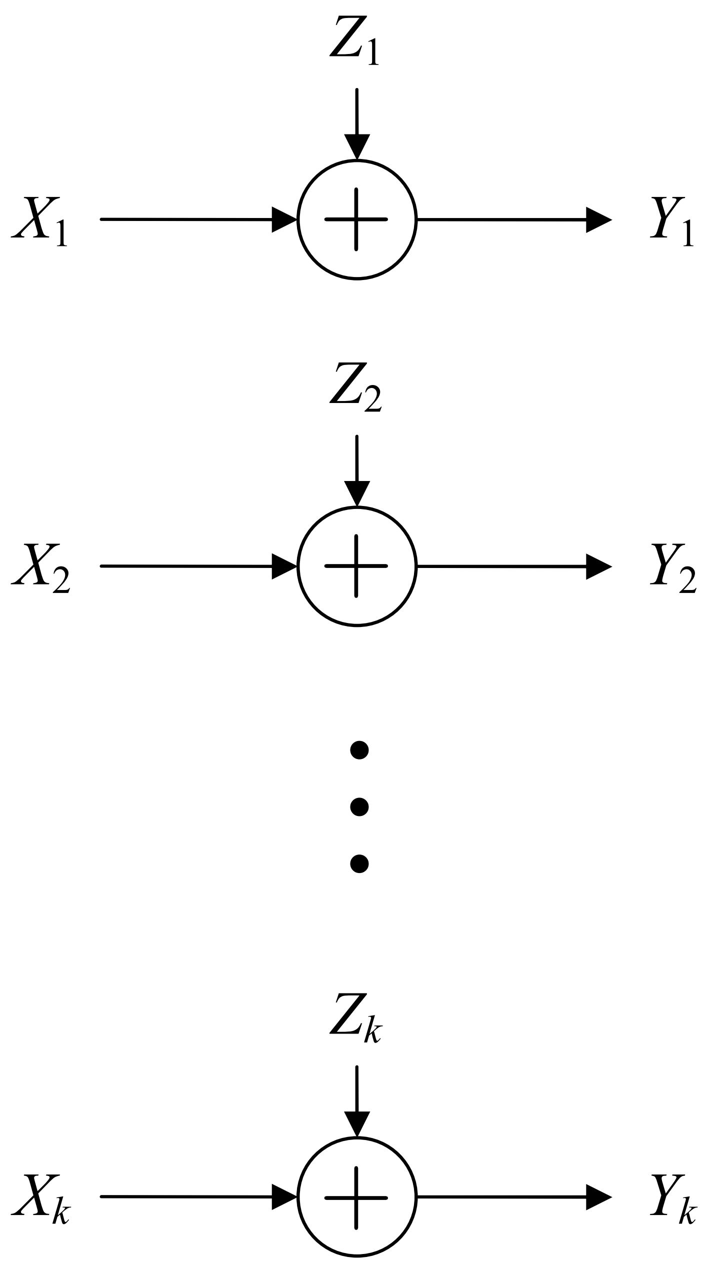
香港中文大學
The Chinese University of Hong Kong

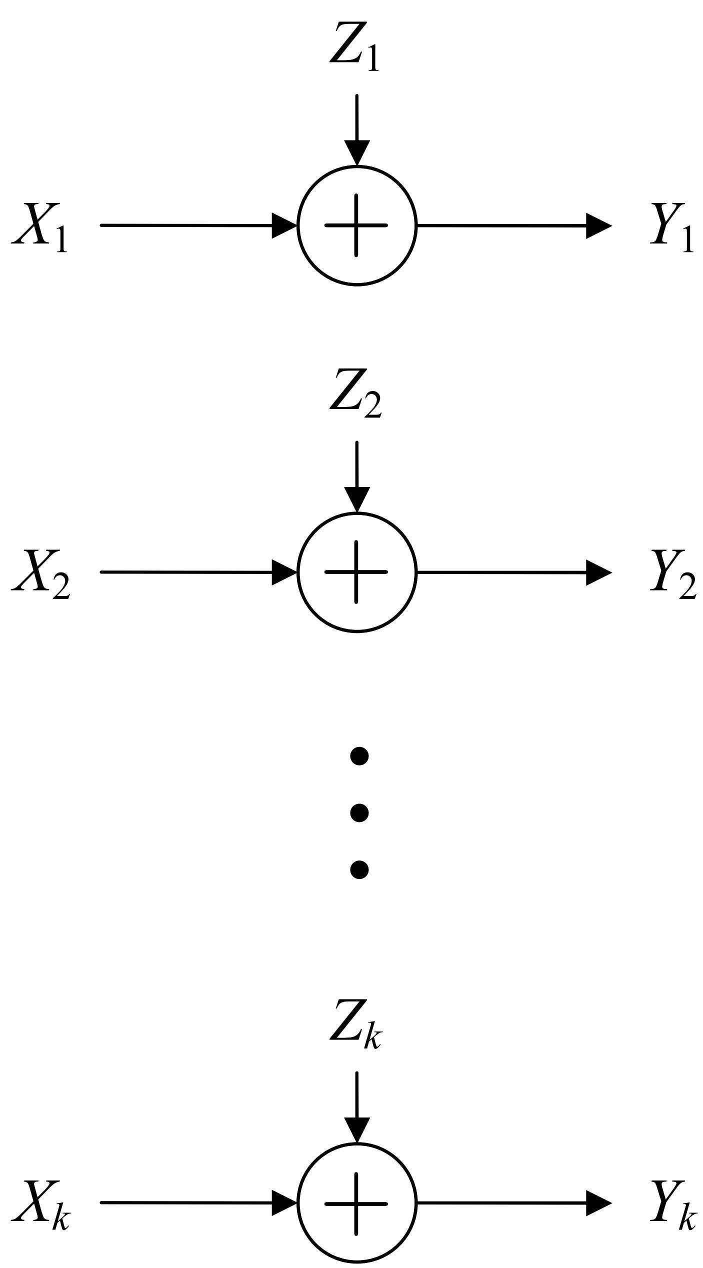
11.6 Correlated Gaussian Channels

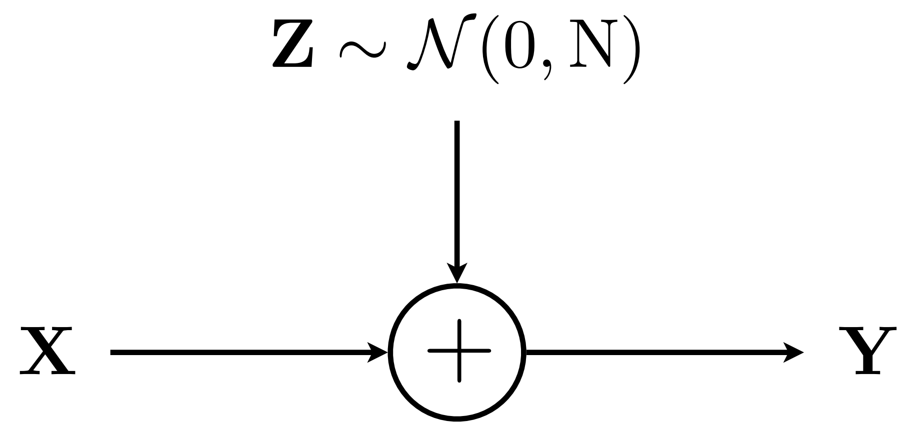
Parallel Gaussian Channels

Parallel Gaussian Channels

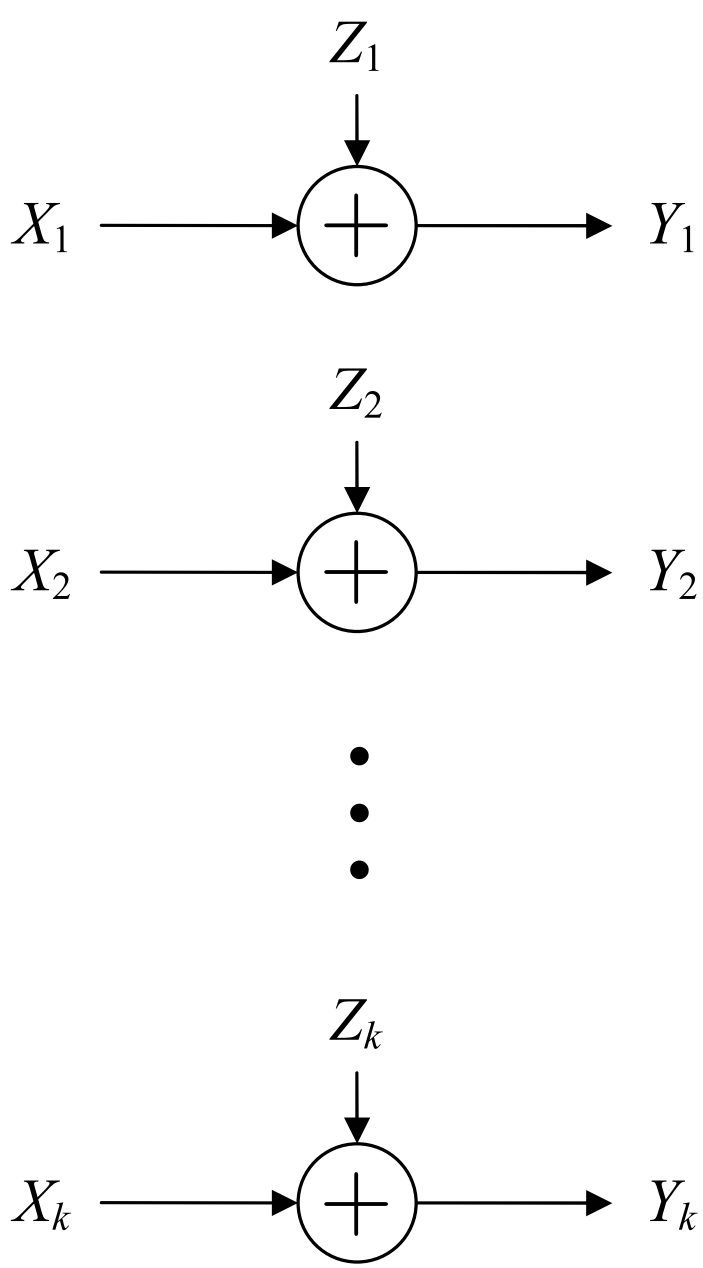


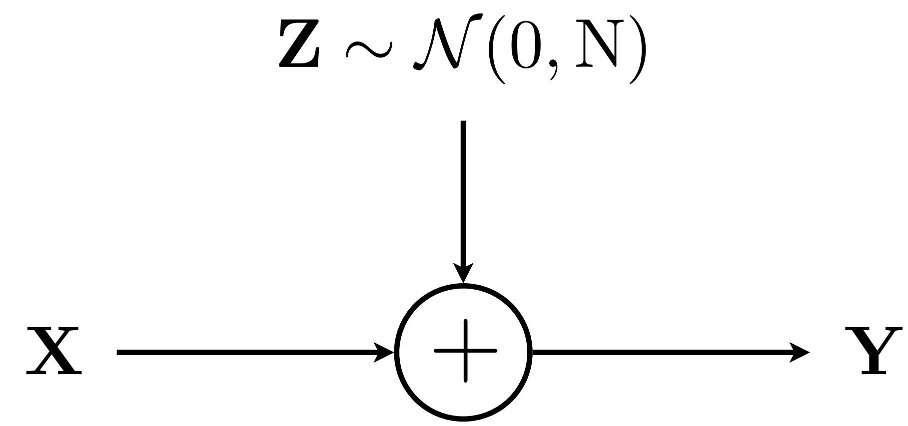
Parallel Gaussian Channels





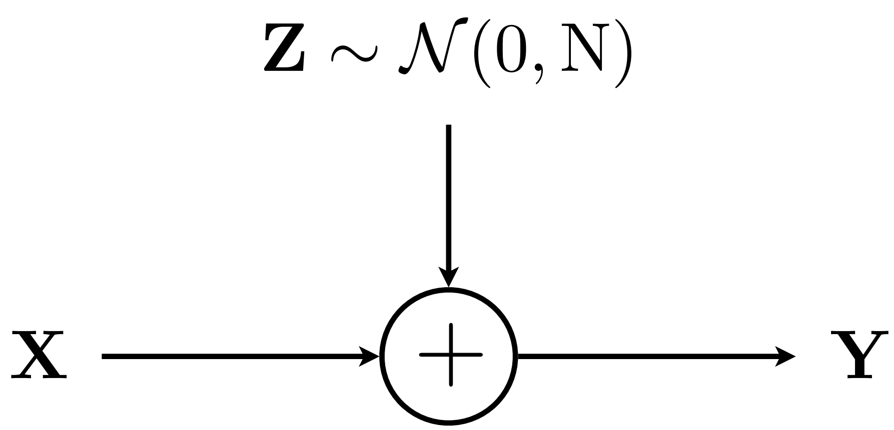
Parallel Gaussian Channels



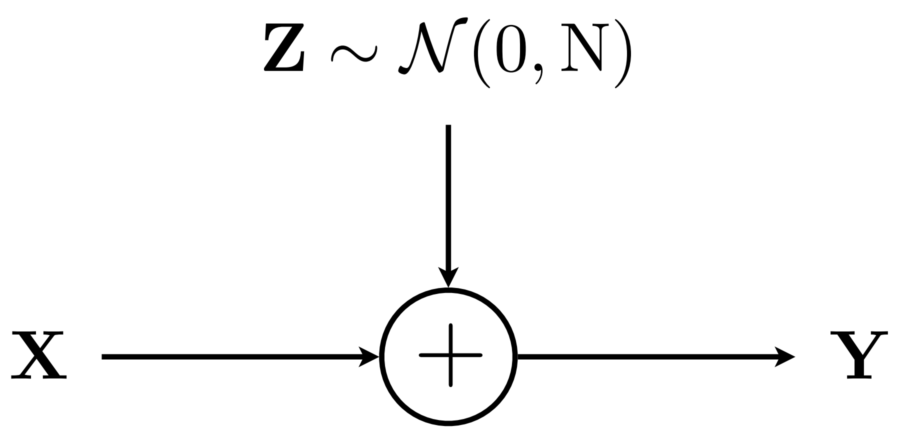


$$\mathrm{N} = \begin{bmatrix} N_1 & & 0 \\ & \ddots & \\ 0 & & N_k \end{bmatrix}$$

