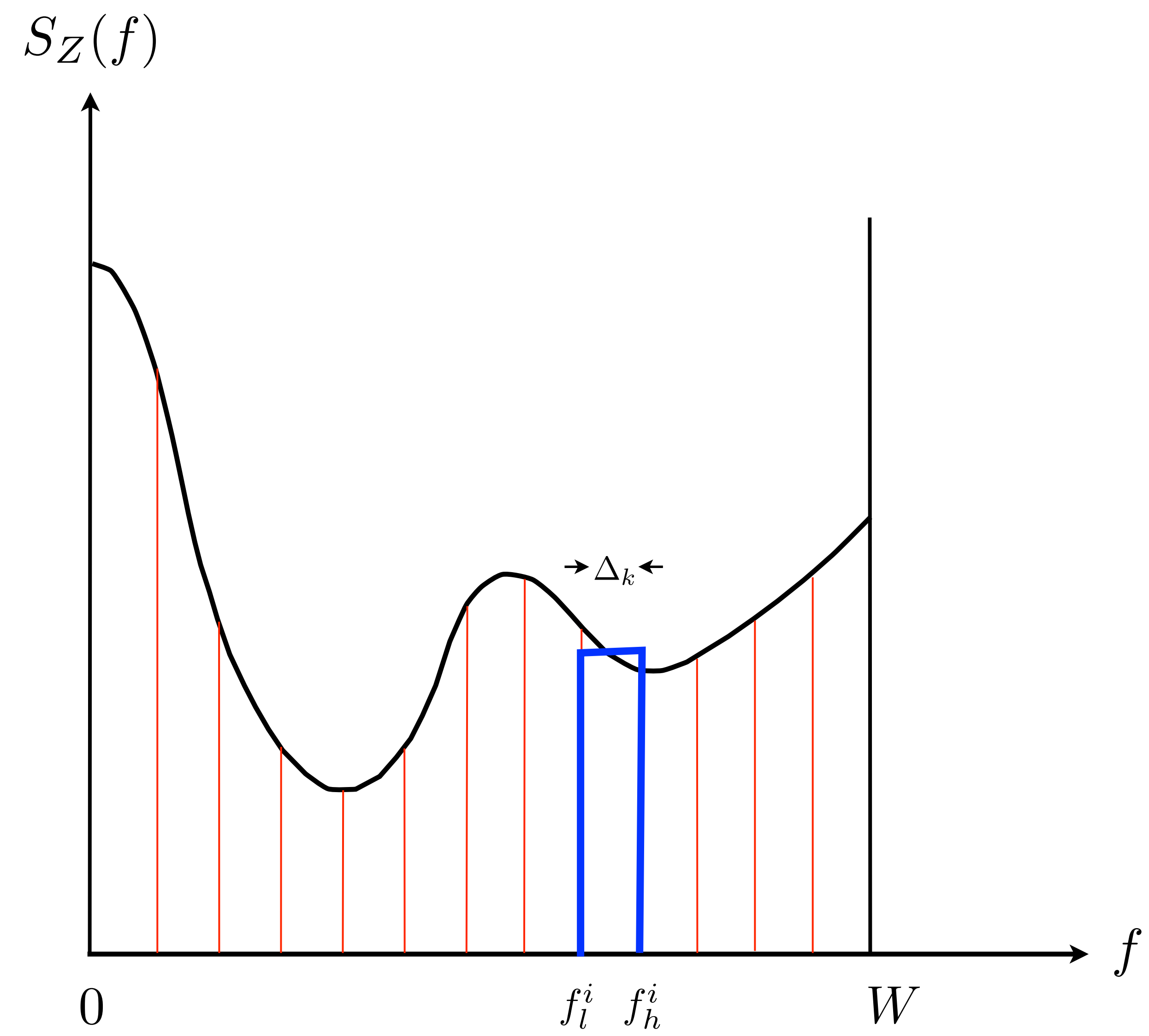

