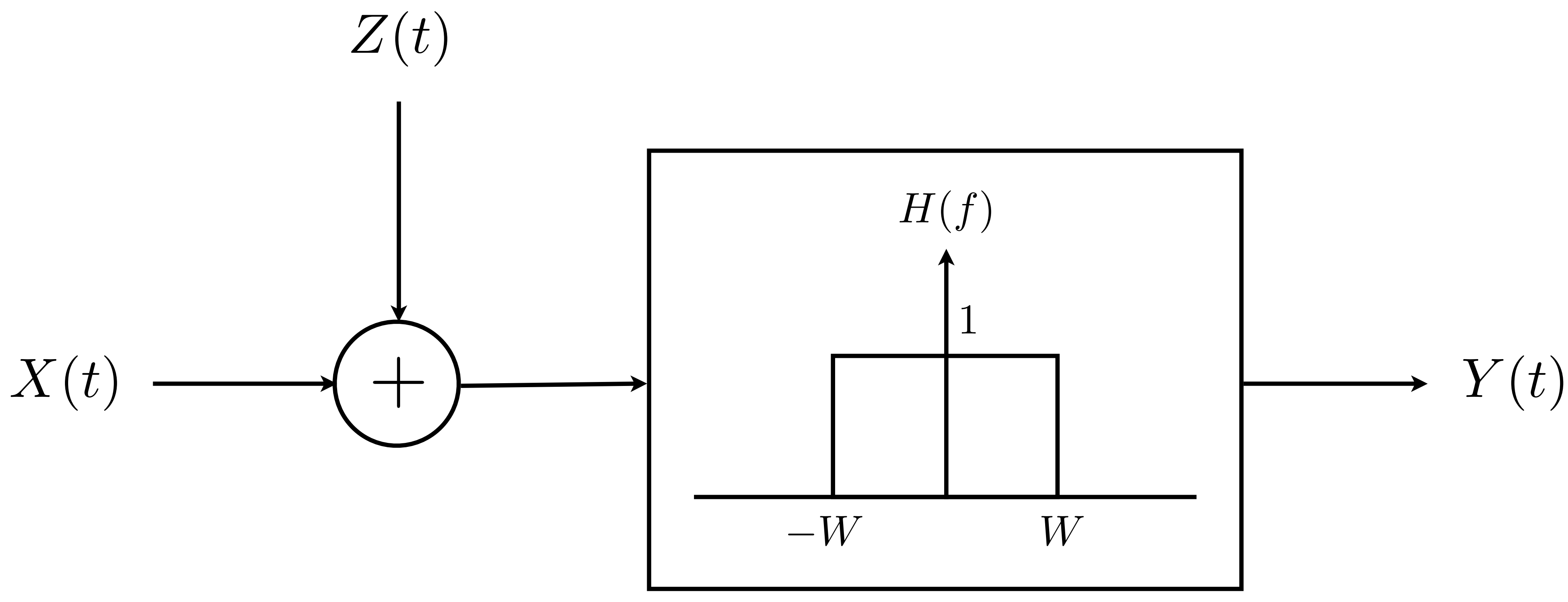
香港中文大學
The Chinese University of Hong Kong

11.7 The Bandlimited White Gaussian Channel


Z(t)
X(t)
H(f)
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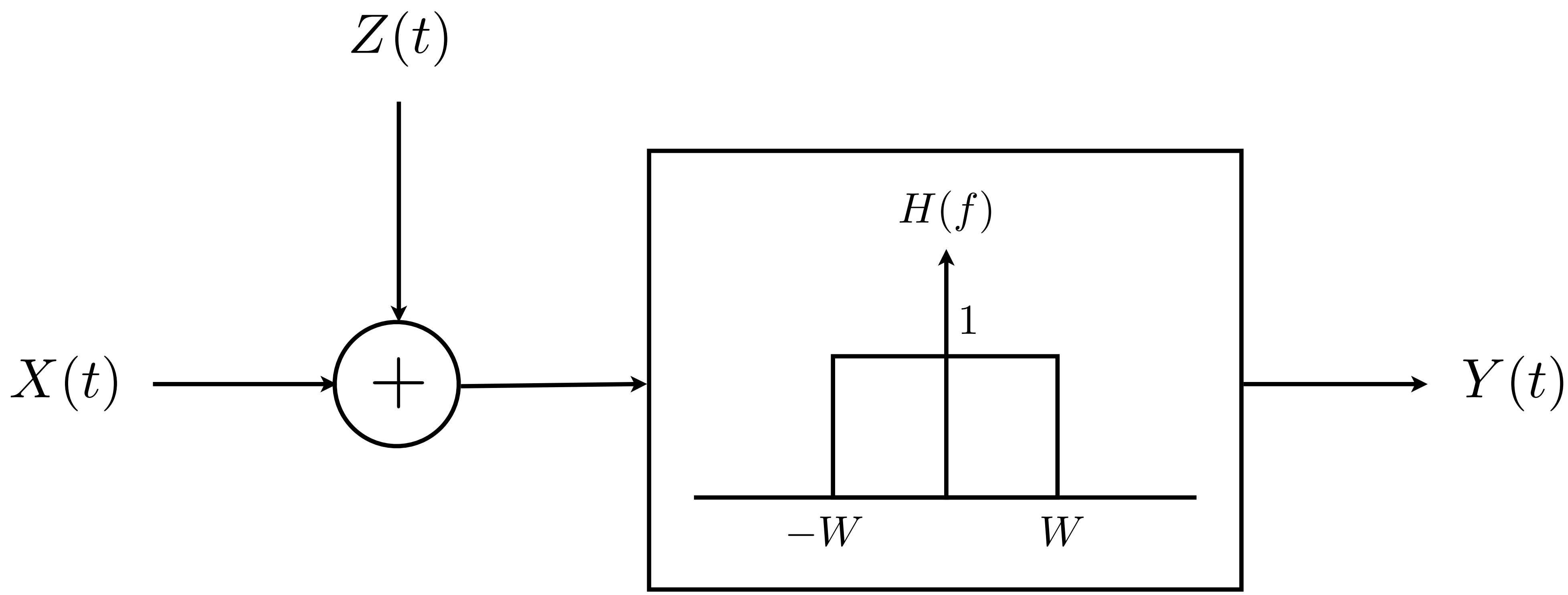
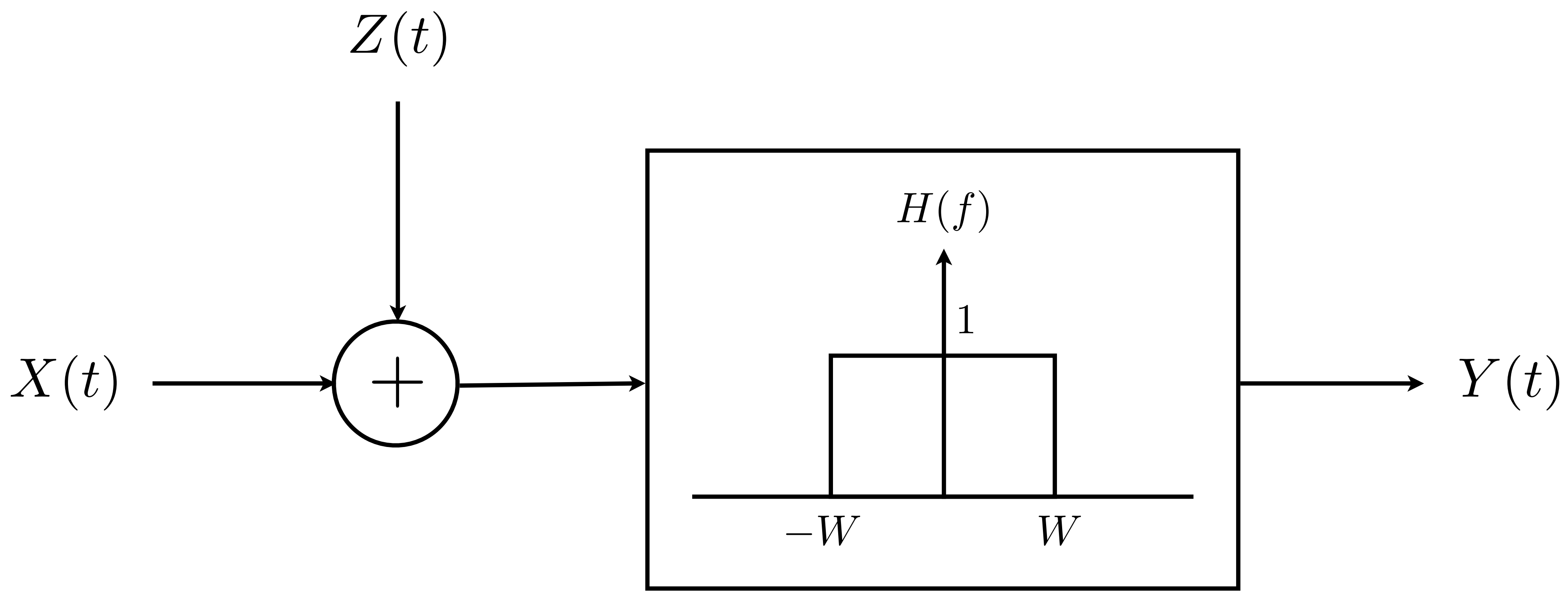
−W
W
Y(t)



- Both input and output are in continuous time.