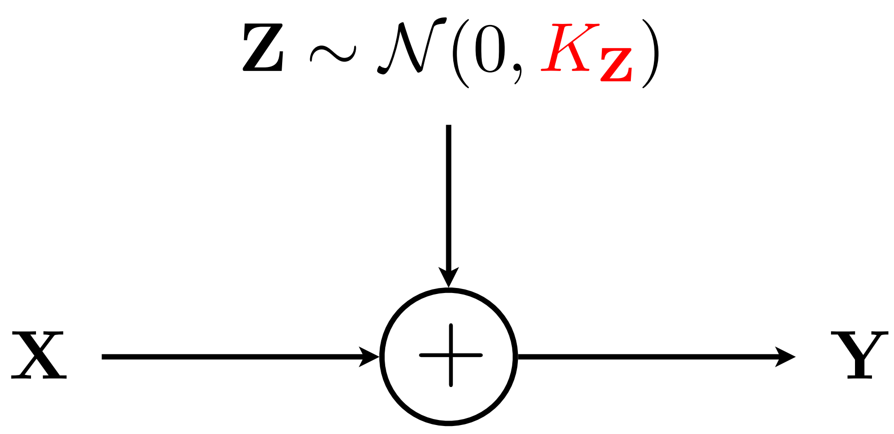
Parallel Gaussian Channels



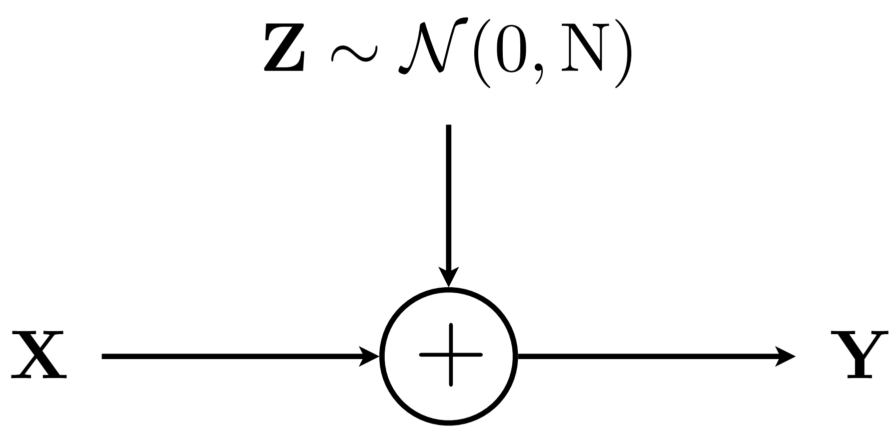
Parallel Gaussian Channels



Correlated Gaussian Channels

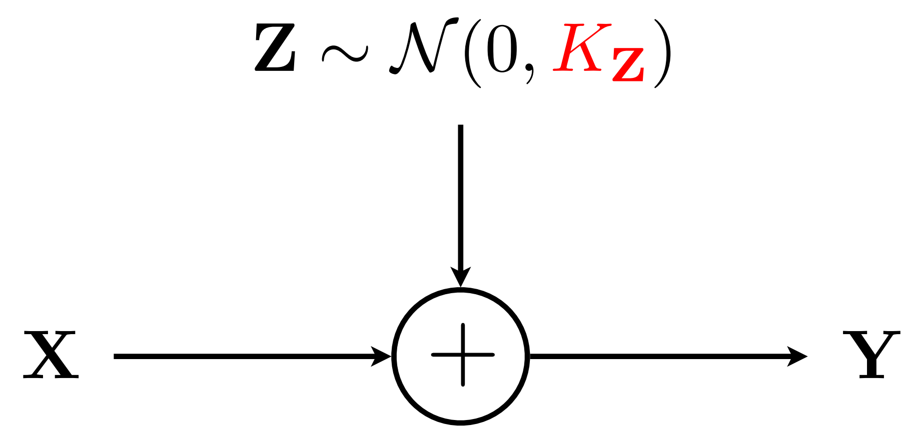


Parallel Gaussian Channels

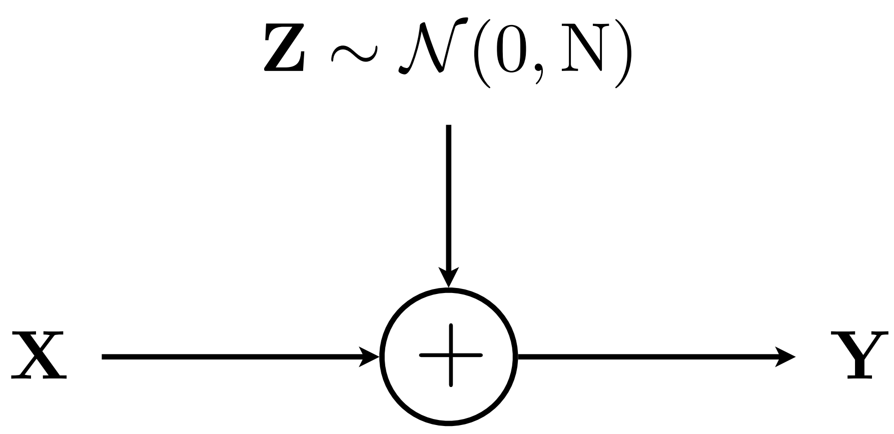


$$\sum_i X_i^2 \leq P$$

Correlated Gaussian Channels

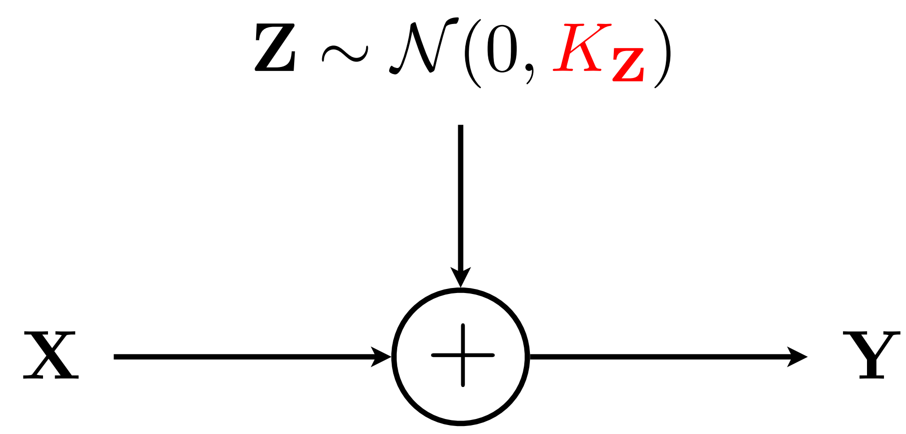


Parallel Gaussian Channels



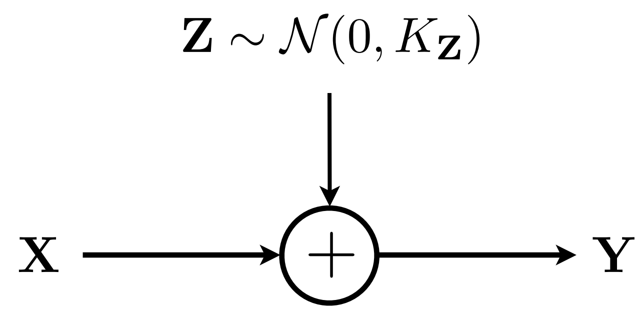
$\sum_i X_i^2 \leq P$

Correlated Gaussian Channels



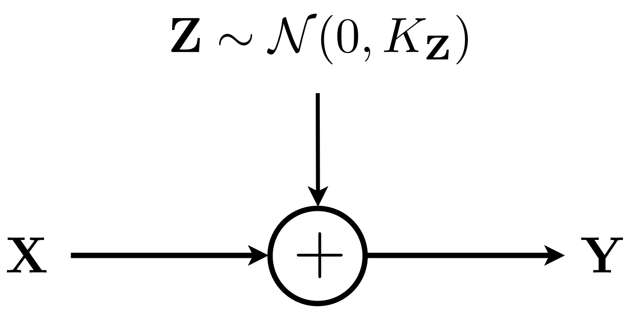
$\sum_i X_i^2 \leq P$

Decorrelation of the Noise Vector



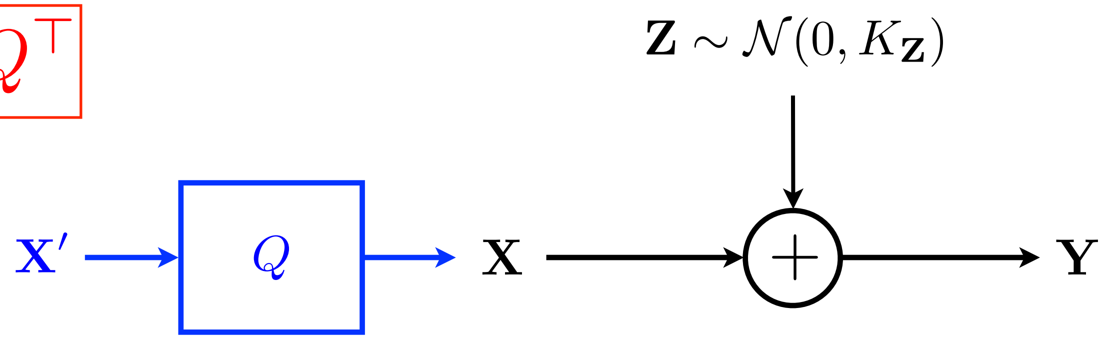
Decorrelation of the Noise Vector

$$K_{\mathbf{Z}} = Q \Lambda Q^{\top}$$



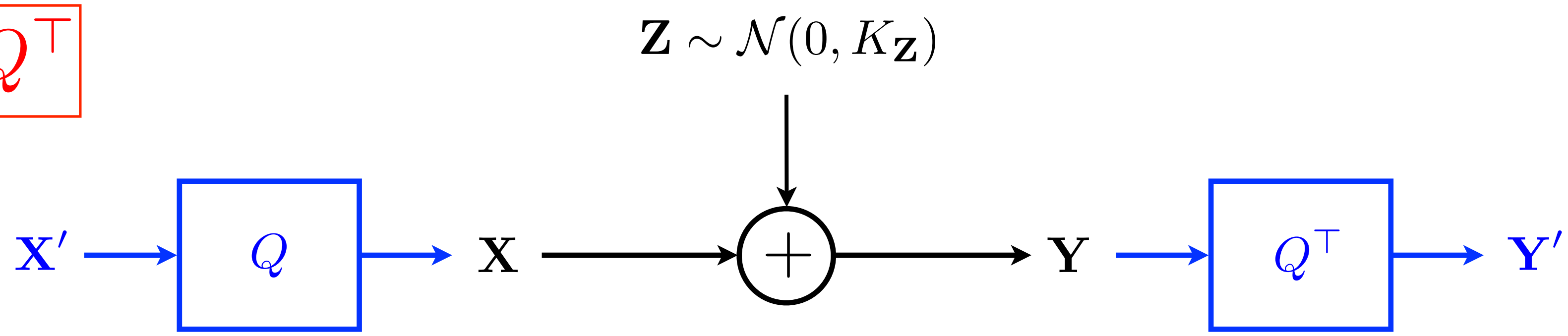
Decorrelation of the Noise Vector

$$K_{\mathbf{Z}} = Q \Lambda Q^{\top}$$



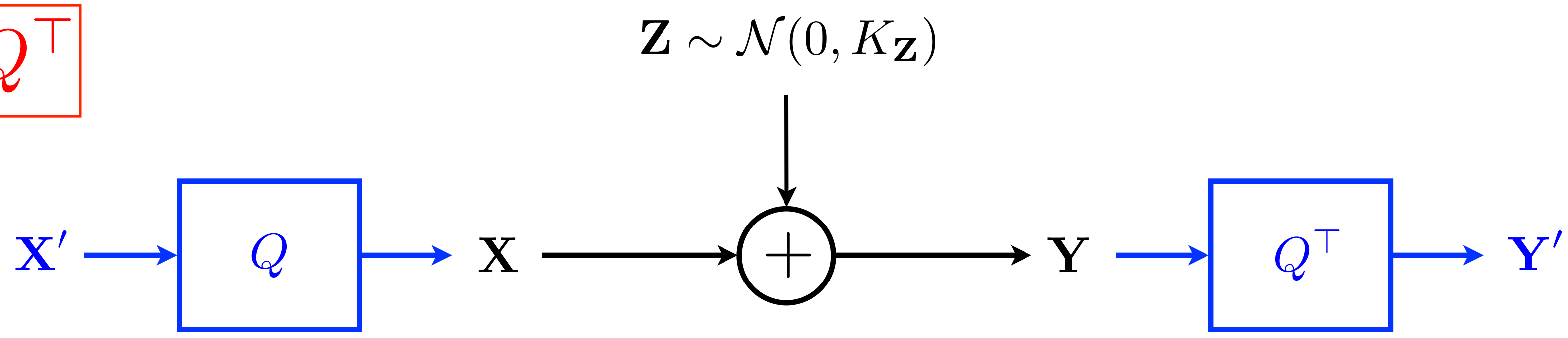
Decorrelation of the Noise Vector

$$K_{\mathbf{Z}} = Q \Lambda Q^\top$$



Decorrelation of the Noise Vector

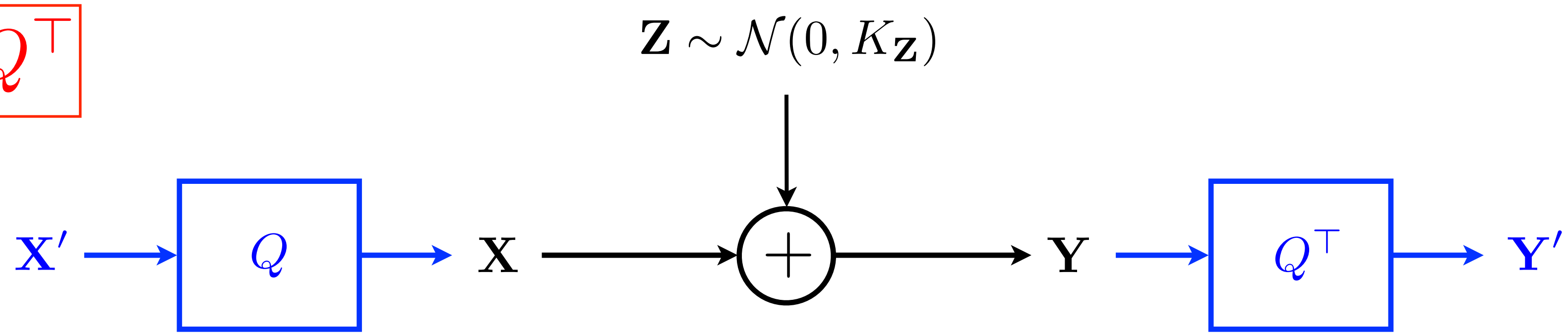
$$K_{\mathbf{Z}} = Q\Lambda Q^\top$$



- $\mathbf{Y}' = Q^\top \mathbf{Y}$ and $\mathbf{X}' = Q^\top \mathbf{X}$ (since $\mathbf{X} = Q\mathbf{X}'$).