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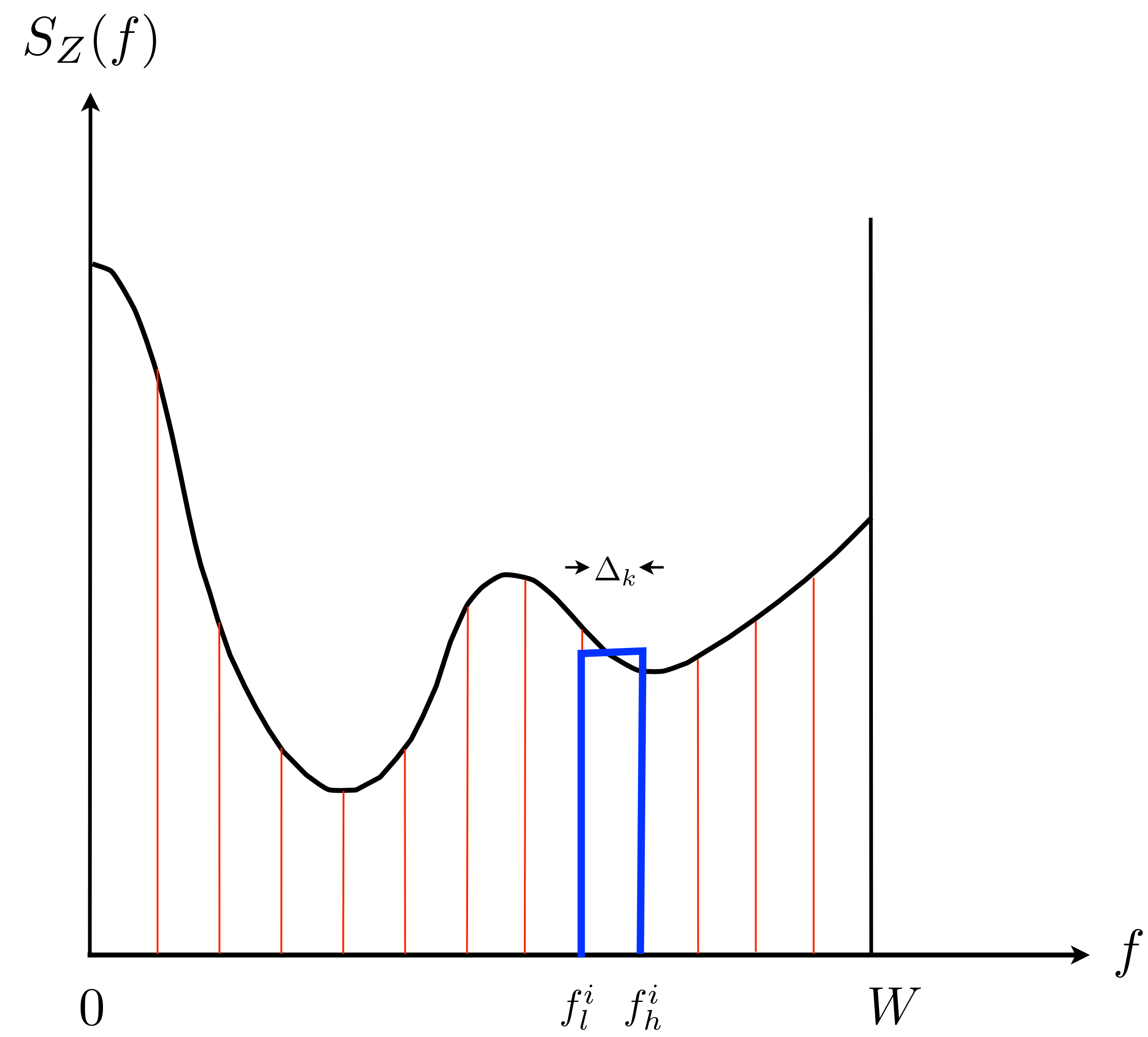