- Both input and output are in continuous time.
- $Z(t)$ is a zero-mean white Gaussian noise process with $S_Z(f) = \frac{N_0}{2}$, called an additive white Gaussian noise (AWGN).

Signal Analysis Preliminaries

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The variables t and f are referred to as time and frequency, respectively.

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Proposition 11.26 For a pair of energy signals $g_1(t)$ and $g_2(t)$

$$R_{12}(\tau) \rightleftharpoons G_1(f) G_2^*(f),$$

where $G_2^*(f)$ denotes the complex conjugate of $G_2(f)$.

Wide-sense Stationary Process

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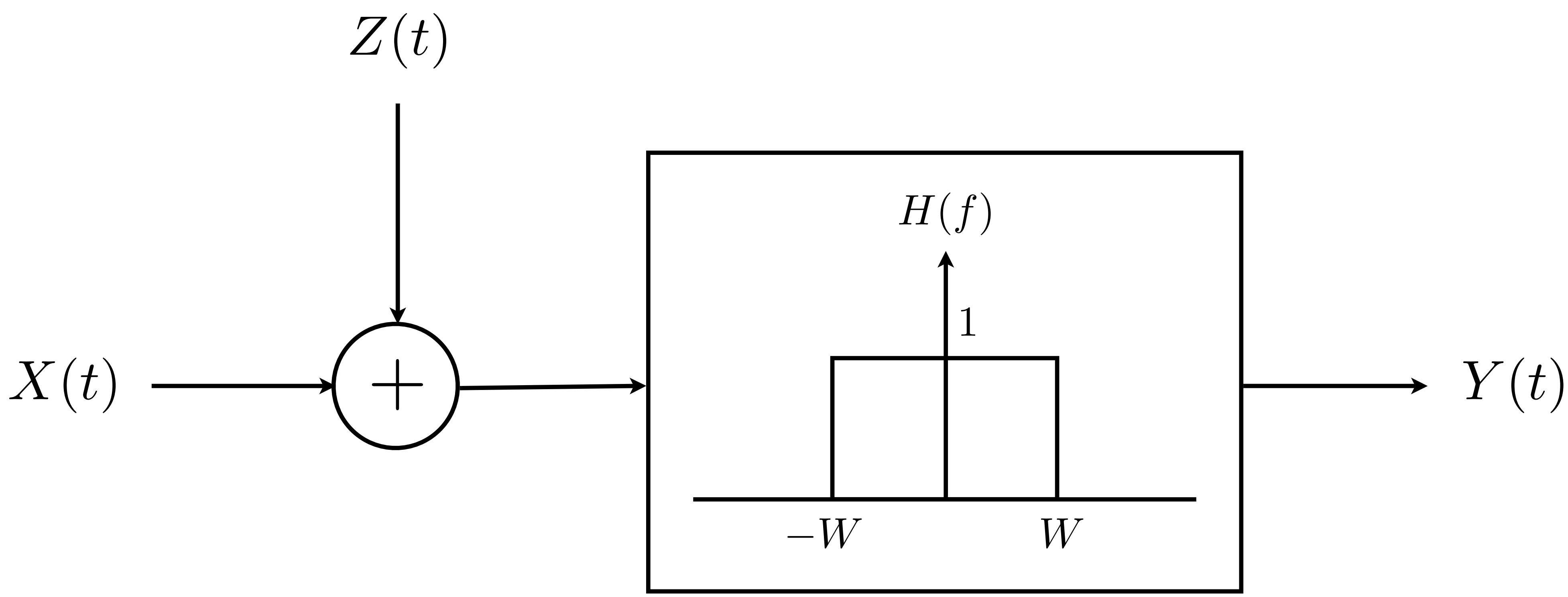
An Equivalent Model



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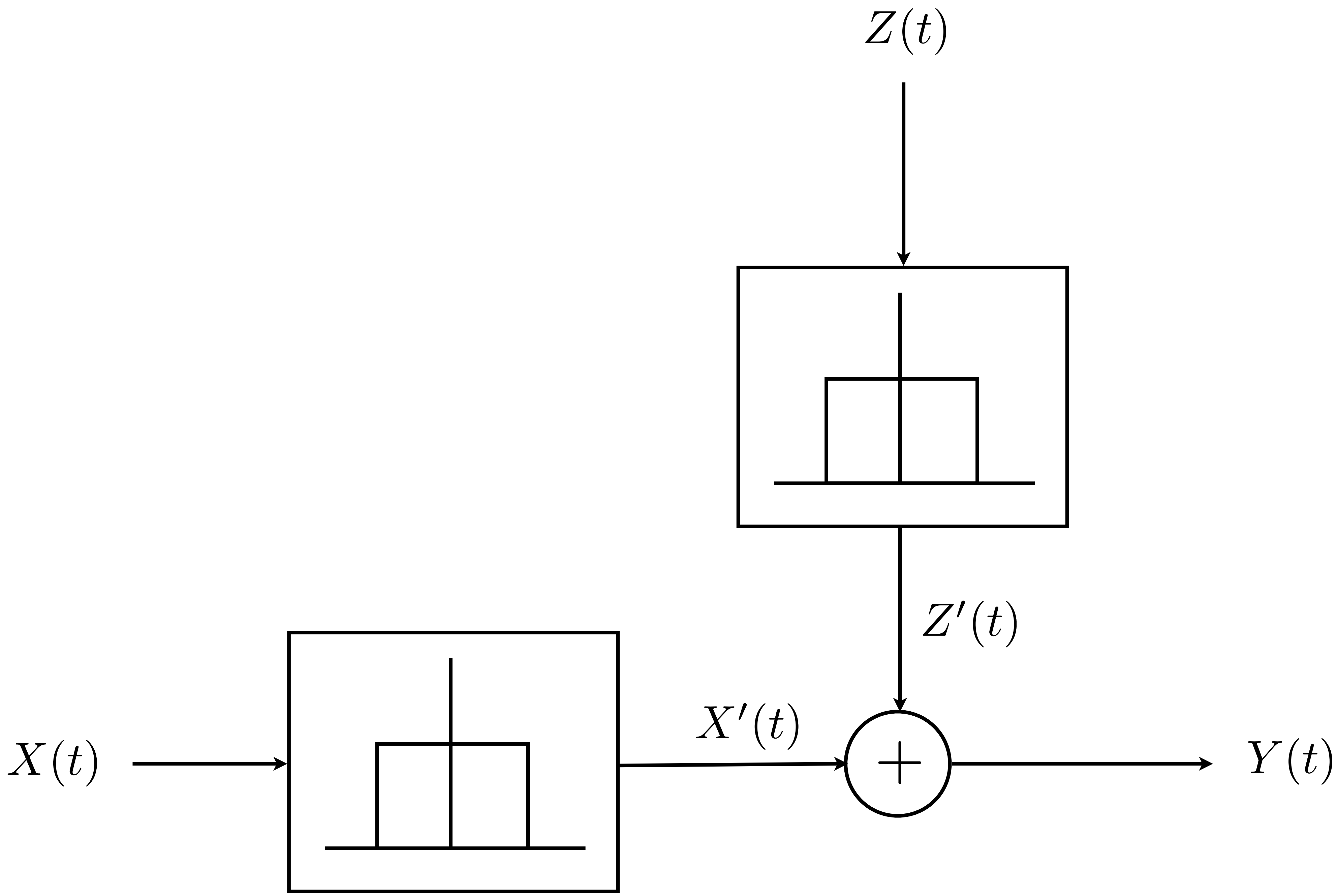

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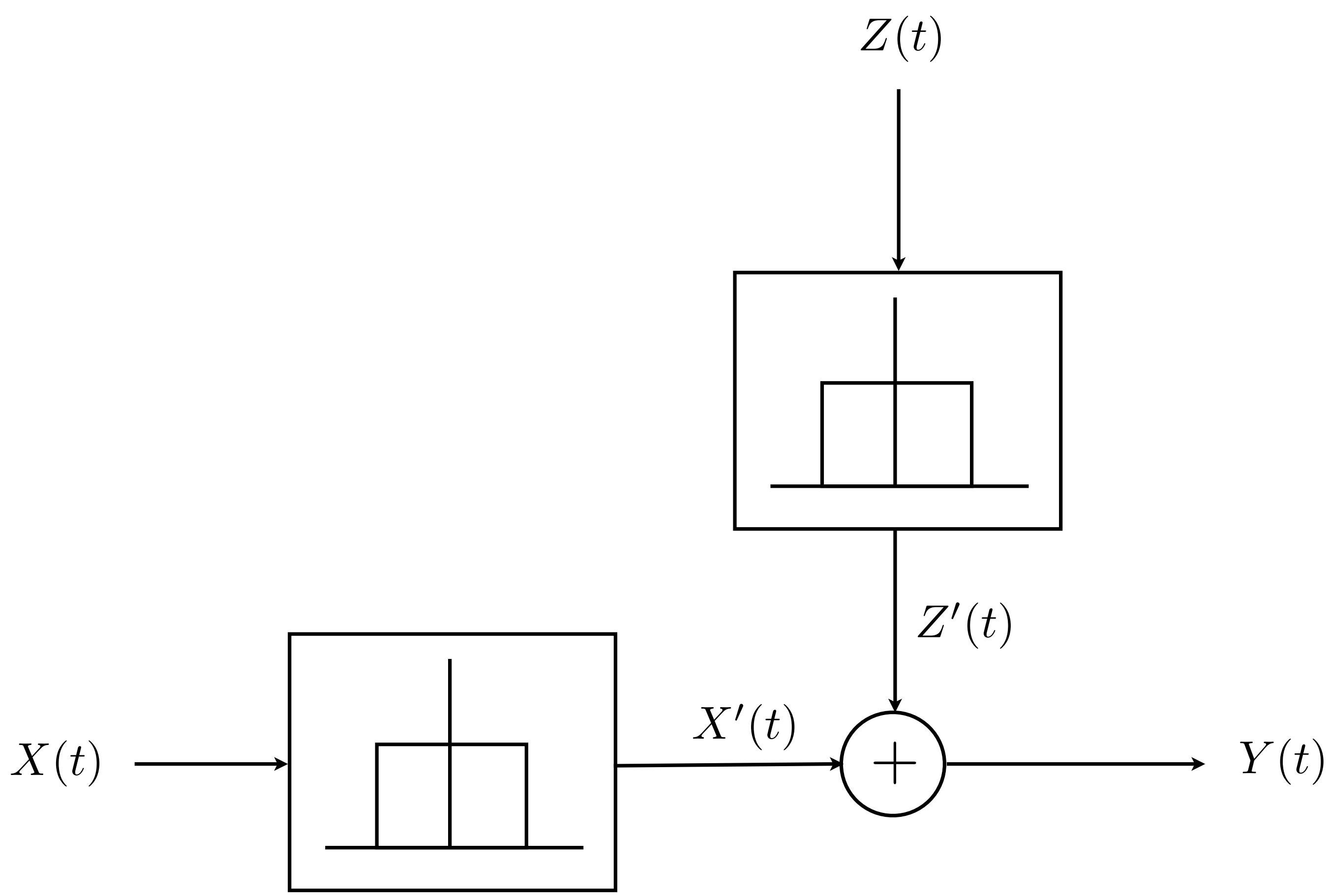