Decorrelation of the Noise Vector

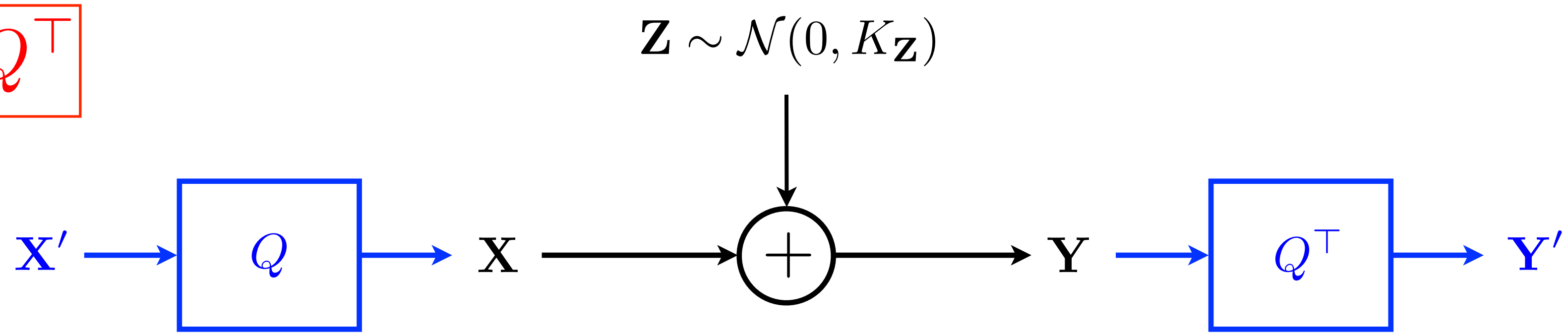
$K_{\mathbf{Z}} = Q \Lambda Q^{\top}$



- $\mathbf{Y}' = Q^{\top}\mathbf{Y}$ and $\mathbf{X}' = Q^{\top}\mathbf{X}$ (since $\mathbf{X} = Q\mathbf{X}'$).

Decorrelation of the Noise Vector

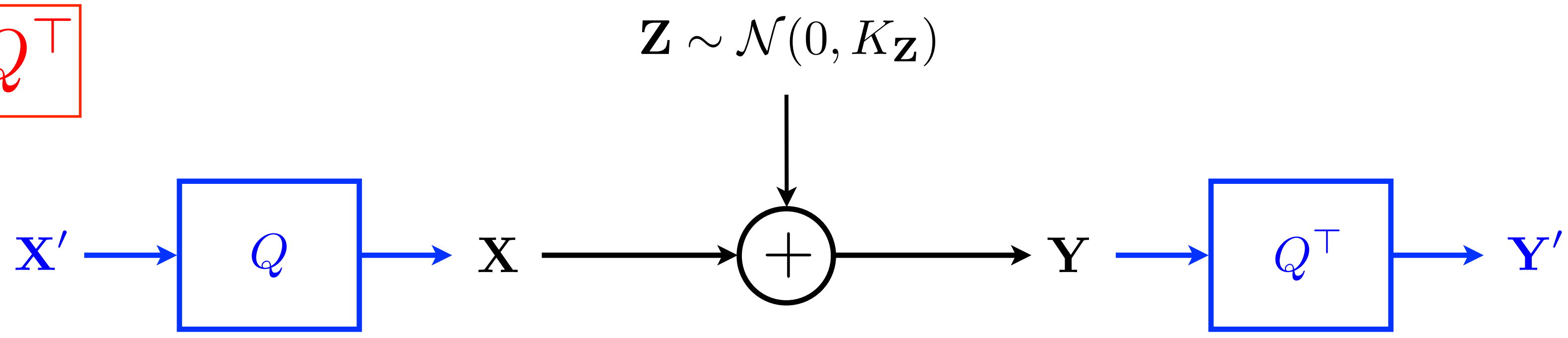
$K_{\mathbf{Z}} = Q \Lambda Q^\top$



- $\mathbf{Y}' = Q^\top \mathbf{Y}$ and $\underline{\mathbf{X}' = Q^\top \mathbf{X}}$ (since $\mathbf{X} = Q\mathbf{X}'$).

Decorrelation of the Noise Vector

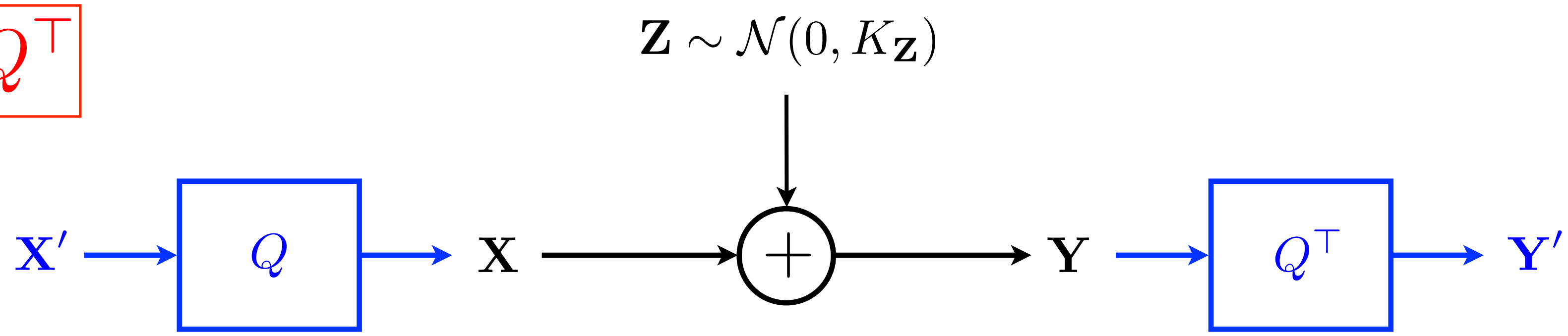
$$K_{\mathbf{Z}} = Q \Lambda Q^\top$$



- $\mathbf{Y}' = Q^\top \mathbf{Y}$ and $\mathbf{X}' = Q^\top \mathbf{X}$ (since $\mathbf{X} = Q\mathbf{X}'$).
- Let $\mathbf{Z}' = Q^\top \mathbf{Z}$, and so $\mathbf{Z}'$ is also Gaussian. Then

Decorrelation of the Noise Vector

$K_{\mathbf{Z}} = Q\Lambda Q^\top$

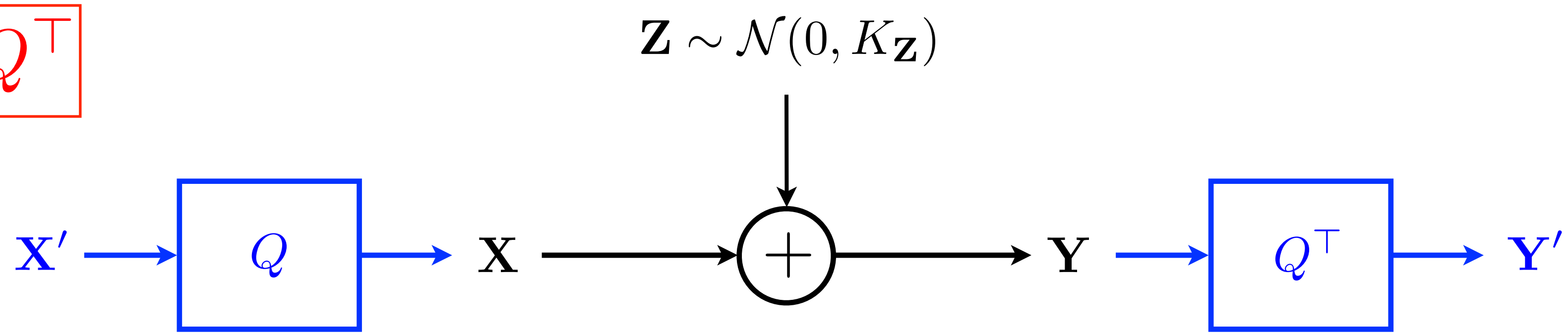


- $\mathbf{Y}' = Q^\top \mathbf{Y}$ and $\mathbf{X}' = Q^\top \mathbf{X}$ (since $\mathbf{X} = Q\mathbf{X}'$).
- Let $\mathbf{Z}' = Q^\top \mathbf{Z}$, and so $\mathbf{Z}'$ is also Gaussian. Then

$$\mathbf{Y}' = Q^\top \mathbf{Y}$$

Decorrelation of the Noise Vector

$K_{\mathbf{Z}} = Q\Lambda Q^\top$

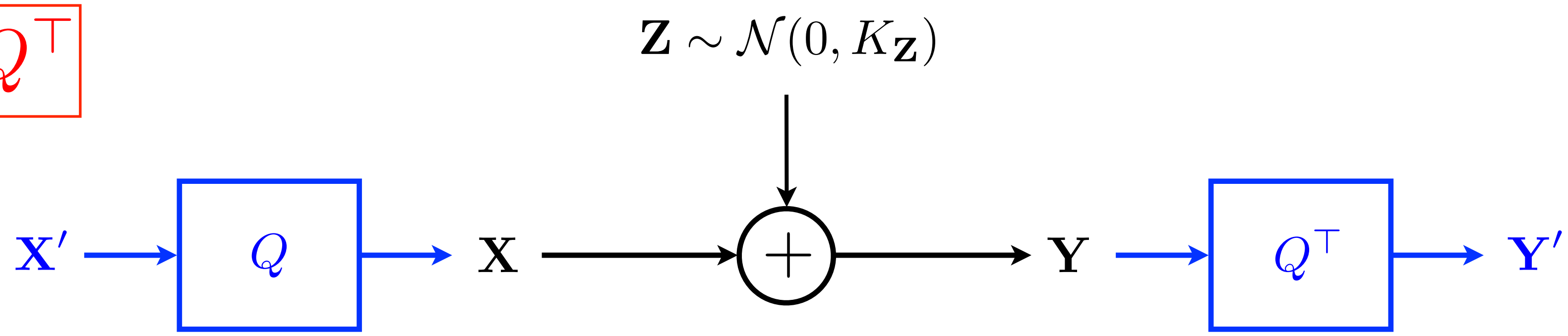


- $\mathbf{Y}' = Q^\top \mathbf{Y}$ and $\mathbf{X}' = Q^\top \mathbf{X}$ (since $\mathbf{X} = Q\mathbf{X}'$).
- Let $\mathbf{Z}' = Q^\top \mathbf{Z}$, and so $\mathbf{Z}'$ is also Gaussian. Then

$$\mathbf{Y}' = Q^\top \mathbf{Y} = Q^\top (\mathbf{X} + \mathbf{Z})$$

Decorrelation of the Noise Vector

$K_{\mathbf{Z}} = Q\Lambda Q^\top$

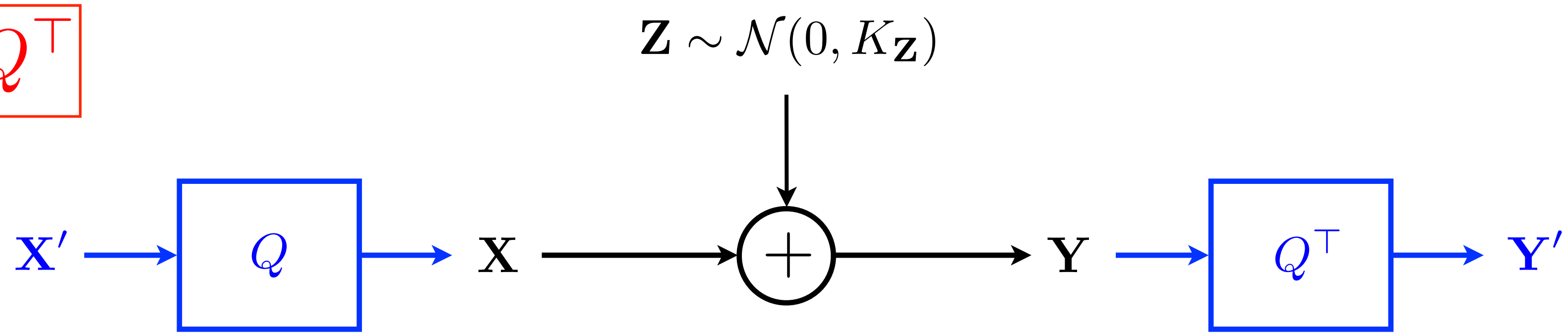


- $\mathbf{Y}' = Q^\top \mathbf{Y}$ and $\mathbf{X}' = Q^\top \mathbf{X}$ (since $\mathbf{X} = Q\mathbf{X}'$).
- Let $\mathbf{Z}' = Q^\top \mathbf{Z}$, and so $\mathbf{Z}'$ is also Gaussian. Then

$$\mathbf{Y}' = Q^\top \mathbf{Y} = Q^\top (\mathbf{X} + \mathbf{Z}) = Q^\top \mathbf{X} + Q^\top \mathbf{Z}$$