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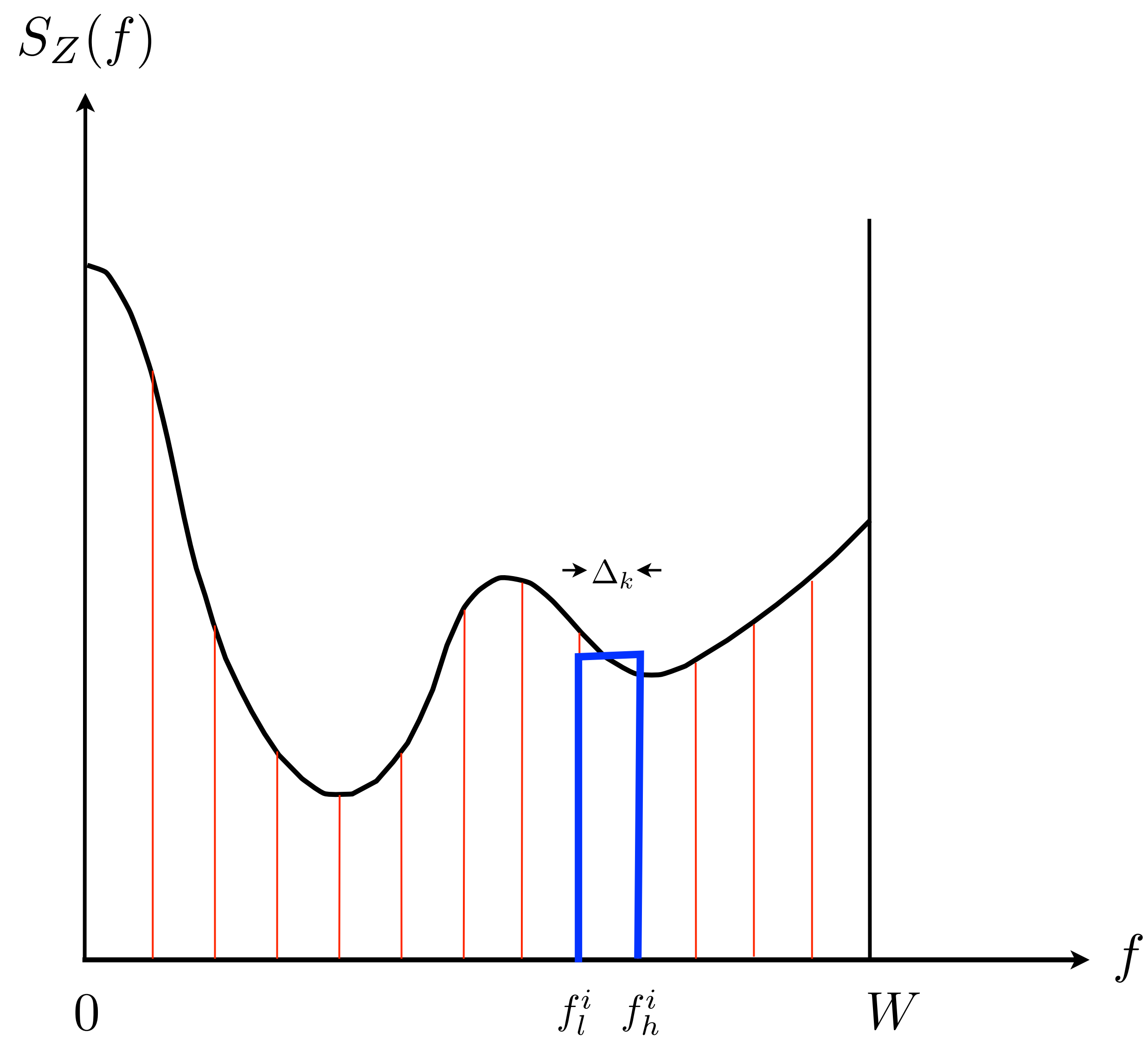