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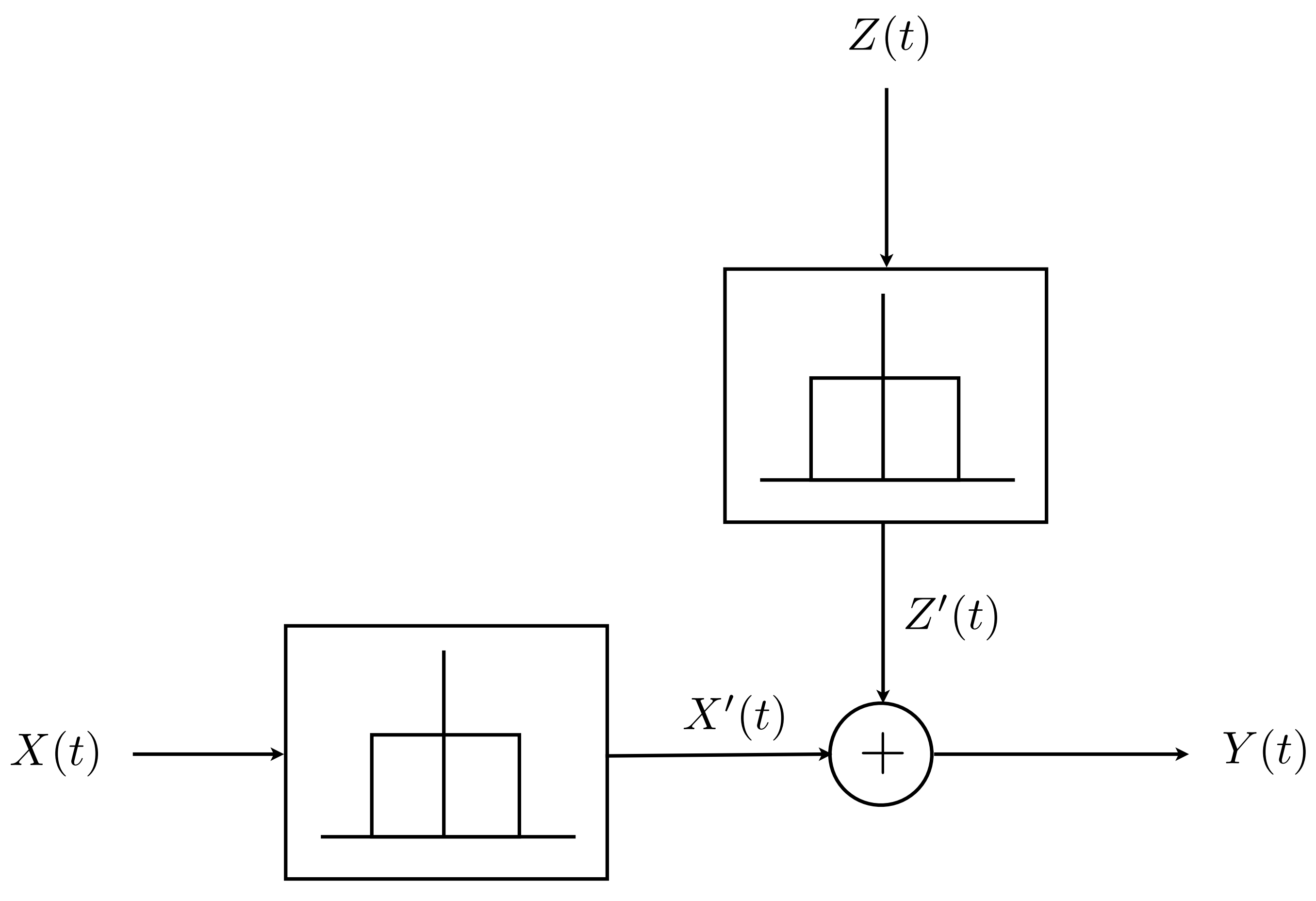

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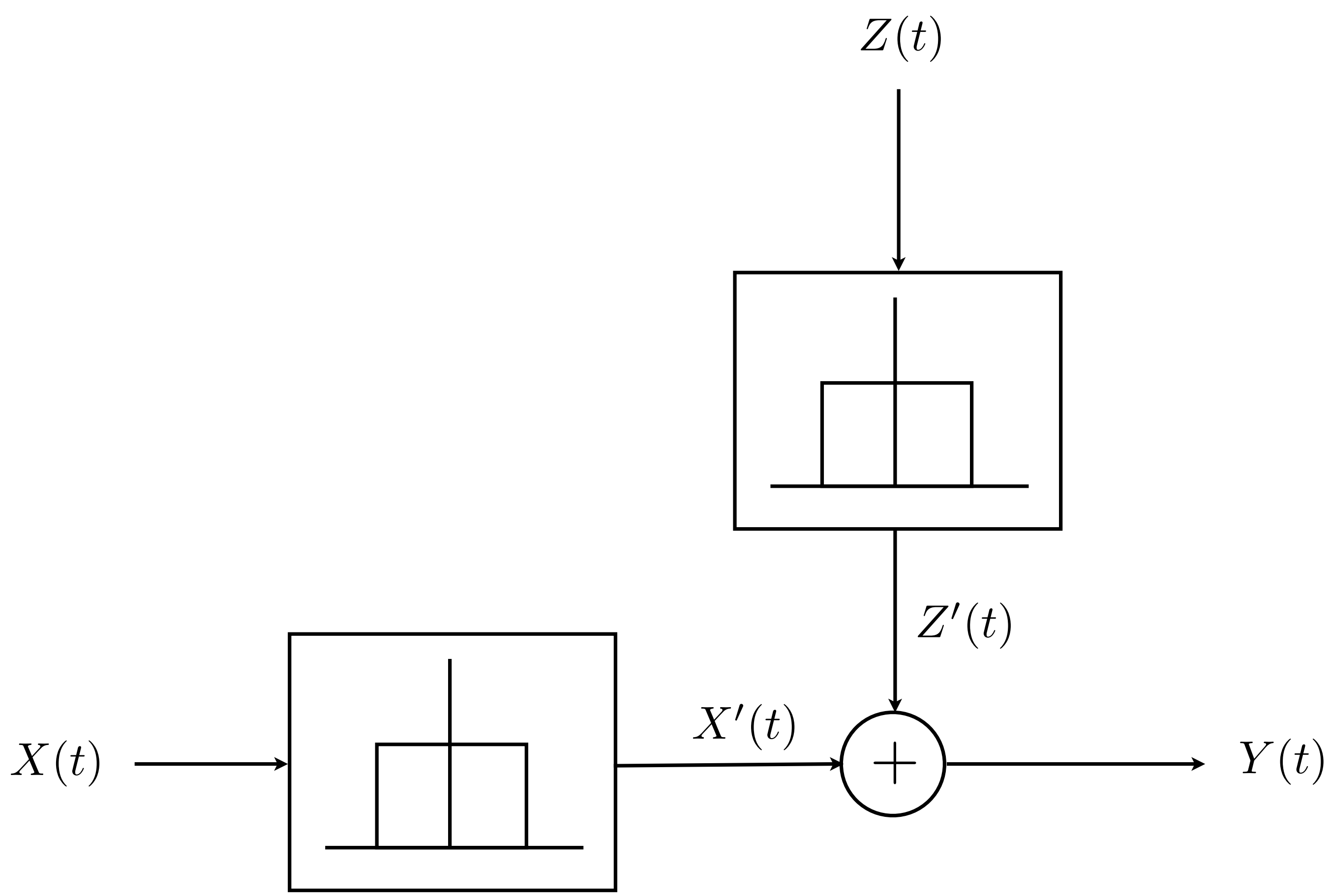