Decorrelation of the Noise Vector

$$K_{\mathbf{Z}} = Q \Lambda Q^{\top}$$

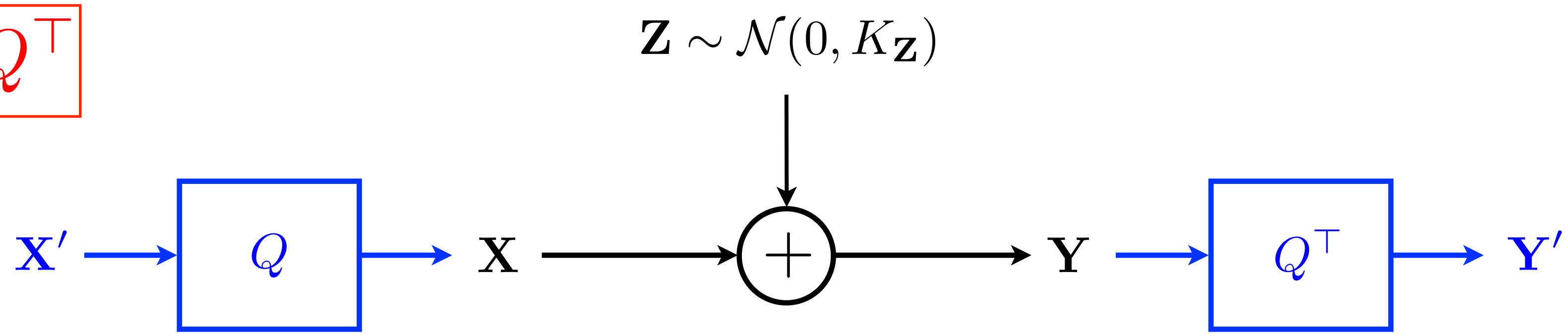


- $\mathbf{Y}' = Q^{\top}\mathbf{Y}$ and $\mathbf{X}' = Q^{\top}\mathbf{X}$ (since $\mathbf{X} = Q\mathbf{X}'$).
- Let $\mathbf{Z}' = Q^{\top}\mathbf{Z}$, and so $\mathbf{Z}'$ is also Gaussian. Then

$$\mathbf{Y}' = Q^{\top}\mathbf{Y} = Q^{\top}(\mathbf{X} + \mathbf{Z}) = \underline{Q^{\top}\mathbf{X}} + Q^{\top}\mathbf{Z}$$

Decorrelation of the Noise Vector

$K_{\mathbf{Z}} = Q\Lambda Q^\top$

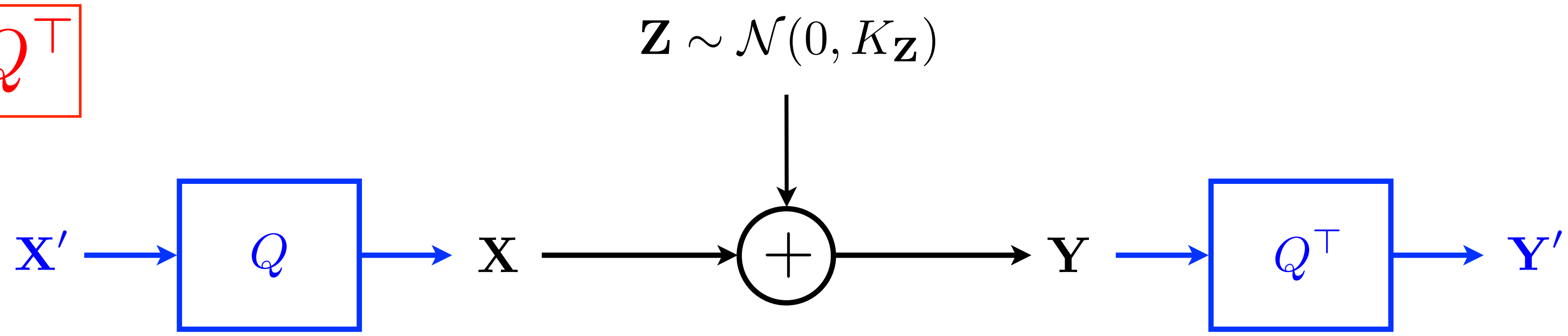


- $\mathbf{Y}' = Q^\top \mathbf{Y}$ and $\mathbf{X}' = Q^\top \mathbf{X}$ (since $\mathbf{X} = Q\mathbf{X}'$).
- Let $\mathbf{Z}' = Q^\top \mathbf{Z}$, and so $\mathbf{Z}'$ is also Gaussian. Then

$$\mathbf{Y}' = Q^\top \mathbf{Y} = Q^\top(\mathbf{X} + \mathbf{Z}) = \underline{Q^\top \mathbf{X}} + Q^\top \mathbf{Z} = \underline{\mathbf{X}'} + \mathbf{Z}'.$$

Decorrelation of the Noise Vector

$$K_{\mathbf{Z}} = Q \Lambda Q^{\top}$$

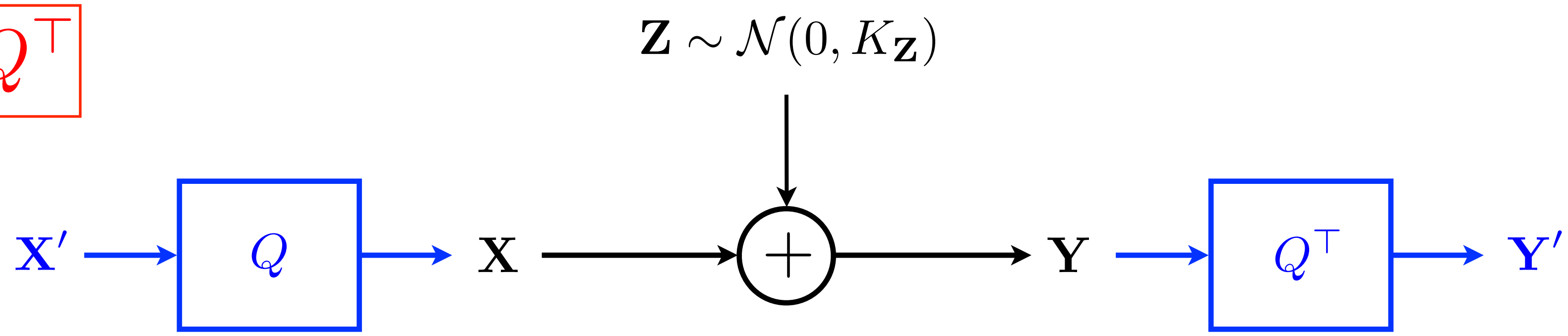


- $\mathbf{Y}' = Q^{\top}\mathbf{Y}$ and $\mathbf{X}' = Q^{\top}\mathbf{X}$ (since $\mathbf{X} = Q\mathbf{X}'$).
- Let $\mathbf{Z}' = Q^{\top}\mathbf{Z}$, and so $\mathbf{Z}'$ is also Gaussian. Then

$$\mathbf{Y}' = Q^{\top}\mathbf{Y} = Q^{\top}(\mathbf{X} + \mathbf{Z}) = Q^{\top}\mathbf{X} + \underline{Q^{\top}\mathbf{Z}} = \mathbf{X}' + \mathbf{Z}'.$$

Decorrelation of the Noise Vector

$K_{\mathbf{Z}} = Q\Lambda Q^\top$

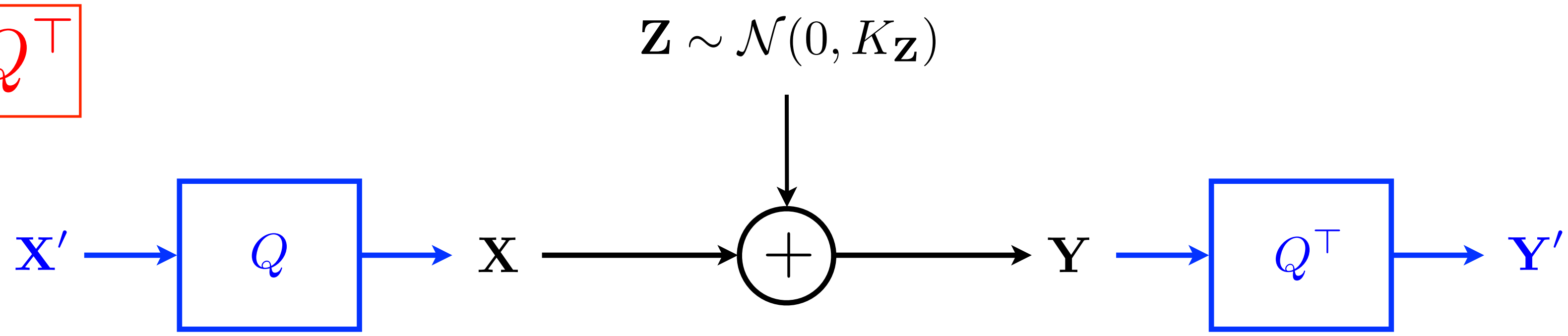


- $\mathbf{Y}' = Q^\top \mathbf{Y}$ and $\mathbf{X}' = Q^\top \mathbf{X}$ (since $\mathbf{X} = Q\mathbf{X}'$).
- Let $\mathbf{Z}' = Q^\top \mathbf{Z}$, and so $\mathbf{Z}'$ is also Gaussian. Then

$$\mathbf{Y}' = Q^\top \mathbf{Y} = Q^\top(\mathbf{X} + \mathbf{Z}) = Q^\top \mathbf{X} + \underline{Q^\top \mathbf{Z}} = \mathbf{X}' + \underline{\mathbf{Z}'}.$$

Decorrelation of the Noise Vector

$$K_{\mathbf{Z}} = Q \Lambda Q^\top$$

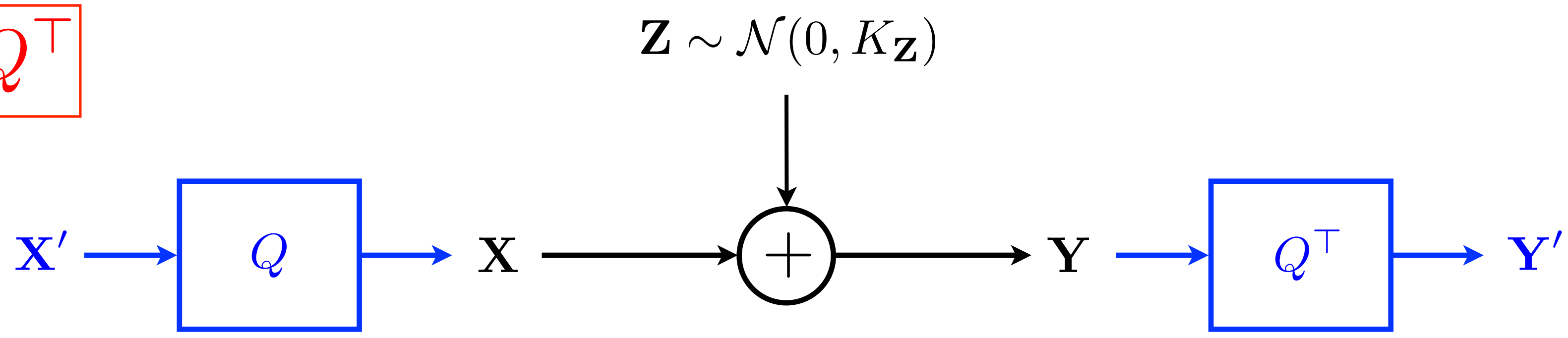


- $\mathbf{Y}' = Q^\top \mathbf{Y}$ and $\mathbf{X}' = Q^\top \mathbf{X}$ (since $\mathbf{X} = Q\mathbf{X}'$).
- Let $\mathbf{Z}' = Q^\top \mathbf{Z}$, and so $\mathbf{Z}'$ is also Gaussian. Then

$$\mathbf{Y}' = Q^\top \mathbf{Y} = Q^\top (\mathbf{X} + \mathbf{Z}) = Q^\top \mathbf{X} + Q^\top \mathbf{Z} = \mathbf{X}' + \mathbf{Z}'.$$

Decorrelation of the Noise Vector

$$K_{\mathbf{Z}} = Q\Lambda Q^\top$$



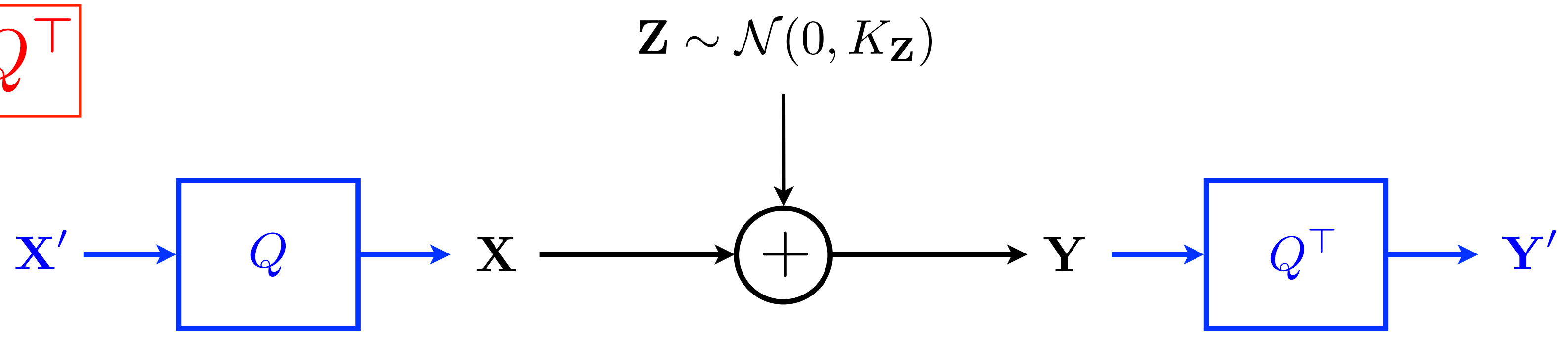
- $\mathbf{Y}' = Q^\top \mathbf{Y}$ and $\mathbf{X}' = Q^\top \mathbf{X}$ (since $\mathbf{X} = Q\mathbf{X}'$).
- Let $\mathbf{Z}' = Q^\top \mathbf{Z}$, and so $\mathbf{Z}'$ is also Gaussian. Then

$$\mathbf{Y}' = Q^\top \mathbf{Y} = Q^\top (\mathbf{X} + \mathbf{Z}) = Q^\top \mathbf{X} + Q^\top \mathbf{Z} = \mathbf{X}' + \mathbf{Z}'.$$

- The equivalent noise vector $\mathbf{Z}'$ is uncorrelated, because

Decorrelation of the Noise Vector

$$K_{\mathbf{Z}} = Q\Lambda Q^{\top}$$



- $\mathbf{Y}' = Q^{\top}\mathbf{Y}$ and $\mathbf{X}' = Q^{\top}\mathbf{X}$ (since $\mathbf{X} = Q\mathbf{X}'$).
- Let $\mathbf{Z}' = Q^{\top}\mathbf{Z}$, and so $\mathbf{Z}'$ is also Gaussian. Then