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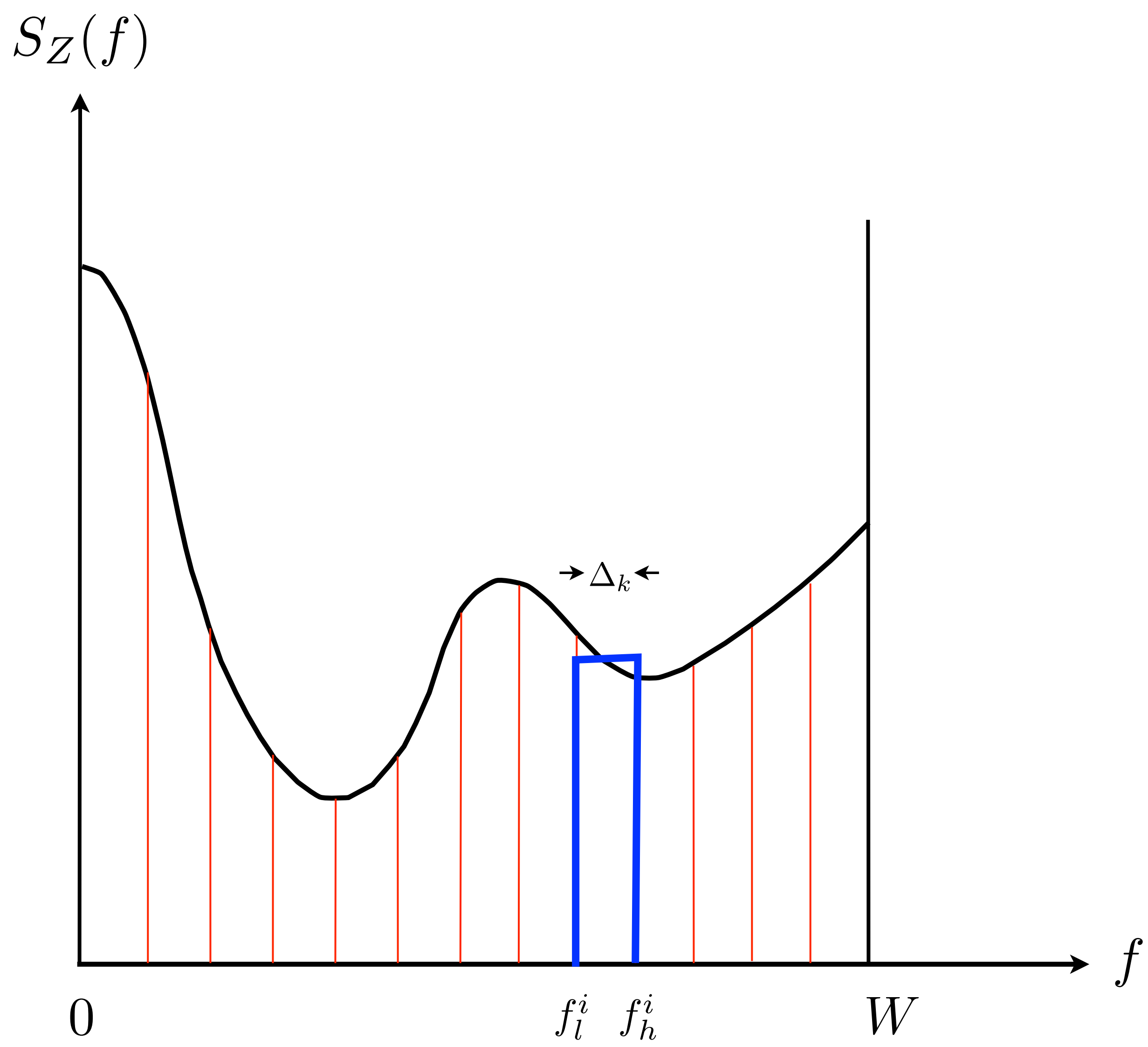