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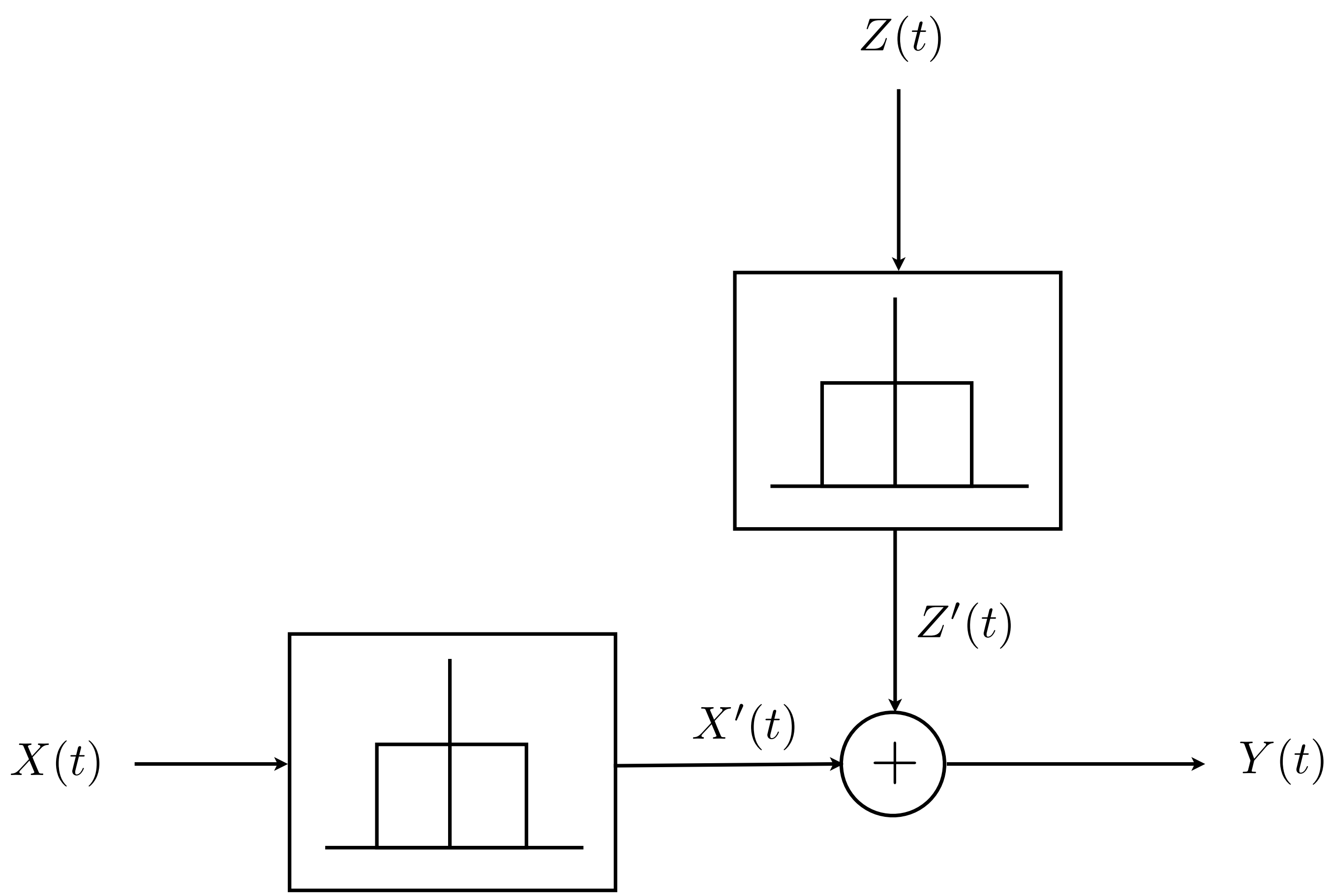

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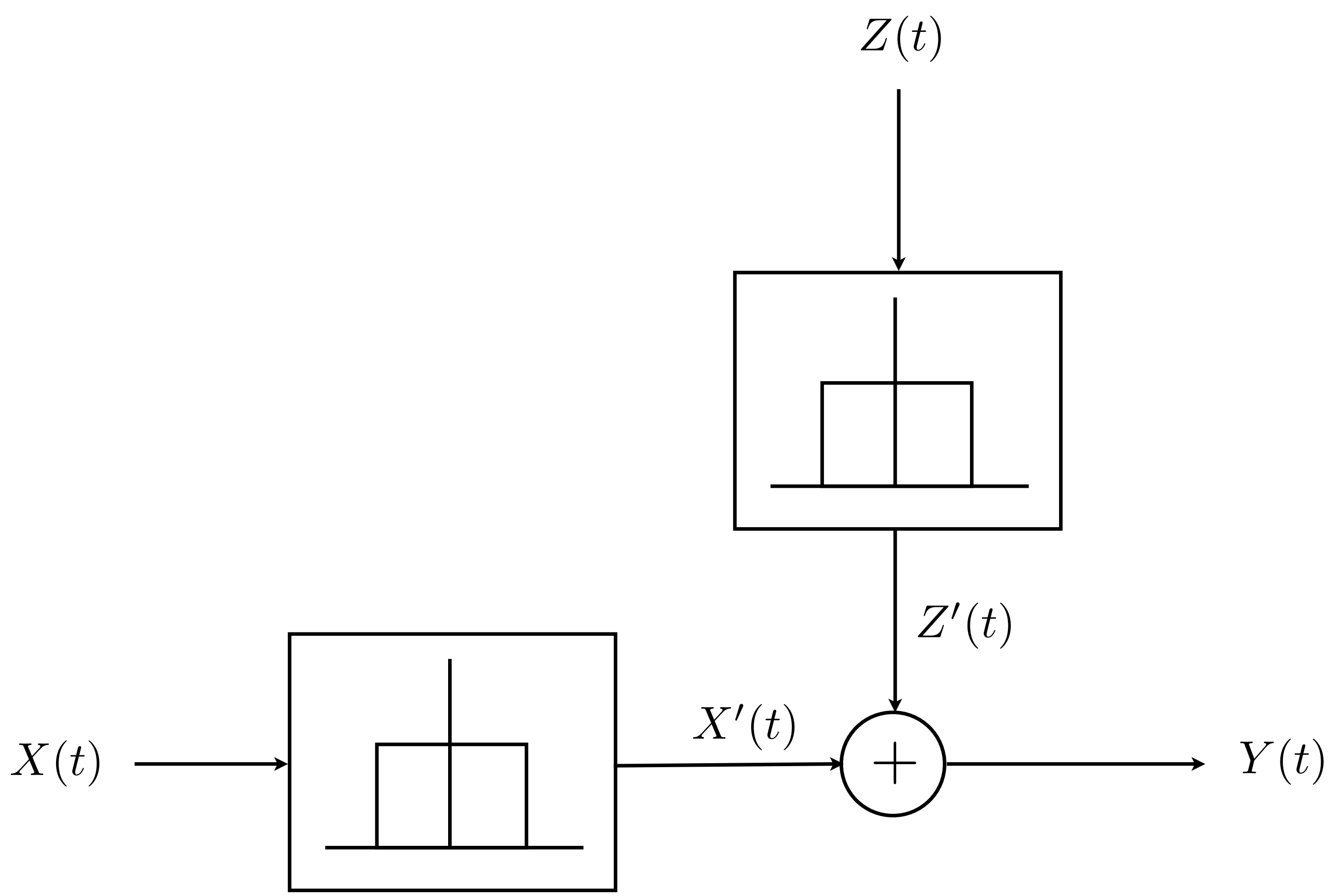