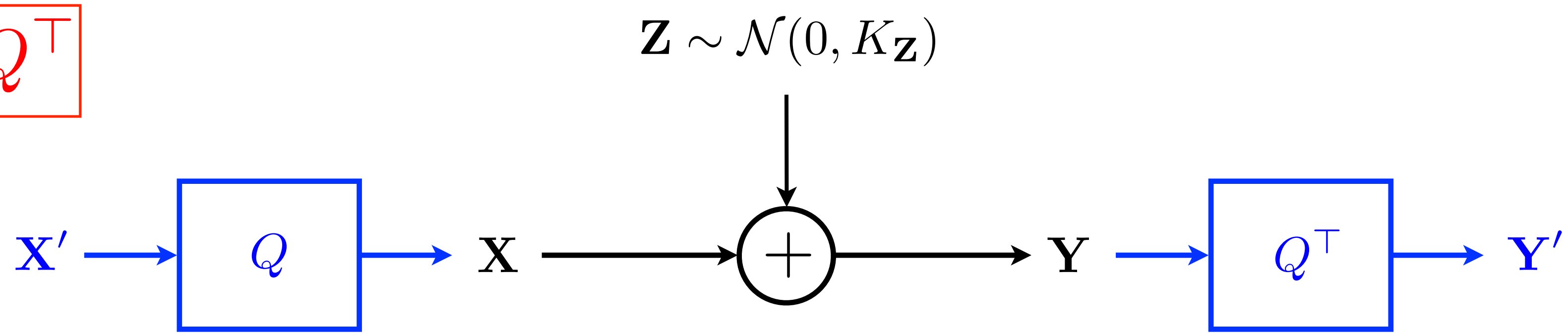
$$\mathbf{Y}' = Q^{\top}\mathbf{Y} = Q^{\top}(\mathbf{X} + \mathbf{Z}) = Q^{\top}\mathbf{X} + Q^{\top}\mathbf{Z} = \mathbf{X}' + \mathbf{Z}'.$$

- The equivalent noise vector $\mathbf{Z}'$ is uncorrelated, because

$$K_{\mathbf{Z}'}$$

Decorrelation of the Noise Vector

$$K_{\mathbf{Z}} = Q\Lambda Q^{\top}$$



- $\mathbf{Y}' = Q^{\top}\mathbf{Y}$ and $\mathbf{X}' = Q^{\top}\mathbf{X}$ (since $\mathbf{X} = Q\mathbf{X}'$).
- Let $\mathbf{Z}' = Q^{\top}\mathbf{Z}$, and so $\mathbf{Z}'$ is also Gaussian. Then

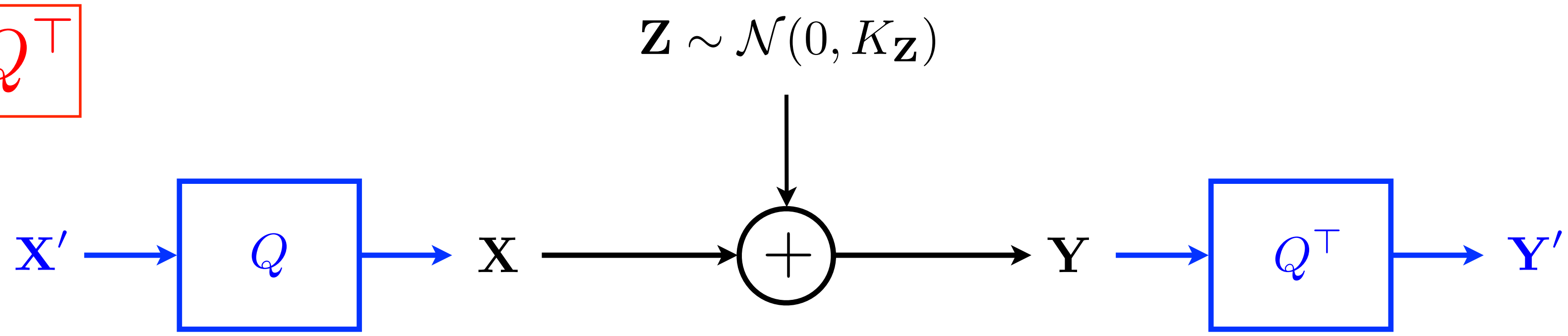
$$\mathbf{Y}' = Q^{\top}\mathbf{Y} = Q^{\top}(\mathbf{X} + \mathbf{Z}) = Q^{\top}\mathbf{X} + Q^{\top}\mathbf{Z} = \mathbf{X}' + \mathbf{Z}'.$$

- The equivalent noise vector $\mathbf{Z}'$ is uncorrelated, because

$$K_{\mathbf{Z}'} = Q^{\top}K_{\mathbf{Z}}Q$$

Decorrelation of the Noise Vector

$$K_{\mathbf{Z}} = Q\Lambda Q^\top$$



- $\mathbf{Y}' = Q^\top \mathbf{Y}$ and $\mathbf{X}' = Q^\top \mathbf{X}$ (since $\mathbf{X} = Q\mathbf{X}'$).
- Let $\mathbf{Z}' = Q^\top \mathbf{Z}$, and so $\mathbf{Z}'$ is also Gaussian. Then

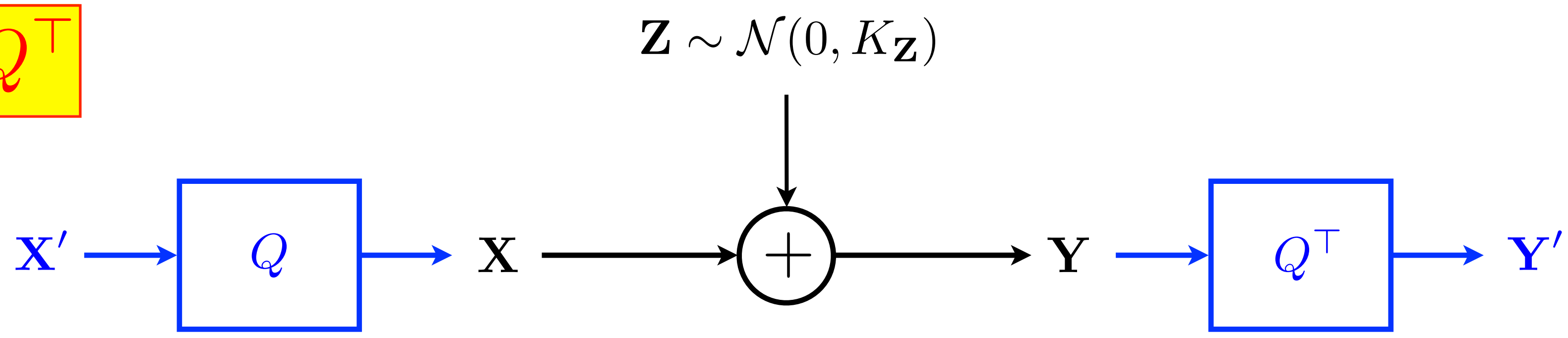
$$\mathbf{Y}' = Q^\top \mathbf{Y} = Q^\top(\mathbf{X} + \mathbf{Z}) = Q^\top \mathbf{X} + Q^\top \mathbf{Z} = \mathbf{X}' + \mathbf{Z}'.$$

- The equivalent noise vector $\mathbf{Z}'$ is uncorrelated, because

$$K_{\mathbf{Z}'} = Q^\top K_{\mathbf{Z}} Q$$

Decorrelation of the Noise Vector

$K_{\mathbf{Z}} = Q\Lambda Q^\top$



- $\mathbf{Y}' = Q^\top \mathbf{Y}$ and $\mathbf{X}' = Q^\top \mathbf{X}$ (since $\mathbf{X} = Q\mathbf{X}'$).
- Let $\mathbf{Z}' = Q^\top \mathbf{Z}$, and so $\mathbf{Z}'$ is also Gaussian. Then

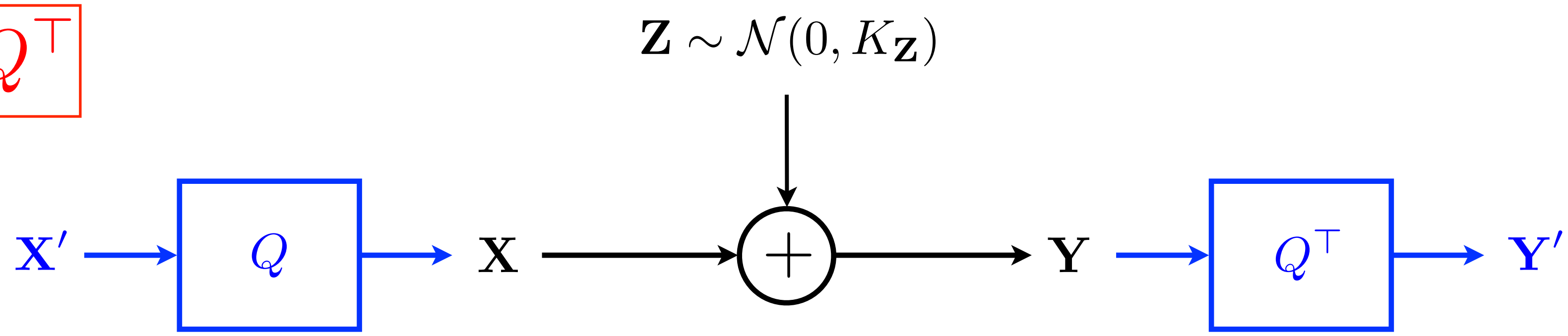
$$\mathbf{Y}' = Q^\top \mathbf{Y} = Q^\top(\mathbf{X} + \mathbf{Z}) = Q^\top \mathbf{X} + Q^\top \mathbf{Z} = \mathbf{X}' + \mathbf{Z}'.$$

- The equivalent noise vector $\mathbf{Z}'$ is uncorrelated, because

$$K_{\mathbf{Z}'} = Q^\top \underline{K_{\mathbf{Z}}} Q = Q^\top \underline{(Q\Lambda Q^\top)} Q$$

Decorrelation of the Noise Vector

$$K_{\mathbf{Z}} = Q\Lambda Q^\top$$



- $\mathbf{Y}' = Q^\top \mathbf{Y}$ and $\mathbf{X}' = Q^\top \mathbf{X}$ (since $\mathbf{X} = Q\mathbf{X}'$).
- Let $\mathbf{Z}' = Q^\top \mathbf{Z}$, and so $\mathbf{Z}'$ is also Gaussian. Then

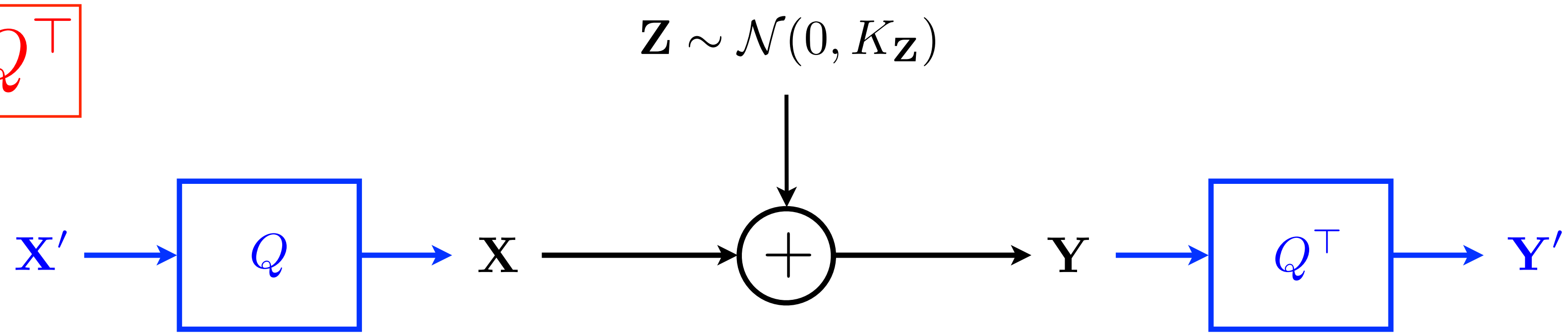
$$\mathbf{Y}' = Q^\top \mathbf{Y} = Q^\top(\mathbf{X} + \mathbf{Z}) = Q^\top \mathbf{X} + Q^\top \mathbf{Z} = \mathbf{X}' + \mathbf{Z}'.$$

- The equivalent noise vector $\mathbf{Z}'$ is uncorrelated, because

$$K_{\mathbf{Z}'} = Q^\top K_{\mathbf{Z}} Q = Q^\top (Q\Lambda Q^\top) Q$$

Decorrelation of the Noise Vector

$$K_{\mathbf{Z}} = Q\Lambda Q^\top$$



- $\mathbf{Y}' = Q^\top \mathbf{Y}$ and $\mathbf{X}' = Q^\top \mathbf{X}$ (since $\mathbf{X} = Q\mathbf{X}'$).
- Let $\mathbf{Z}' = Q^\top \mathbf{Z}$, and so $\mathbf{Z}'$ is also Gaussian. Then

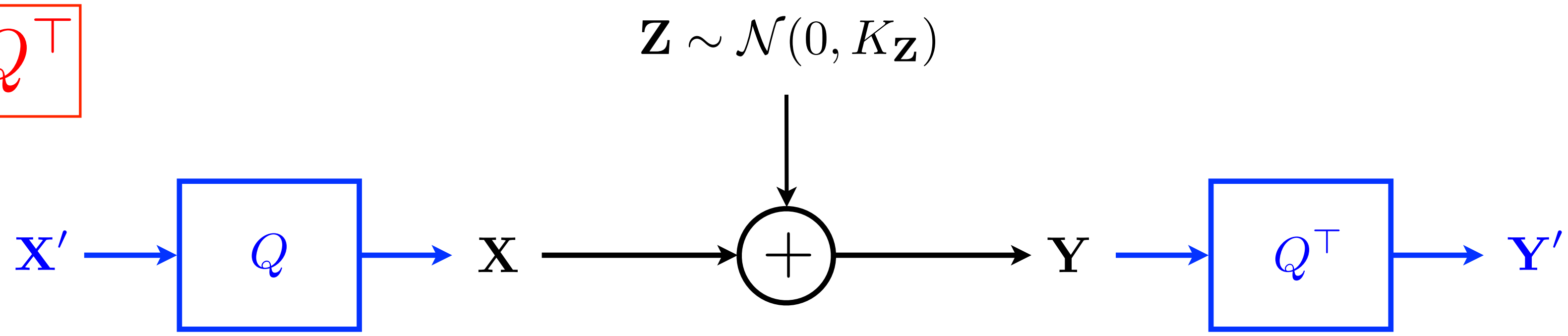
$$\mathbf{Y}' = Q^\top \mathbf{Y} = Q^\top (\mathbf{X} + \mathbf{Z}) = Q^\top \mathbf{X} + Q^\top \mathbf{Z} = \mathbf{X}' + \mathbf{Z}'.$$

- The equivalent noise vector $\mathbf{Z}'$ is uncorrelated, because

$$K_{\mathbf{Z}'} = Q^\top K_{\mathbf{Z}} Q = \cancel{Q^\top}(Q\Lambda Q^\top)Q$$