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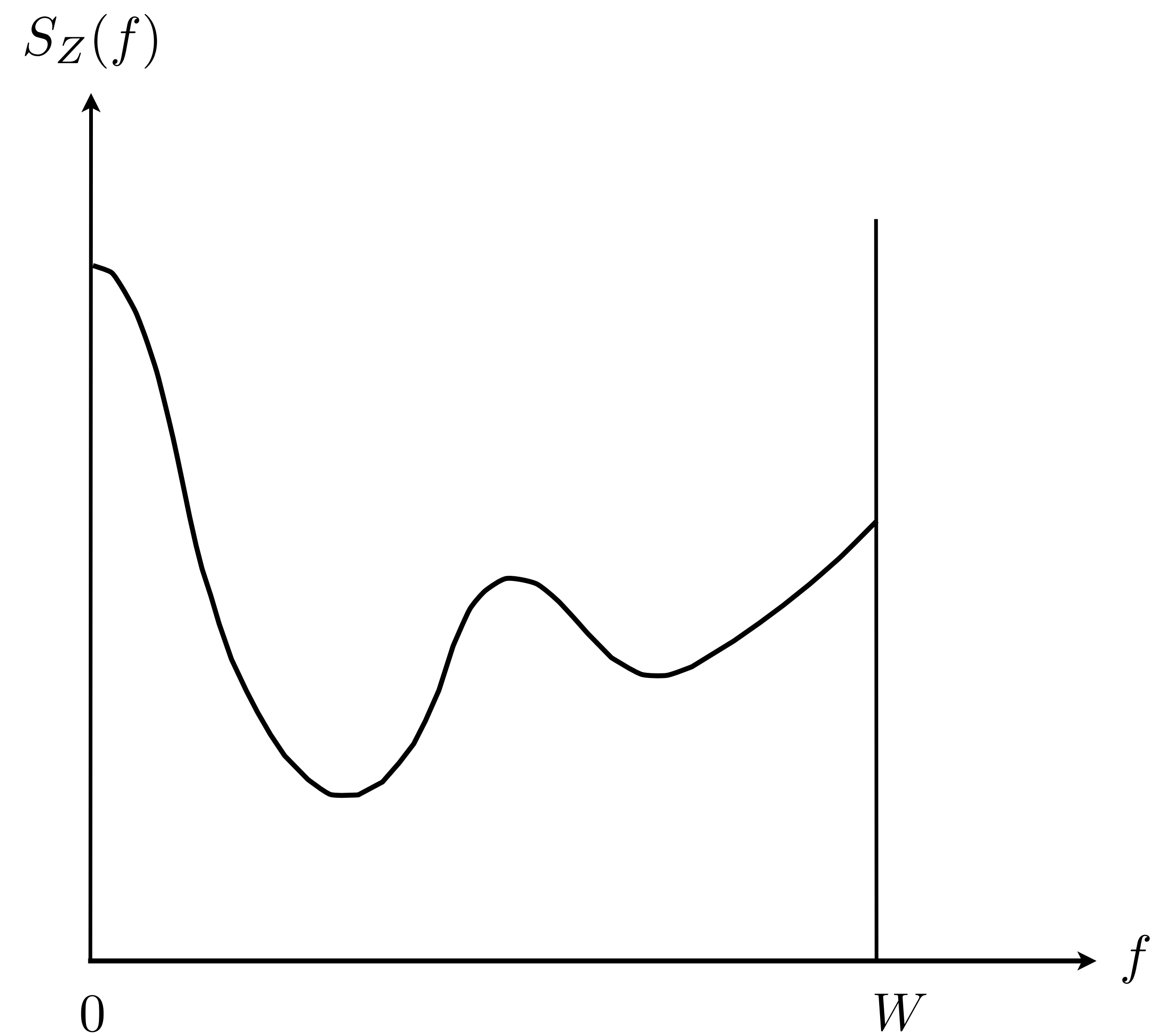

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1. Divide $[0, W]$ into k subintervals, each with width $\Delta_k = \frac{W}{k}$.

2. Let the ith subinterval be $\left[f_l^i, f_h^i\right]$, $1 \le i \le k$.

3. As an approximation, assume that the noise power over the ith subinterval is a constant $S_{Z,i}$.

4. Then the channel consists of k sub-channels, with the ith sub-channel being a bandpass white Gaussian channel occupying the frequency band $\left[f_l^i, f_h^i\right]$.

5. Let P_i be the power allocated to the ith sub-channel. Then the capacity of the ith sub-channel is

$$\Delta_k \log\left(1 + \frac{P_i}{2S_{Z,i}\Delta_k}\right).$$

6. The noise process $Z_i'(t)$ of the ith sub-channel is obtained by passing $Z(t)$ through the corresponding ideal bandpass filter bandlimited to $\left[f_l^i, f_h^i\right]$.