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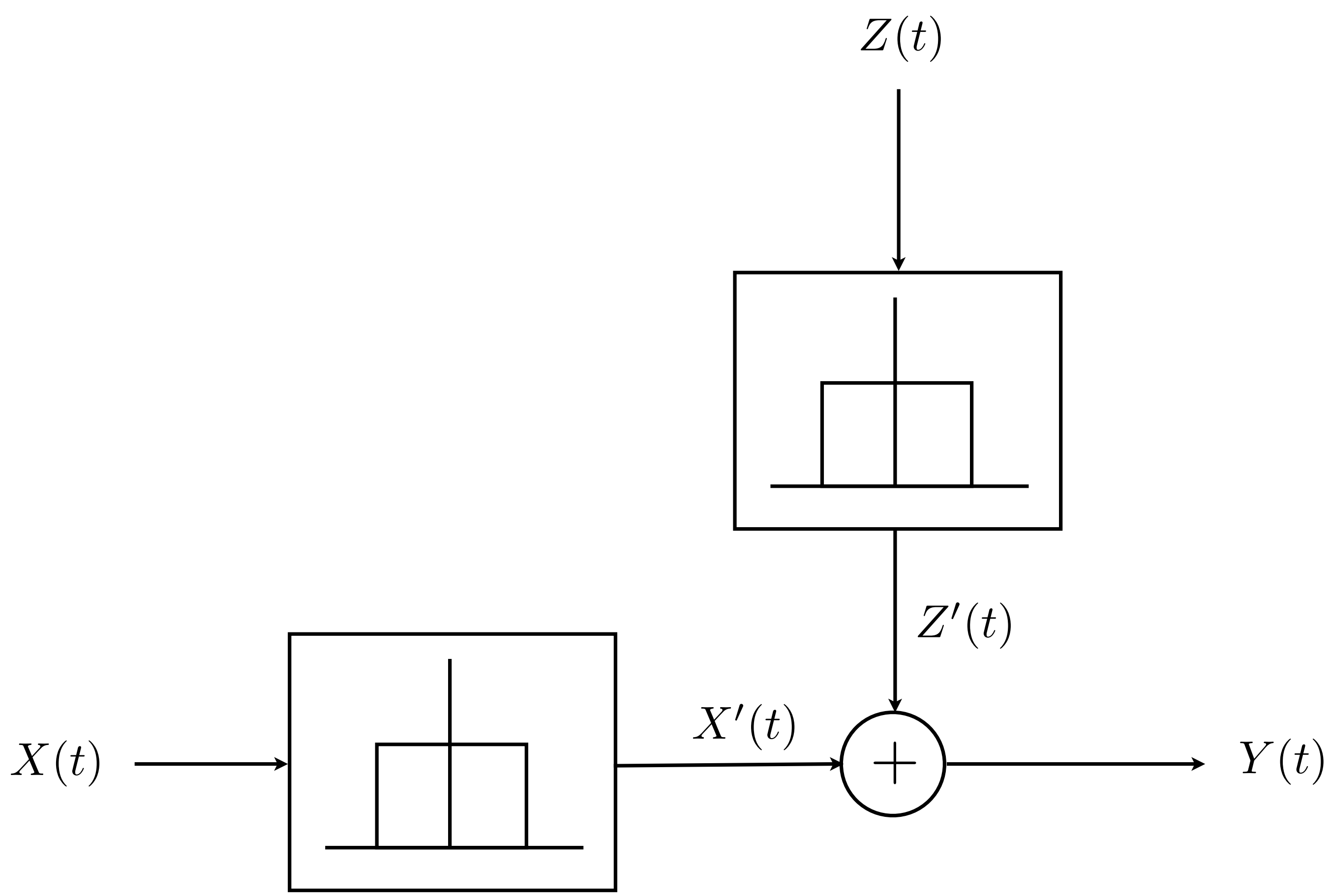

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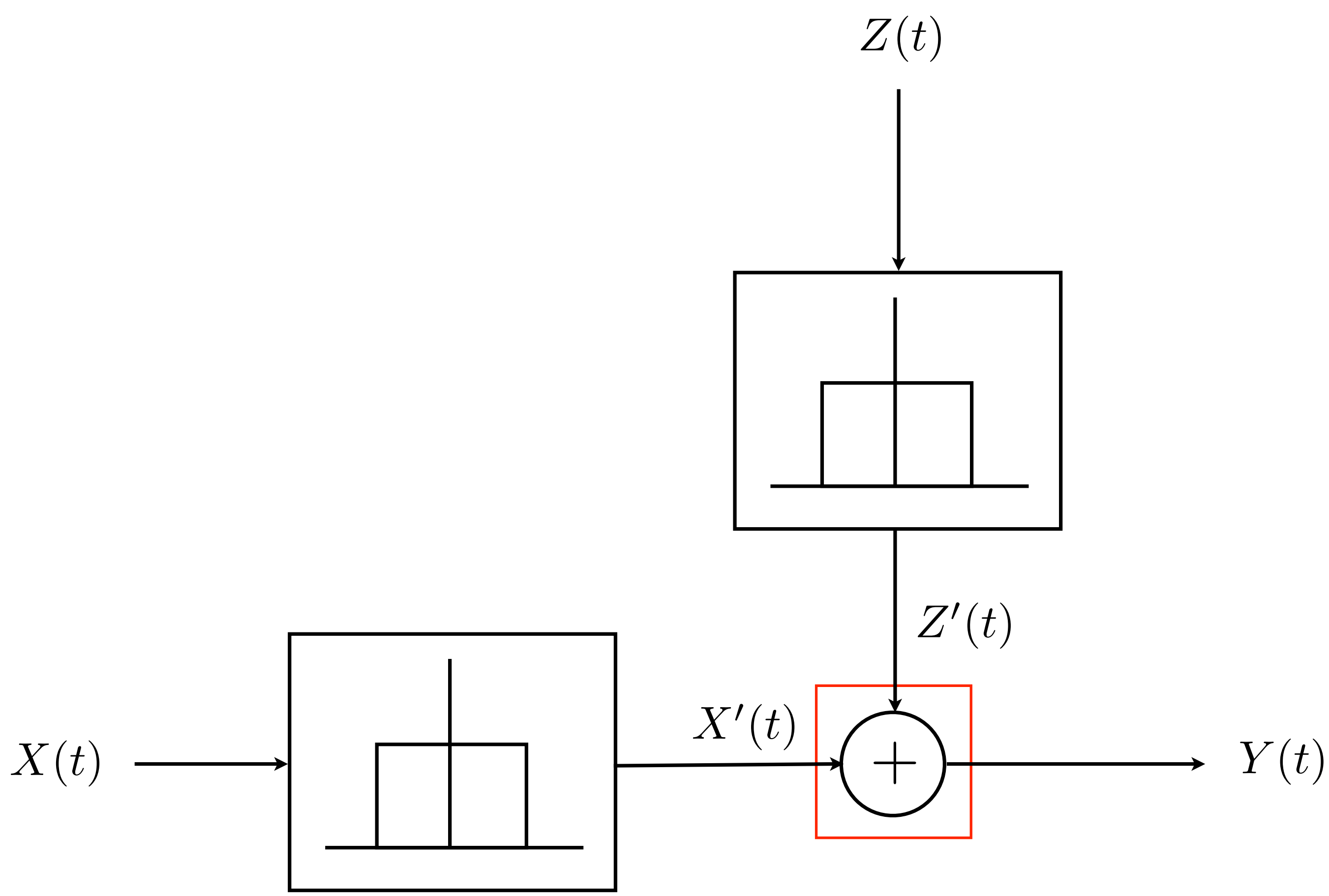

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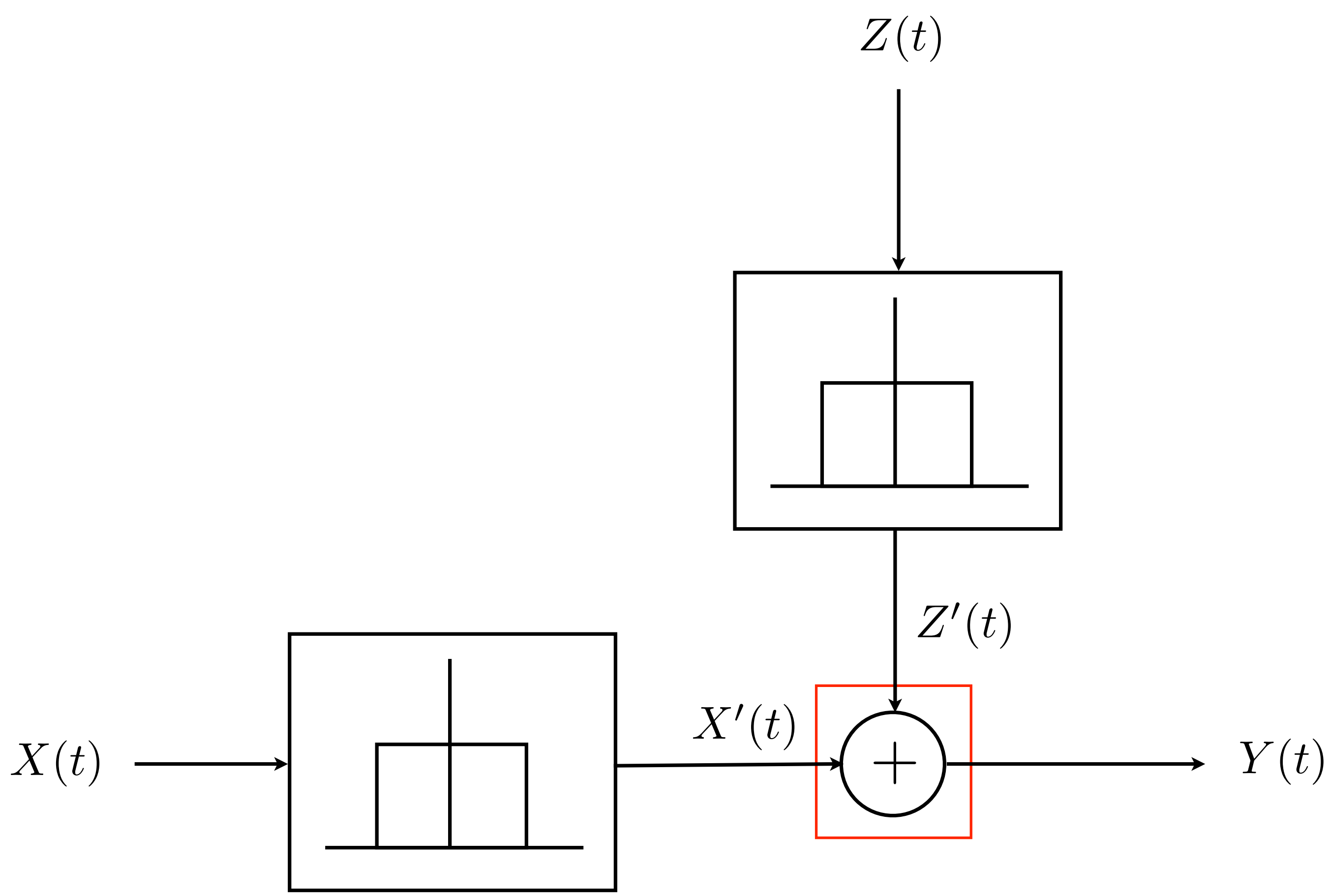

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- Regard $X'(t)$ as the channel input and $Z'(t)$ as the additive noise process.
- Impose a suitable power constraint on $X'(t)$.