Decorrelation of the Noise Vector

$$K_{\mathbf{Z}} = Q \Lambda Q^\top$$



- $\mathbf{Y}' = Q^\top \mathbf{Y}$ and $\mathbf{X}' = Q^\top \mathbf{X}$ (since $\mathbf{X} = Q\mathbf{X}'$).
- Let $\mathbf{Z}' = Q^\top \mathbf{Z}$, and so $\mathbf{Z}'$ is also Gaussian. Then

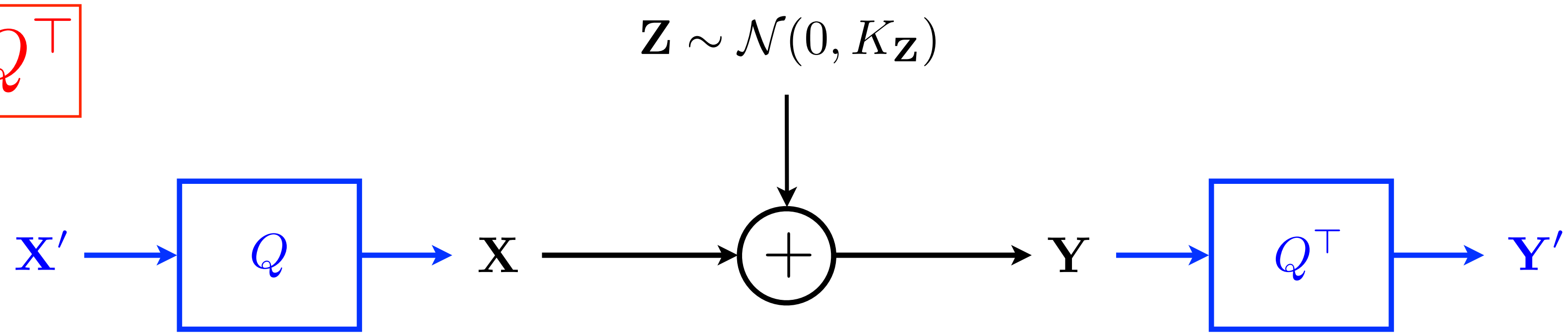
$$\mathbf{Y}' = Q^\top \mathbf{Y} = Q^\top(\mathbf{X} + \mathbf{Z}) = Q^\top \mathbf{X} + Q^\top \mathbf{Z} = \mathbf{X}' + \mathbf{Z}'.$$

- The equivalent noise vector $\mathbf{Z}'$ is uncorrelated, because

$$K_{\mathbf{Z}'} = Q^\top K_{\mathbf{Z}} Q = \cancel{Q^\top}(\cancel{Q}\Lambda Q^\top)Q$$

Decorrelation of the Noise Vector

$K_{\mathbf{Z}} = Q\Lambda Q^\top$



- $\mathbf{Y}' = Q^\top \mathbf{Y}$ and $\mathbf{X}' = Q^\top \mathbf{X}$ (since $\mathbf{X} = Q\mathbf{X}'$).
- Let $\mathbf{Z}' = Q^\top \mathbf{Z}$, and so $\mathbf{Z}'$ is also Gaussian. Then

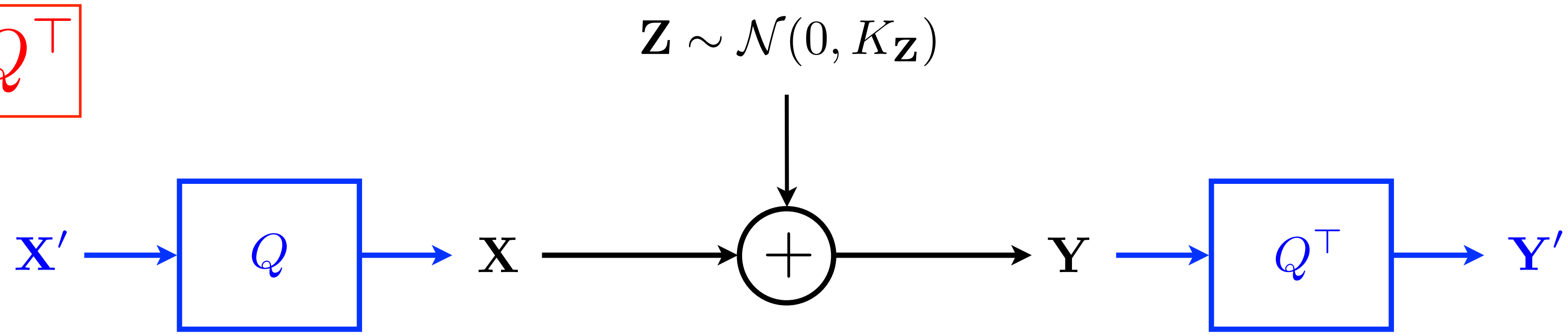
$$\mathbf{Y}' = Q^\top \mathbf{Y} = Q^\top(\mathbf{X} + \mathbf{Z}) = Q^\top \mathbf{X} + Q^\top \mathbf{Z} = \mathbf{X}' + \mathbf{Z}'.$$

- The equivalent noise vector $\mathbf{Z}'$ is uncorrelated, because

$$K_{\mathbf{Z}'} = Q^\top K_{\mathbf{Z}} Q = Q^\top (Q \Lambda Q^\top) Q$$

Decorrelation of the Noise Vector

$$K_{\mathbf{Z}} = Q\Lambda Q^\top$$



- $\mathbf{Y}' = Q^\top \mathbf{Y}$ and $\mathbf{X}' = Q^\top \mathbf{X}$ (since $\mathbf{X} = Q\mathbf{X}'$).
- Let $\mathbf{Z}' = Q^\top \mathbf{Z}$, and so $\mathbf{Z}'$ is also Gaussian. Then

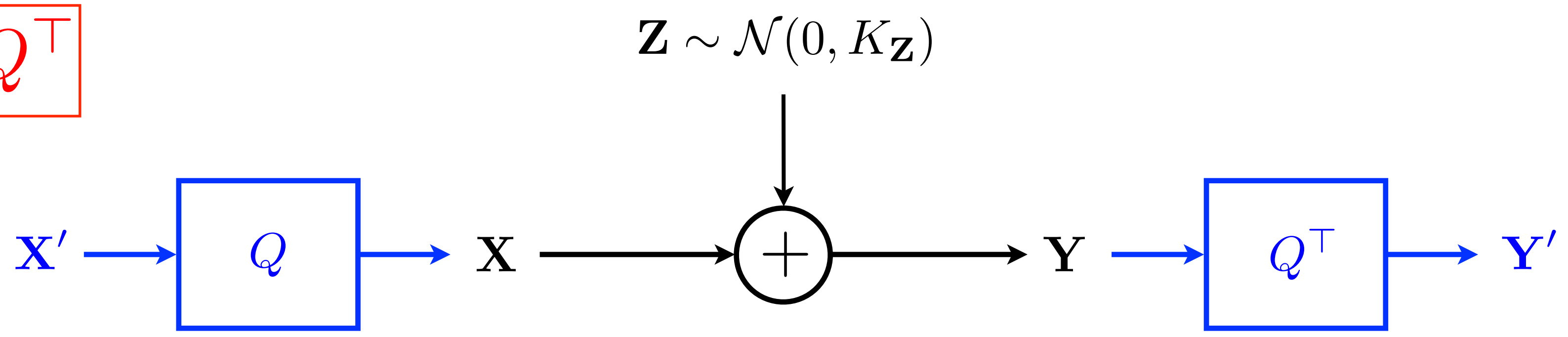
$$\mathbf{Y}' = Q^\top \mathbf{Y} = Q^\top(\mathbf{X} + \mathbf{Z}) = Q^\top \mathbf{X} + Q^\top \mathbf{Z} = \mathbf{X}' + \mathbf{Z}'.$$

- The equivalent noise vector $\mathbf{Z}'$ is uncorrelated, because

$$K_{\mathbf{Z}'} = Q^\top K_{\mathbf{Z}} Q = \cancel{Q}^\top (\cancel{Q}\Lambda \cancel{Q}^\top)\cancel{Q}$$

Decorrelation of the Noise Vector

$$K_{\mathbf{Z}} = Q \Lambda Q^{\top}$$



- $\mathbf{Y}' = Q^{\top}\mathbf{Y}$ and $\mathbf{X}' = Q^{\top}\mathbf{X}$ (since $\mathbf{X} = Q\mathbf{X}'$).
- Let $\mathbf{Z}' = Q^{\top}\mathbf{Z}$, and so $\mathbf{Z}'$ is also Gaussian. Then

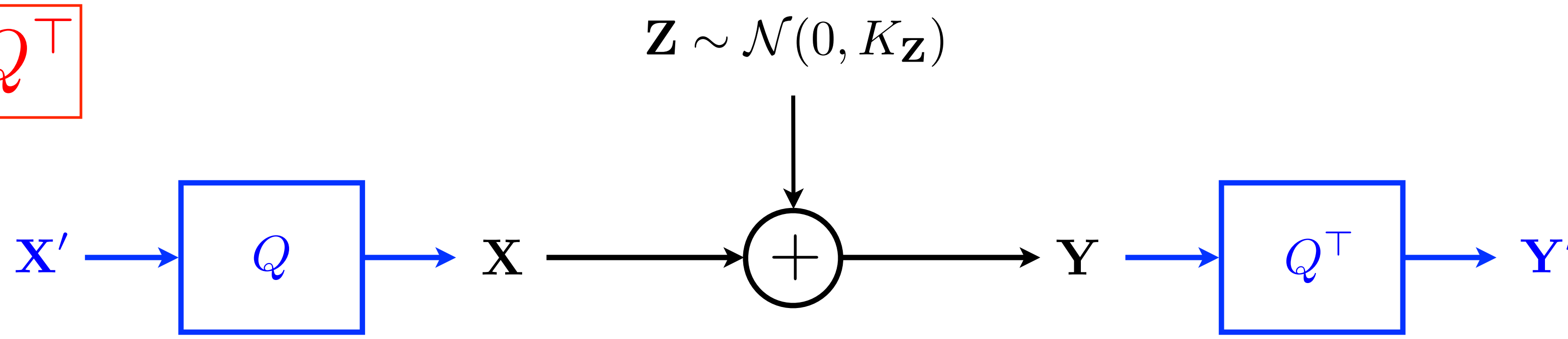
$$\mathbf{Y}' = Q^{\top}\mathbf{Y} = Q^{\top}(\mathbf{X} + \mathbf{Z}) = Q^{\top}\mathbf{X} + Q^{\top}\mathbf{Z} = \mathbf{X}' + \mathbf{Z}'.$$

- The equivalent noise vector $\mathbf{Z}'$ is uncorrelated, because

$$K_{\mathbf{Z}'} = Q^{\top} K_{\mathbf{Z}} Q = Q^{\top}(Q \Lambda Q^{\top}) Q = \Lambda,$$

Decorrelation of the Noise Vector

$$K_{\mathbf{Z}} = Q\Lambda Q^\top$$



- $\mathbf{Y}' = Q^\top \mathbf{Y}$ and $\mathbf{X}' = Q^\top \mathbf{X}$ (since $\mathbf{X} = Q\mathbf{X}'$).
- Let $\mathbf{Z}' = Q^\top \mathbf{Z}$, and so $\mathbf{Z}'$ is also Gaussian. Then

$$\mathbf{Y}' = Q^\top \mathbf{Y} = Q^\top(\mathbf{X} + \mathbf{Z}) = Q^\top \mathbf{X} + Q^\top \mathbf{Z} = \mathbf{X}' + \mathbf{Z}'.$$

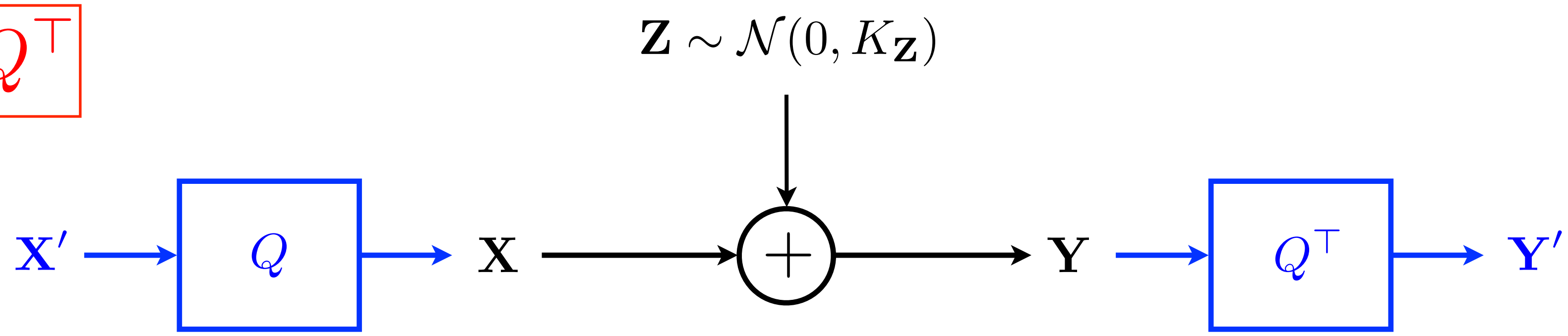
- The equivalent noise vector $\mathbf{Z}'$ is uncorrelated, because

$$K_{\mathbf{Z}'} = Q^\top K_{\mathbf{Z}} Q = Q^\top (Q\Lambda Q^\top) Q = \Lambda,$$

i.e., $Z_i' \sim \mathcal{N}(0, \lambda_i)$, and Z_i', $1 \le i \le k$ are mutually independent.