7. It can be shown (see Problem 9) that the noise processes $Z_i'(t)$, $1 \le i \le k$ are independent.

8. By sampling the sub-channels at the Nyquist rate $2\Delta_k$, the k sub-channels can be regarded as a system of parallel Gaussian channels.

9. Thus the capacity of the channel is equal to the sum of the capacities of the individual sub-channels when the power allocation among the k sub-channels is optimal.

10. Let P_i^* be the optimal power allocation for the ith sub-channel.

11. The channel capacity is equal to

$$\sum_{i=1}^{k} \Delta_k \log\left(1 + \frac{P_i^*}{2S_{Z,i}\Delta_k}\right) = \sum_{i=1}^{k} \Delta_k \log\left(1 + \frac{\frac{P_i^*}{2\Delta_k}}{S_{Z,i}}\right)$$

where by Proposition 11.23,

$$\frac{P_i^*}{2\Delta_k} = (\nu - S_{Z,i})^+ \quad \text{with} \quad \sum_{i=1}^{k} P_i^* = P.$$

12. As $k \to \infty$,

$$\sum_{i=1}^{k} \Delta_k \log\left(1 + \frac{\frac{P_i^*}{2\Delta_k}}{S_{Z,i}}\right)$$

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12. As $k \to \infty$,

$$\sum_{i=1}^{k} \Delta_k \log\left(1 + \frac{\frac{P_i^*}{2\Delta_k}}{S_{Z,i}}\right) \to \int_0^W \log\left(1 + \frac{(\nu - S_Z(f))^+}{S_Z(f)}\right) df$$

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&\quad \text{bits per unit time}
\end{aligned}$$

since $S_Z(-f) = S_Z(f)$ (see Problem 8).

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$$\sum_{i=1}^{k} \underline{P_i^*} = \sum_{i=1}^{k} \underline{2\Delta_k(\nu - S_{Z,i})^+}$$

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14. Therefore,

$$\sum_{i=1}^{k} P_i^* = P \quad \to \quad \int_{-W}^{W} (\nu - S_Z(f))^+ df = P. \quad (1)$$

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since $S_Z(-f) = S_Z(f)$ (see Problem 8).

13. As $k \to \infty$,

$$\begin{aligned} \sum_{i=1}^{k} P_i^* \quad &= \quad \sum_{i=1}^{k} 2\Delta_k (\nu - S_{Z,i})^+ \\ &\to \quad 2\int_0^W (\nu - S_Z(f))^+ df \\ &\to \quad \int_{-W}^{W} (\nu - S_Z(f))^+ df \end{aligned}$$

since $S_Z(f) = S_Z(-f)$.

14. Therefore,

$$\sum_{i=1}^{k} P_i^* = P \quad \to \quad \int_{-W}^{W} (\nu - S_Z(f))^+ df = P. \qquad (1)$$

The Capacity of the Bandlimited Colored Gaussian Channel

11. The channel capacity is equal to

$$\sum_{i=1}^{k} \Delta_k \log\left(1 + \frac{P_i^*}{2S_{Z,i}\Delta_k}\right) = \sum_{i=1}^{k} \Delta_k \log\left(1 + \frac{\frac{P_i^*}{2\Delta_k}}{S_{Z,i}}\right)$$

where by Proposition 11.23,

$$\frac{P_i^*}{2\Delta_k} = (\nu - S_{Z,i})^+ \quad \text{with} \quad \sum_{i=1}^{k} P_i^* = P.$$

12. As $k \to \infty$,

$$\begin{aligned} & \sum_{i=1}^{k} \Delta_k \log\left(1 + \frac{\frac{P_i^*}{2\Delta_k}}{S_{Z,i}}\right) \\ \to \quad & \int_0^W \log\left(1 + \frac{(\nu - S_Z(f))^+}{S_Z(f)}\right) df \\ = \quad & \frac{1}{2}\int_{-W}^{W} \log\left(1 + \frac{(\nu - S_Z(f))^+}{S_Z(f)}\right) df \end{aligned}$$