Theorem 11.29 (Nyquist-Shannon Sampling Theorem)

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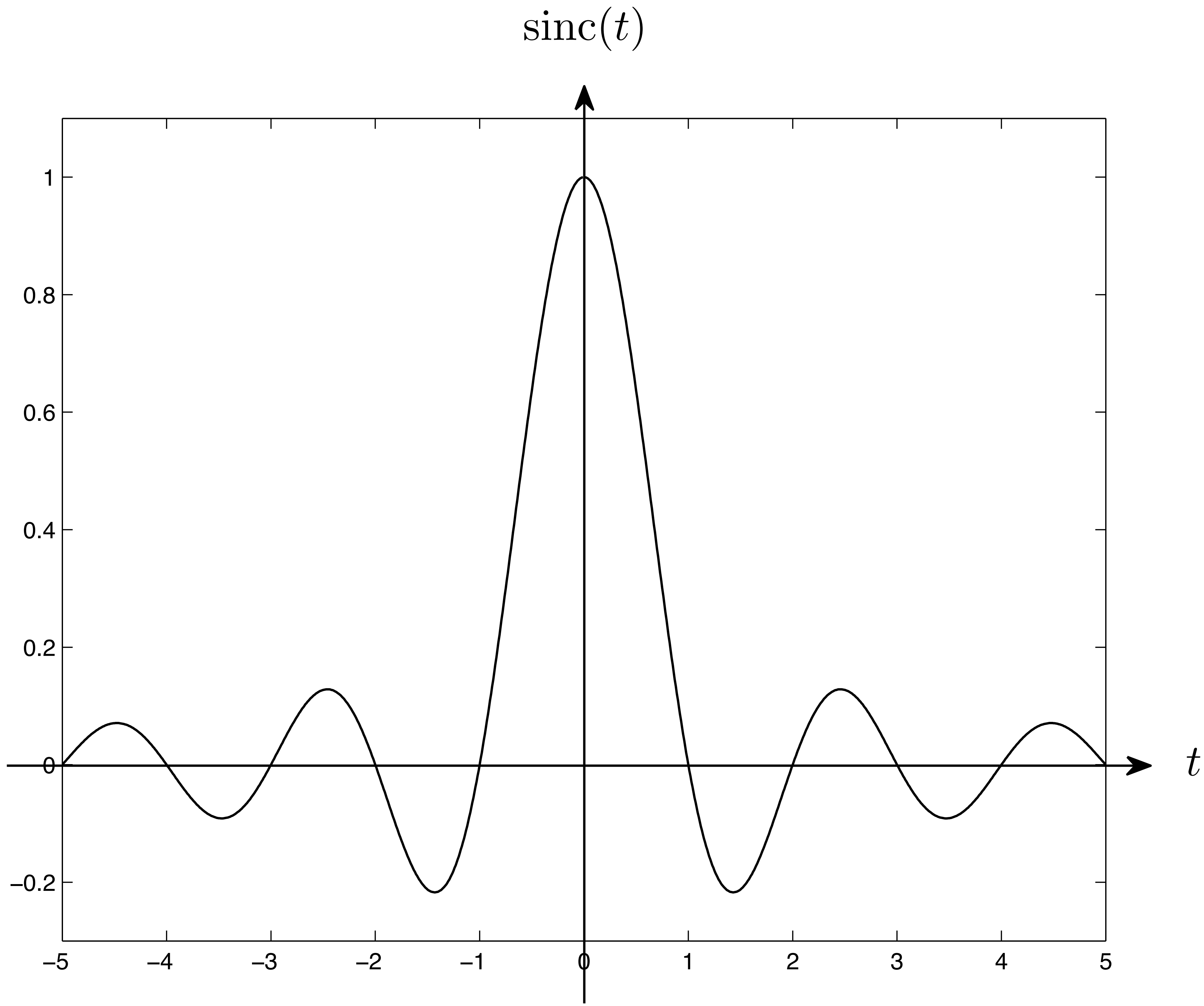
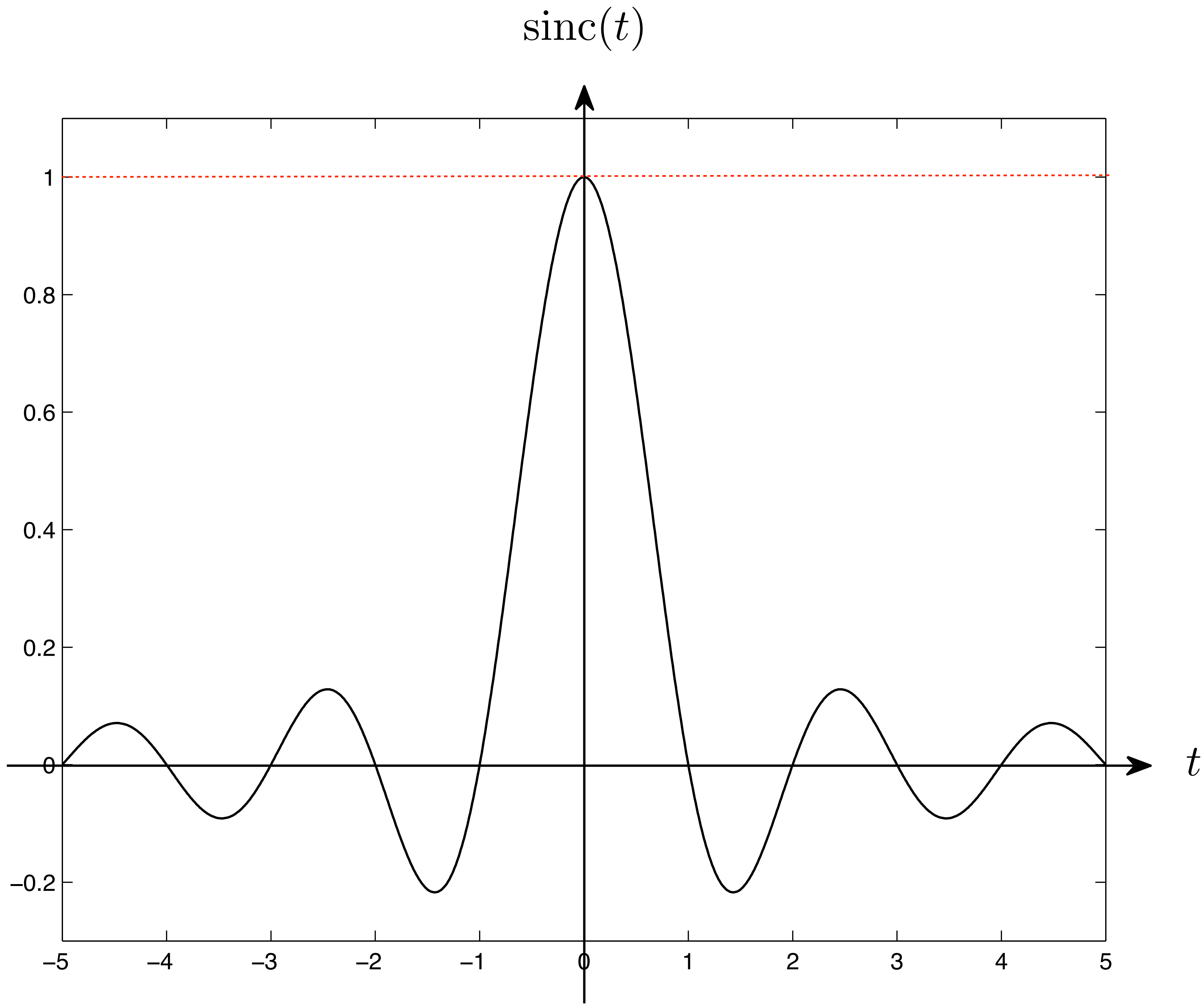