Decorrelation of the Noise Vector

$$K_{\mathbf{Z}} = Q\Lambda Q^\top$$



- $\mathbf{Y}' = Q^\top \mathbf{Y}$ and $\mathbf{X}' = Q^\top \mathbf{X}$ (since $\mathbf{X} = Q\mathbf{X}'$).
- Let $\mathbf{Z}' = Q^\top \mathbf{Z}$, and so $\mathbf{Z}'$ is also Gaussian. Then

$$\mathbf{Y}' = Q^\top \mathbf{Y} = Q^\top(\mathbf{X} + \mathbf{Z}) = Q^\top \mathbf{X} + Q^\top \mathbf{Z} = \mathbf{X}' + \mathbf{Z}'.$$

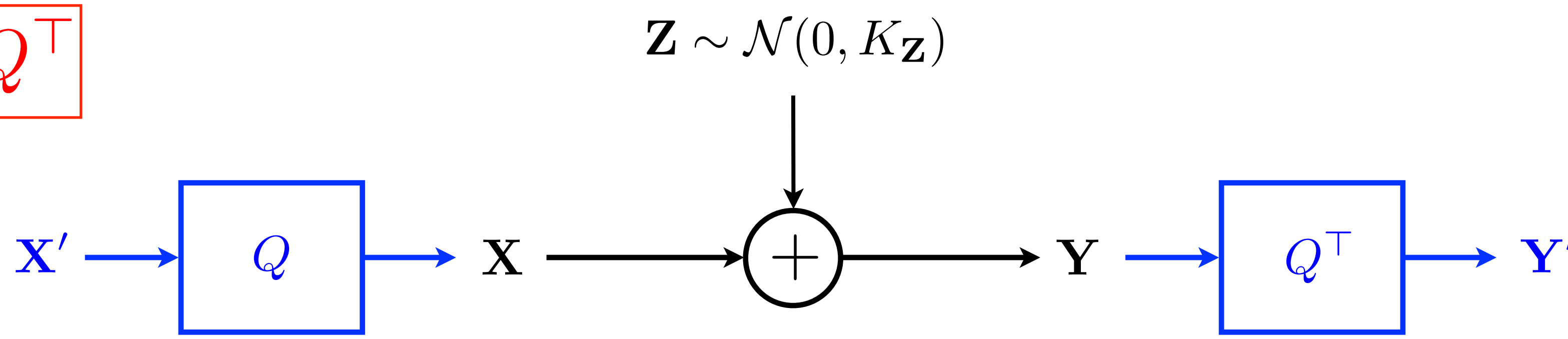
- The equivalent noise vector $\mathbf{Z}'$ is uncorrelated, because

$$K_{\mathbf{Z}'} = Q^\top K_{\mathbf{Z}} Q = Q^\top (Q\Lambda Q^\top) Q = \Lambda,$$

i.e., $Z_i' \sim \mathcal{N}(0, \lambda_i)$, and Z_i', $1 \leq i \leq k$ are mutually independent.

Decorrelation of the Noise Vector

$K_{\mathbf{Z}} = Q \Lambda Q^\top$



- $\mathbf{Y}' = Q^\top \mathbf{Y}$ and $\mathbf{X}' = Q^\top \mathbf{X}$ (since $\mathbf{X} = Q\mathbf{X}'$).
- Let $\mathbf{Z}' = Q^\top \mathbf{Z}$, and so $\mathbf{Z}'$ is also Gaussian. Then

$$\mathbf{Y}' = Q^\top \mathbf{Y} = Q^\top(\mathbf{X} + \mathbf{Z}) = Q^\top \mathbf{X} + Q^\top \mathbf{Z} = \mathbf{X}' + \mathbf{Z}'.$$

- The equivalent noise vector $\mathbf{Z}'$ is uncorrelated, because

$$K_{\mathbf{Z}'} = Q^\top K_{\mathbf{Z}} Q = Q^\top (Q \Lambda Q^\top) Q = \Lambda,$$

  i.e., $Z'_i \sim \mathcal{N}(0, \lambda_i)$, and Z'_i, $1 \le i \le k$ are mutually independent.

- The "equivalent system" is a system of parallel Gaussian channels.

Decorrelation of the Noise Vector

- $\mathbf{Y}' = Q^\top \mathbf{Y}$ and $\mathbf{X}' = Q^\top \mathbf{X}$ (since $\mathbf{X} = Q\mathbf{X}'$).
- Let $\mathbf{Z}' = Q^\top \mathbf{Z}$, and so $\mathbf{Z}'$ is also Gaussian. Then

$$\mathbf{Y}' = Q^\top \mathbf{Y} = Q^\top(\mathbf{X} + \mathbf{Z}) = Q^\top \mathbf{X} + Q^\top \mathbf{Z} = \mathbf{X}' + \mathbf{Z}'.$$

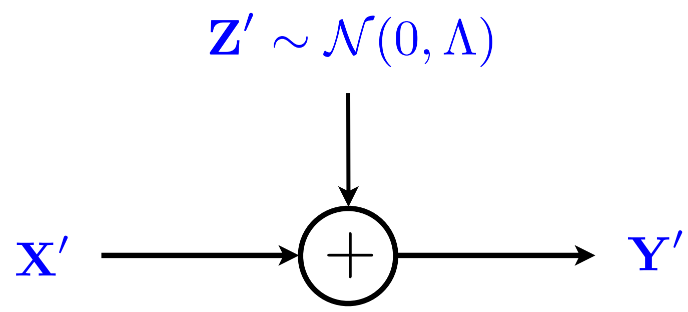
- The equivalent noise vector $\mathbf{Z}'$ is uncorrelated, because

$$K_{\mathbf{Z}'} = Q^\top K_{\mathbf{Z}} Q = Q^\top (Q \Lambda Q^\top) Q = \Lambda,$$

i.e., $Z_i' \sim \mathcal{N}(0, \lambda_i)$, and Z_i', $1 \leq i \leq k$ are mutually independent.

- The "equivalent system" is a system of parallel Gaussian channels.

Decorrelation of the Noise Vector



- $\mathbf{Y}' = Q^\top \mathbf{Y}$ and $\mathbf{X}' = Q^\top \mathbf{X}$ (since $\mathbf{X} = Q\mathbf{X}'$).
- Let $\mathbf{Z}' = Q^\top \mathbf{Z}$, and so $\mathbf{Z}'$ is also Gaussian. Then

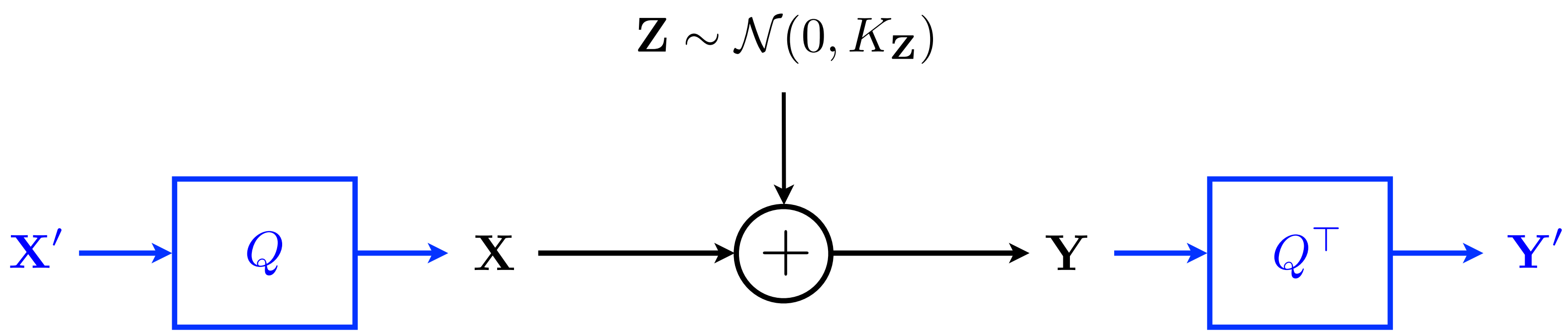
$$\mathbf{Y}' = Q^\top \mathbf{Y} = Q^\top(\mathbf{X} + \mathbf{Z}) = Q^\top \mathbf{X} + Q^\top \mathbf{Z} = \mathbf{X}' + \mathbf{Z}'.$$

- The equivalent noise vector $\mathbf{Z}'$ is uncorrelated, because

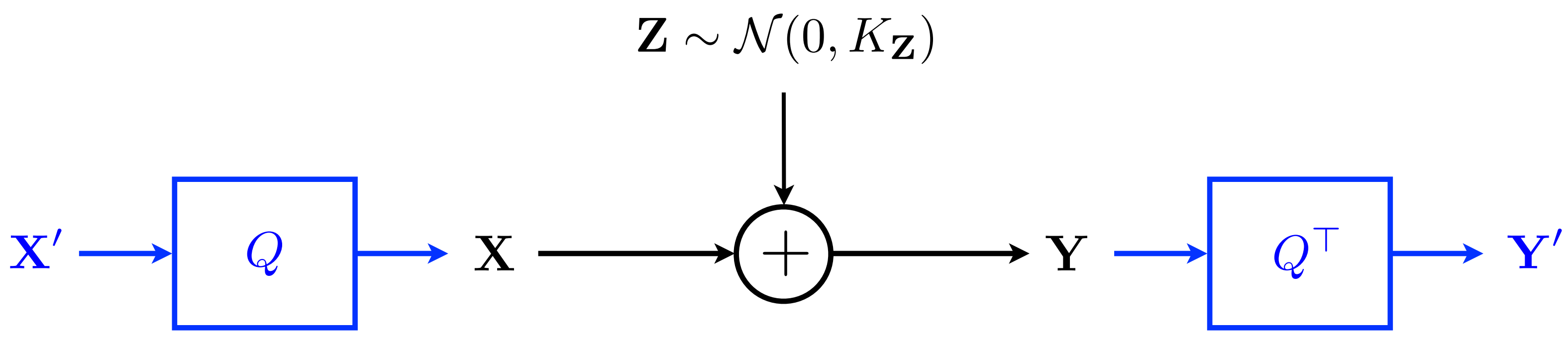
$$K_{\mathbf{Z}'} = Q^\top K_{\mathbf{Z}} Q = Q^\top (Q \Lambda Q^\top) Q = \Lambda,$$

  i.e., $Z_i' \sim \mathcal{N}(0, \lambda_i)$, and Z_i', $1 \leq i \leq k$ are mutually independent.
- The "equivalent system" is a system of parallel Gaussian channels.

Power Constraint

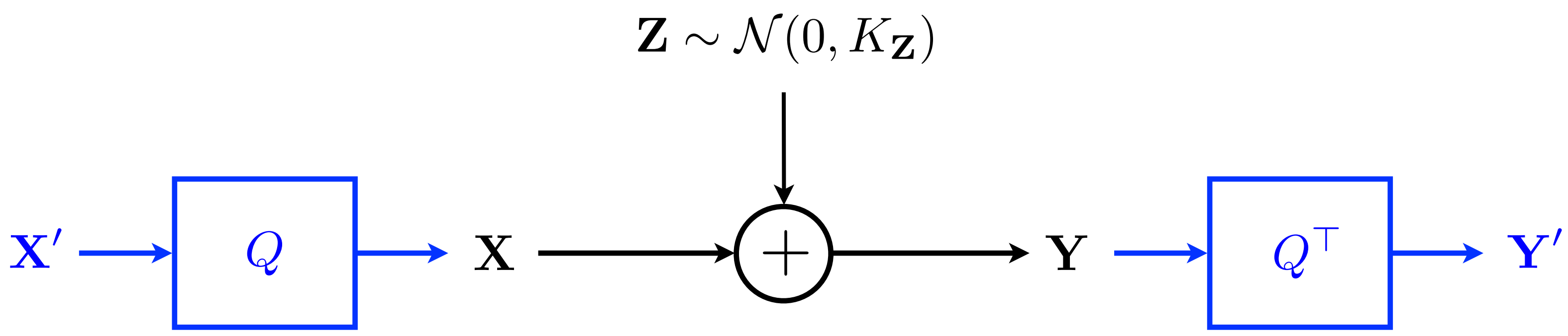


Power Constraint



- Since $\mathbf{X}' = Q^\top \mathbf{X}$ and $Q^\top$ is an orthogonal matrix, by Proposition 10.9,

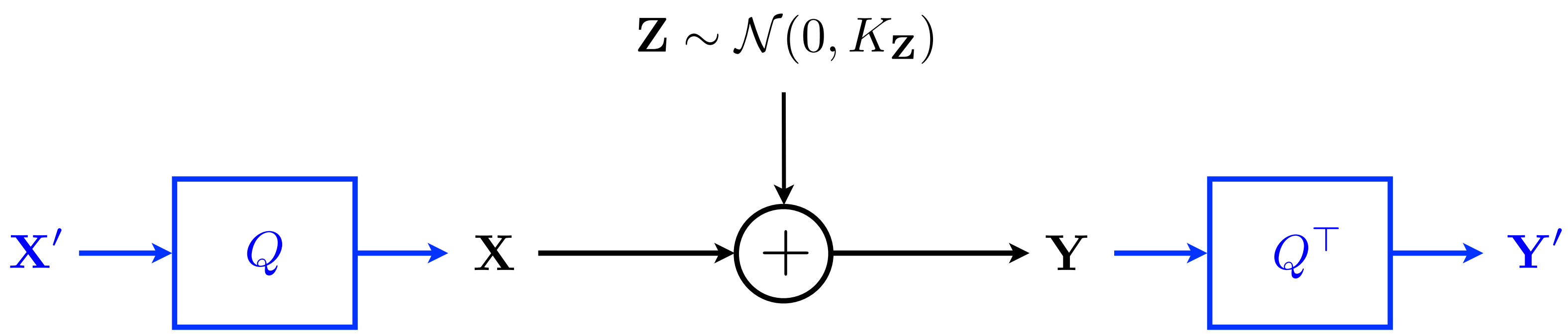
Power Constraint



- Since $\mathbf{X}' = Q^\top \mathbf{X}$ and $Q^\top$ is an orthogonal matrix, by Proposition 10.9,

$$E \sum_i (X_i')^2 = E \sum_i X_i^2.$$

Power Constraint



- Since $\mathbf{X}' = Q^\top \mathbf{X}$ and $Q^\top$ is an orthogonal matrix, by Proposition 10.9,