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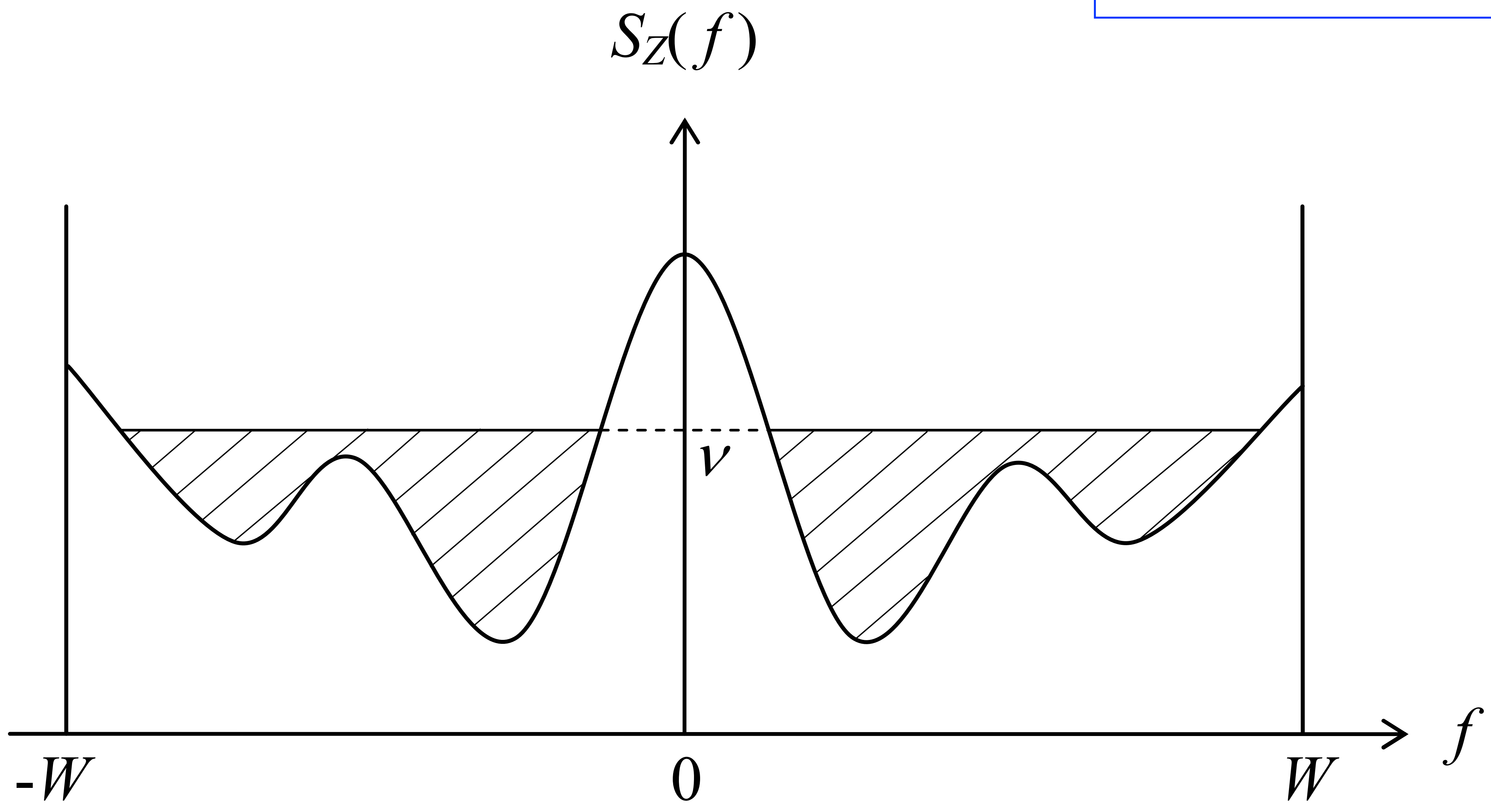
15. The optimal power allocation given in (1) has a water-filling interpretation.

Water-Filling

$$\int_{-W}^{W} (\nu - S_Z(f))^+ \, df = P$$



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