$\mathrm{sinc}(t)$
1
0.8
0.6
0.4
0.2
0
–0.2
–5
–4
–3
–2
–1
0
1
2
3
4
5
t



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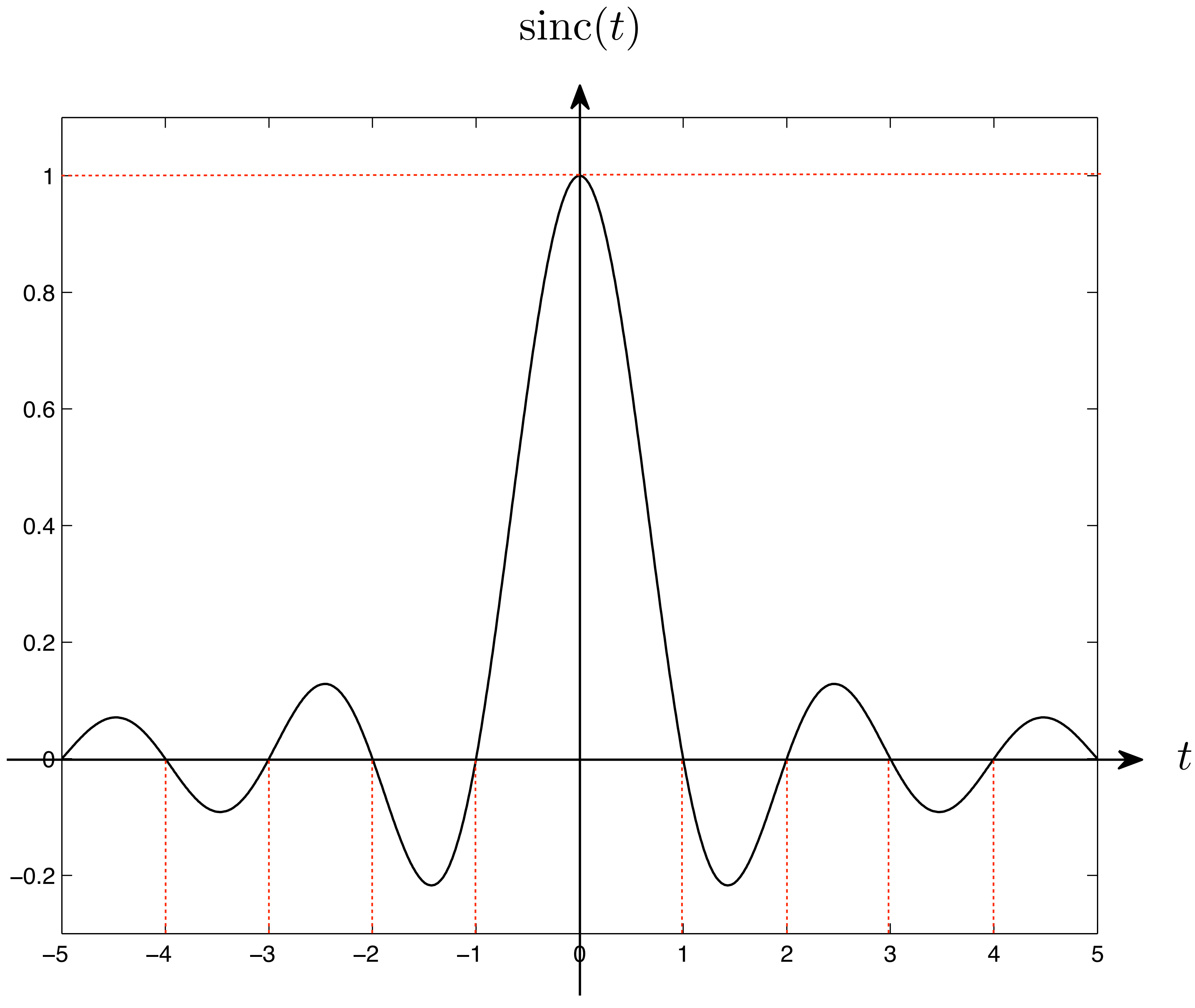

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- $$\operatorname{sinc}(2Wt - i) = \operatorname{sinc}\left(2W\left(t - \frac{i}{2W}\right)\right) = \begin{cases} 1 & t = \frac{i}{2W} \\ 0 & t = \frac{j}{2W},\ j \neq i \end{cases}$$

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called the sinc function, is defined to be 1 at $t = 0$ by continuity.

Remarks

- $\text{sinc}(t) = 0$ for every integer $i \neq 0$.
- $$\text{sinc}(2Wt - i) = \text{sinc}\left(2W\left(t - \frac{i}{2W}\right)\right) = \begin{cases} 1 & t = \frac{i}{2W} \\ 0 & t = \frac{j}{2W},\ j \neq i \end{cases}$$

Theorem 11.29 (Nyquist-Shannon Sampling Theorem) Let $g(t)$ be a signal with Fourier transform $G(f)$ that vanishes for $f \notin [-W, W]$. Then

$$g(t) = \sum_{i=-\infty}^{\infty} g\left(\frac{i}{2W}\right) \operatorname{sinc}(2Wt - i)$$

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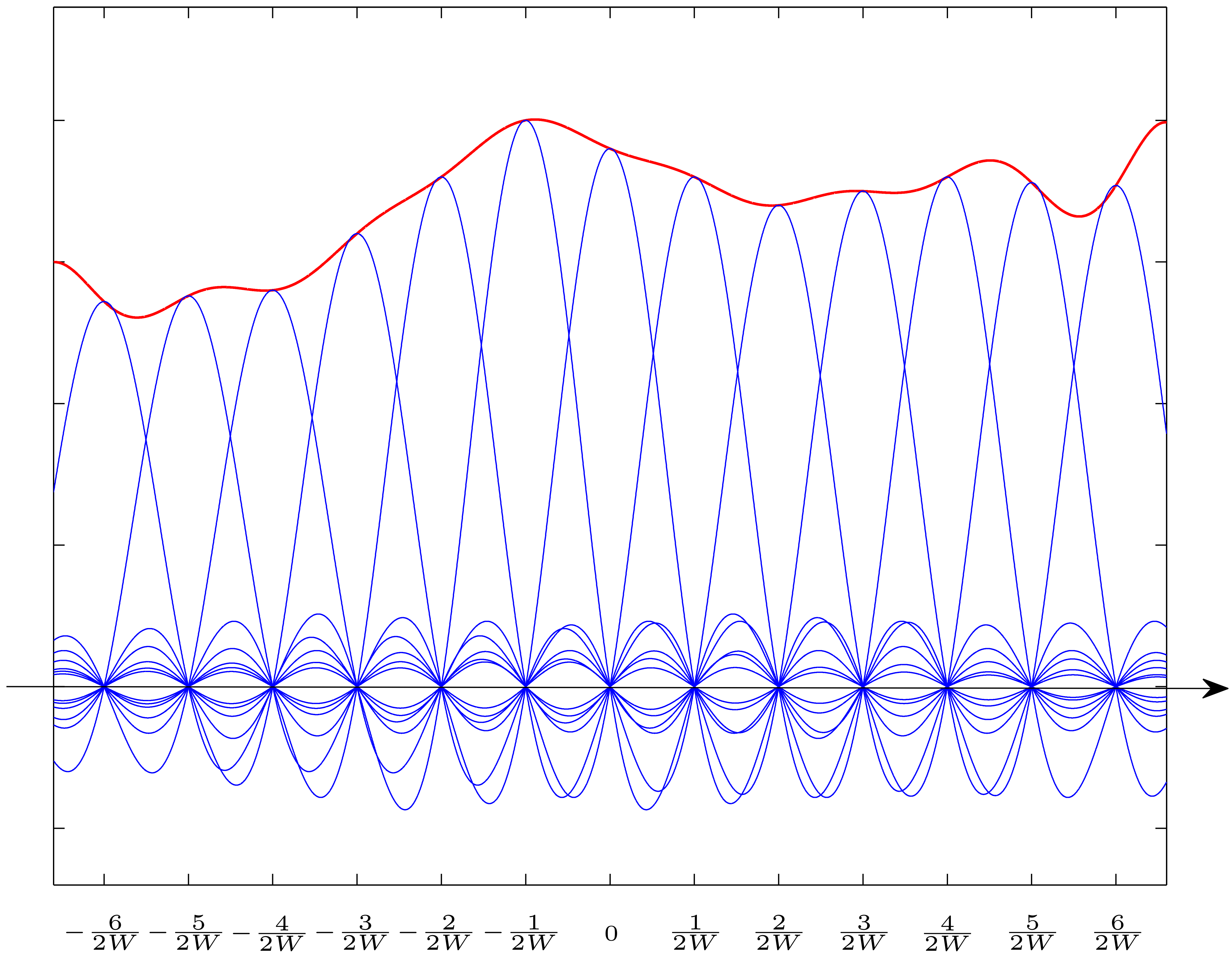
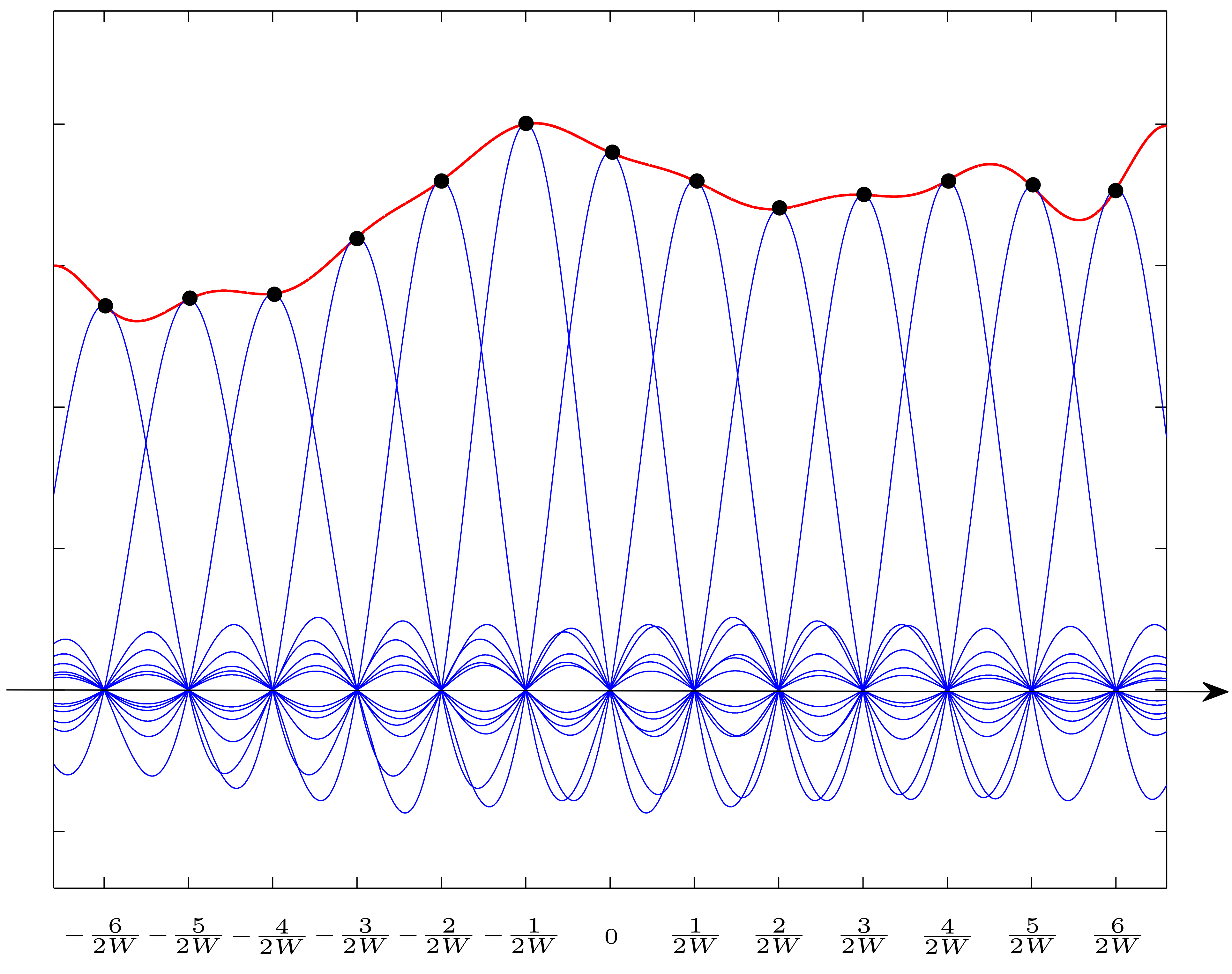


$-\frac{6}{2W}$ $-\frac{5}{2W}$ $-\frac{4}{2W}$ $-\frac{3}{2W}$ $-\frac{2}{2W}$ $-\frac{1}{2W}$ 0 $\frac{1}{2W}$ $\frac{2}{2W}$ $\frac{3}{2W}$ $\frac{4}{2W}$ $\frac{5}{2W}$ $\frac{6}{2W}$



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In particular,

$$R_{ii'}(0) = \int_{-\infty}^{\infty} \operatorname{sinc}(2Wt - i)\operatorname{sinc}(2Wt - i')dt.$$

6. Now

$$\begin{aligned}&\operatorname{sinc}(2Wt - i) \\ &\rightleftharpoons \quad e^{-j2\pi f\left(\frac{i}{2W}\right)} \left(\frac{1}{2W}\right) \operatorname{rect}\left(\frac{f}{2W}\right) := G_i(f)\end{aligned}$$

and

$$\begin{aligned}&\operatorname{sinc}(2Wt - i') \\ &\rightleftharpoons \quad e^{-j2\pi f\left(\frac{i'}{2W}\right)} \left(\frac{1}{2W}\right) \operatorname{rect}\left(\frac{f}{2W}\right) := G_{i'}(f).\end{aligned}$$

7. Then we have

$$\underline{R_{ii'}(\tau)} \rightleftharpoons G_i(f)G_{i'}^*(f),$$

3. Then by Rayleigh's energy theorem, we have

$$\begin{aligned}\int_{-\infty}^{\infty} \text{sinc}^2(2Wt)dt &= \left(\frac{1}{2W}\right)^2 \int_{-\infty}^{\infty} \text{rect}^2\left(\frac{f}{2W}\right) df \\ &= \left(\frac{1}{2W}\right)^2 (2W) \\ &= \frac{1}{2W}.\end{aligned}$$

4. It then follows that

$$\int_{-\infty}^{\infty} \psi_i^2(t)dt = \int_{-\infty}^{\infty} \psi_0^2(t)dt = 1. \tag{1}$$

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and so

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$$\int_{-\infty}^{\infty} \text{sinc}^2(2Wt)dt = \left(\frac{1}{2W}\right)^2 \int_{-\infty}^{\infty} \text{rect}^2\left(\frac{f}{2W}\right) df$$
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8. Therefore, for $i \neq i'$,

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$\psi_i(t) = \sqrt{2W}\operatorname{sinc}(2Wt - i)$

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- With these assumptions, the waveform channel can be regarded as a discrete-time channel defined at $t = \frac{i}{2W}$, with the ith input and output of the channel being X'_i and Y_i, respectively.

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Sampling the Noise Process Z'(t)

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If X and Y are zero-mean, then

$$\operatorname{cov}(X, Y) = E(XY) - (EX)(EY)$$

Proposition 11.31 $Z'\left(\frac{i}{2W}\right)$, $-\infty < i < \infty$ are i.i.d. Gaussian random variables with zero mean and variance $N_0 W$.

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8. Therefore, $Z'\left(\frac{i}{2W}\right)$, $-\infty < i < \infty$ are uncorrelated and hence independent because they are jointly Gaussian.

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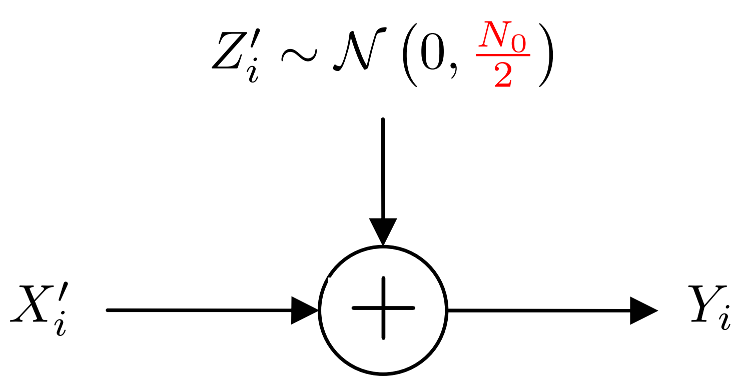

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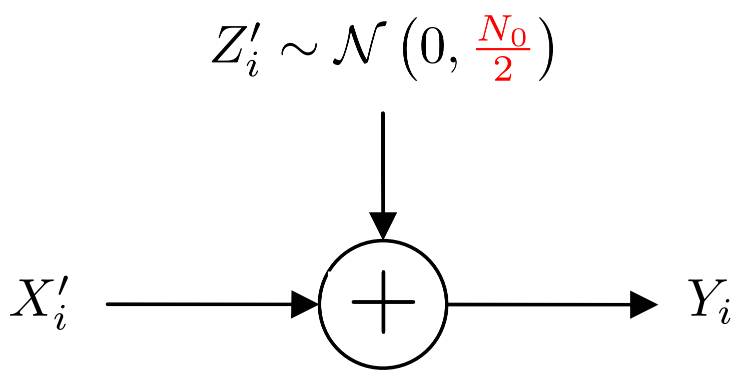

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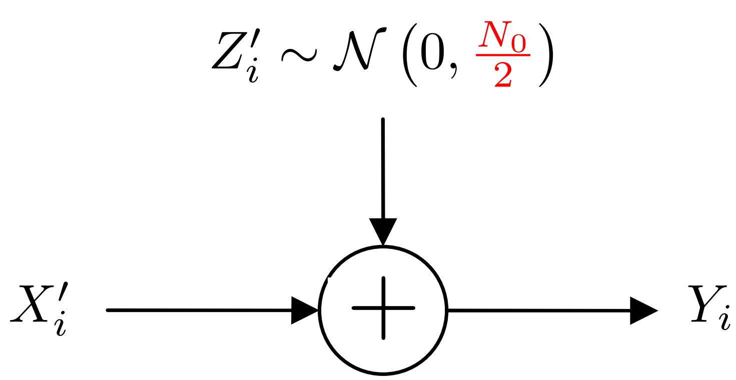

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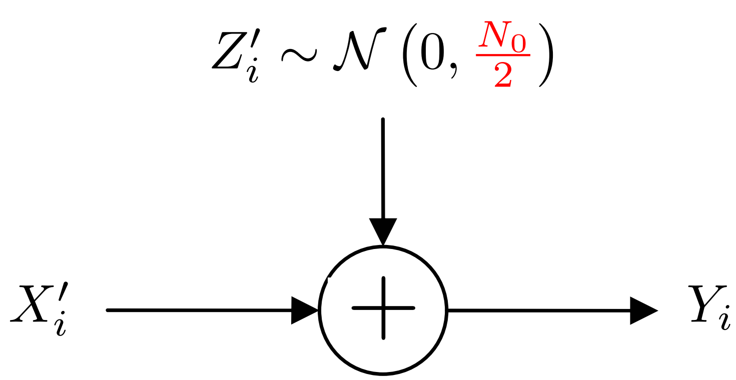
- The capacity of the discrete-time channel is

$$\frac{1}{2}\log\left(1+\frac{P/2W}{N_0/2}\right)=\frac{1}{2}\log\left(1+\frac{P}{N_0W}\right) \text{ bits per sample.}$$

Capacity of the Bandlimited White Gaussian Channel



Input power constraint $= \frac{P}{2W}$

- The capacity of the discrete-time channel is

$$\frac{1}{2}\log\left(1+\frac{P/2W}{N_0/2}\right) = \frac{1}{2}\log\left(1+\frac{P}{N_0 W}\right) \text{ bits per sample.}$$

- Since there are $2W$ samples per unit time, the capacity is

$$W\log\left(1+\frac{P}{N_0 W}\right) \text{ bits per unit time.}$$