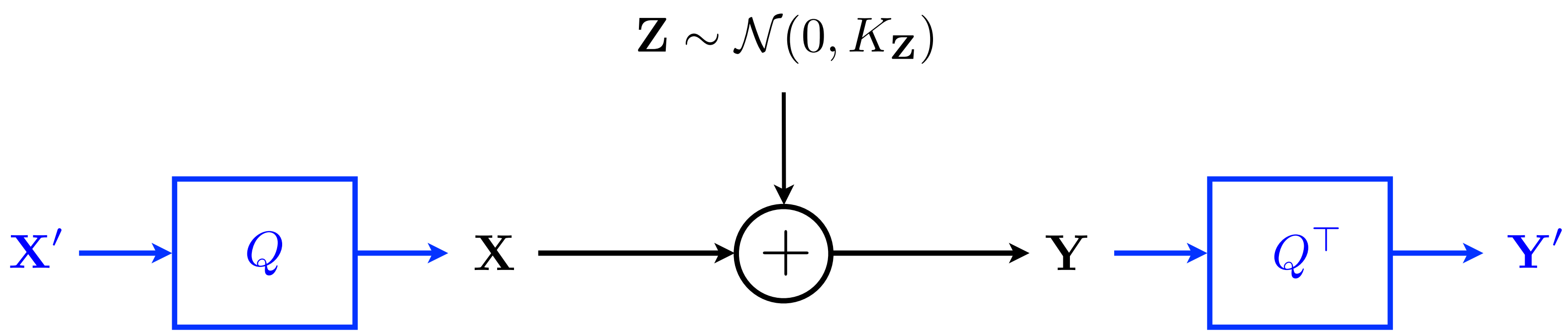
$$E \sum_i (X_i')^2 = E \sum_i X_i^2.$$

- Therefore, the input power constraint

$$E \sum_i X_i^2 \leq P$$

of the original system translates to the input power constraint

Power Constraint



- Since $\mathbf{X}' = Q^\top \mathbf{X}$ and $Q^\top$ is an orthogonal matrix, by Proposition 10.9,

$$E \sum_i (X_i')^2 = E \sum_i X_i^2.$$

- Therefore, the input power constraint

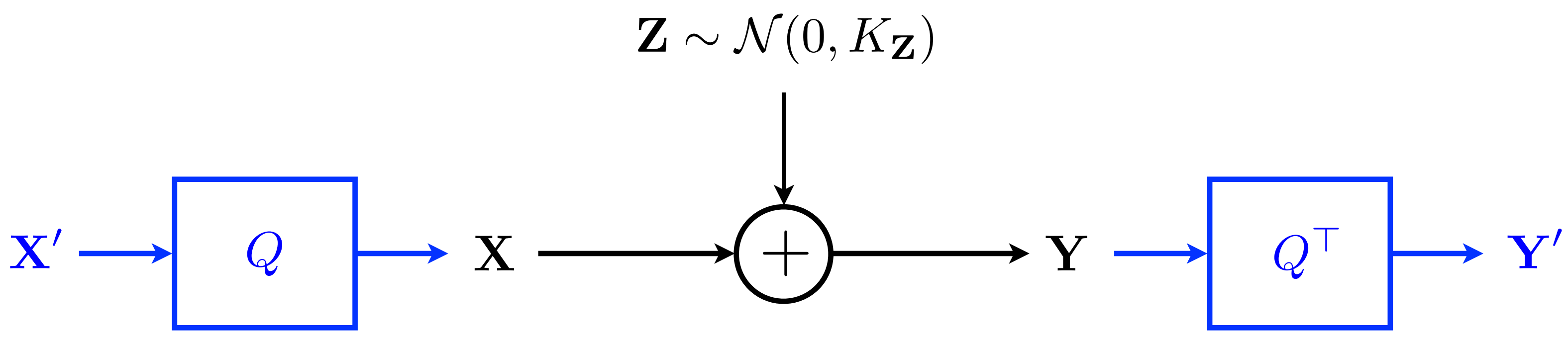
$$E \sum_i X_i^2 \leq P$$

of the original system translates to the input power constraint

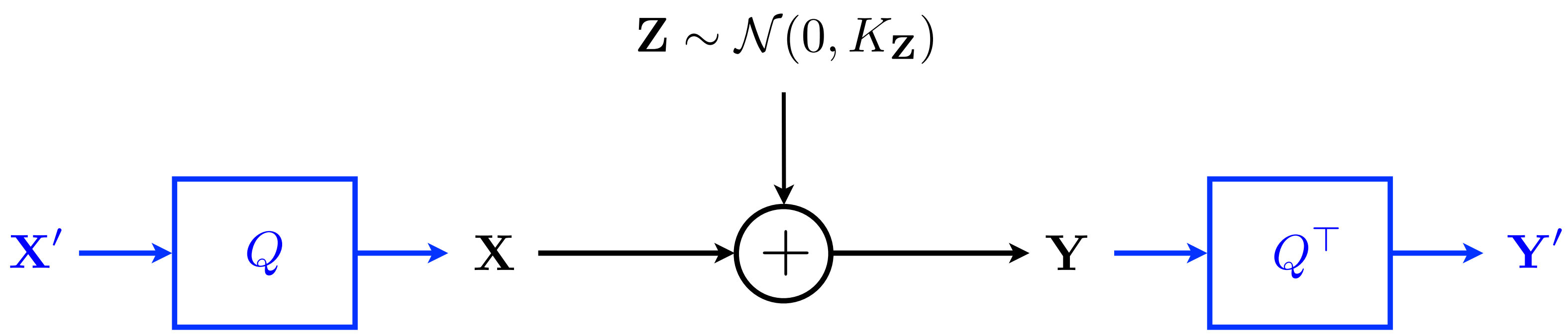
$$E \sum_i (X_i')^2 \leq P$$

of the equivalent system.

Equivalence of Capacity

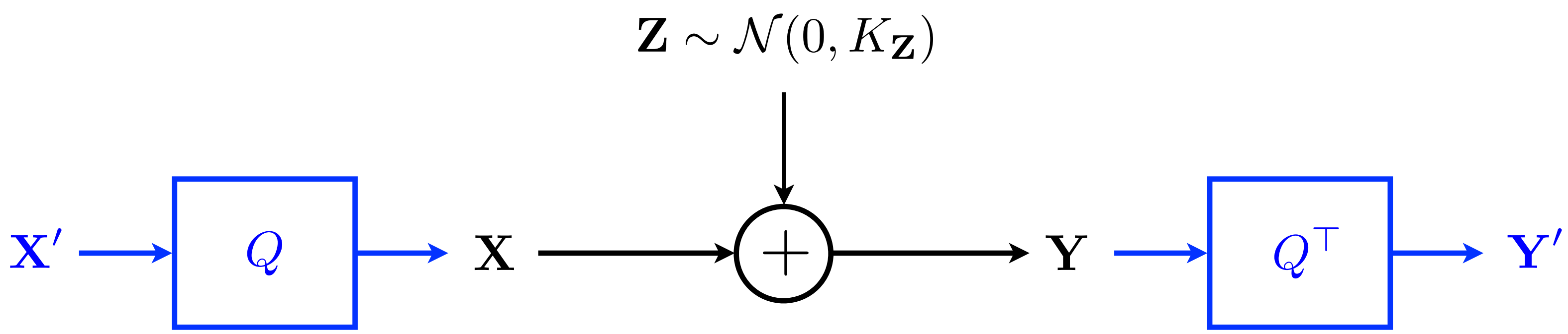


Equivalence of Capacity



Proposition $I(\mathbf{X}'; \mathbf{Y}') = I(\mathbf{X}; \mathbf{Y})$.

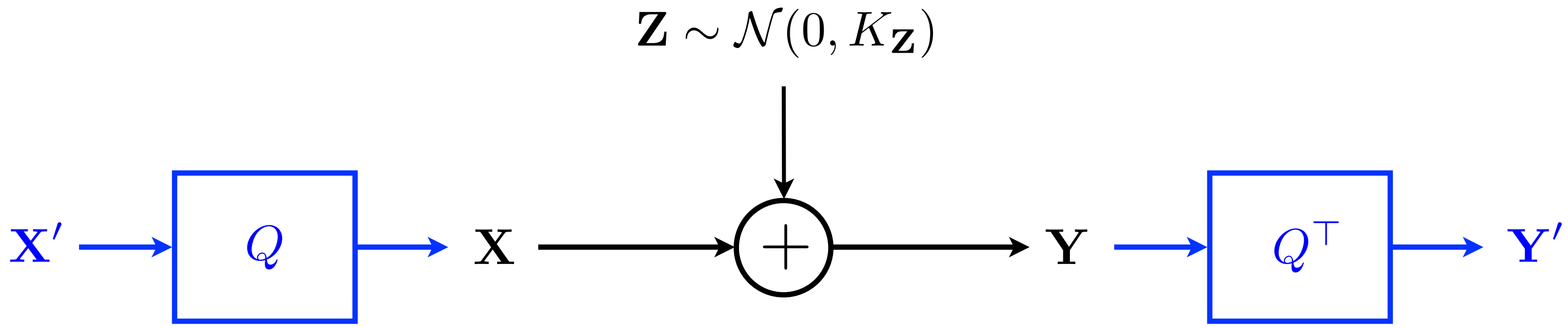
Equivalence of Capacity



Proposition $I(\mathbf{X}'; \mathbf{Y}') = I(\mathbf{X}; \mathbf{Y})$.

Proof

Equivalence of Capacity

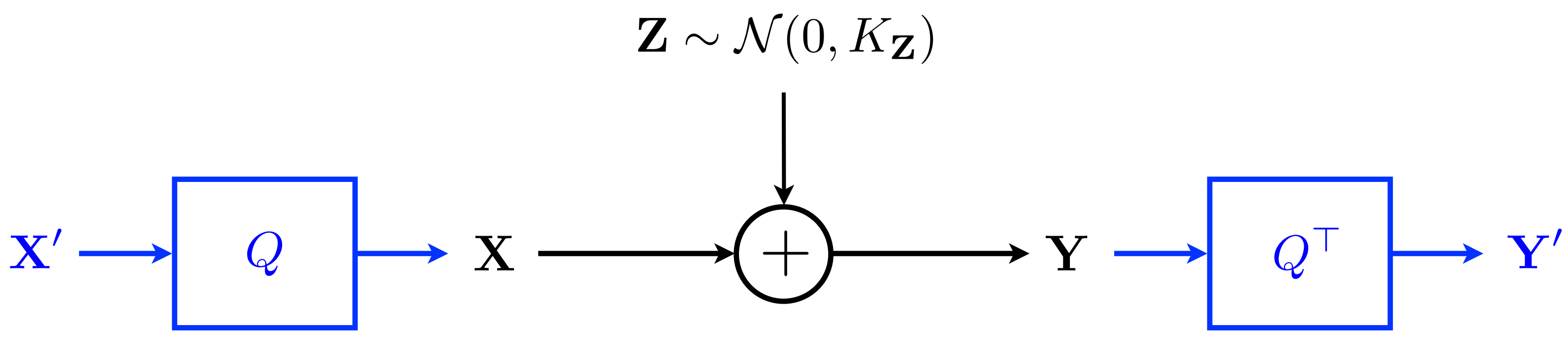


Proposition $I(\mathbf{X}';\mathbf{Y}') = I(\mathbf{X};\mathbf{Y})$.

Proof

$I(\mathbf{X}';\mathbf{Y}')$

Equivalence of Capacity

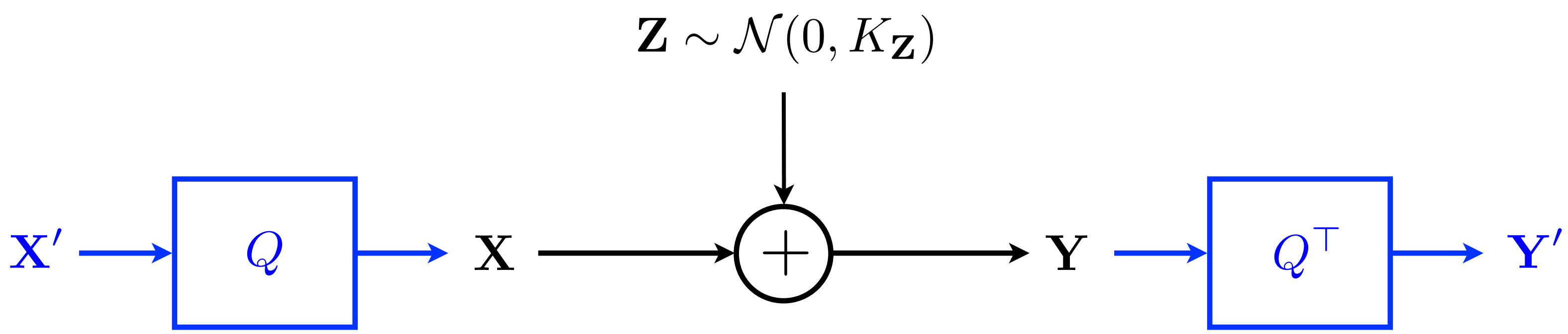


Proposition $I(\mathbf{X}'; \mathbf{Y}') = I(\mathbf{X}; \mathbf{Y})$.

Proof

$$
\begin{aligned}
& I(\mathbf{X}'; \mathbf{Y}') \\
&\quad = \quad h(\mathbf{Y}') - h(\mathbf{Y}'|\mathbf{X}')
\end{aligned}
$$

Equivalence of Capacity

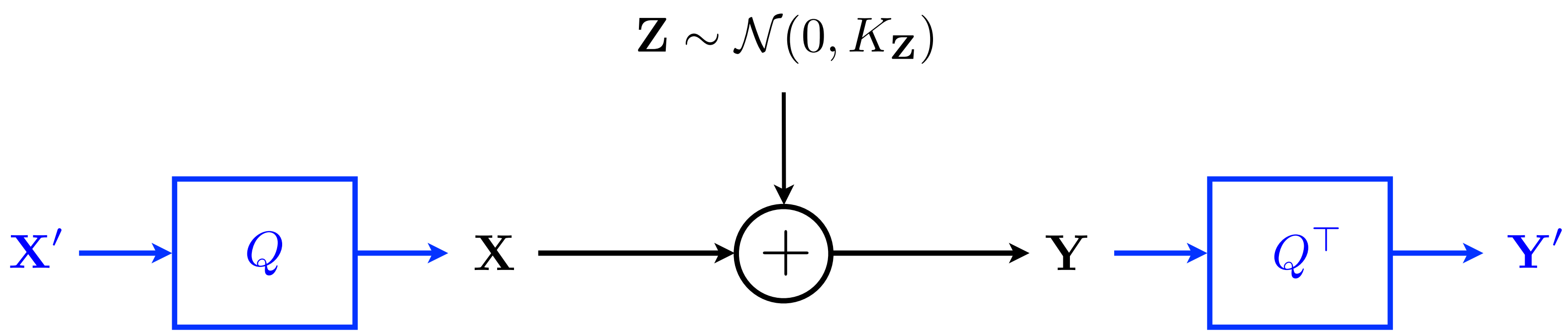


Proposition $I(\mathbf{X}'; \mathbf{Y}') = I(\mathbf{X}; \mathbf{Y})$.

Proof

$$
\begin{aligned}
& I(\mathbf{X}'; \mathbf{Y}') \\
& \quad = \quad h(\mathbf{Y}') - \underline{h(\mathbf{Y}'|\mathbf{X}')}
\end{aligned}
$$

Equivalence of Capacity



Proposition $I(\mathbf{X}'; \mathbf{Y}') = I(\mathbf{X}; \mathbf{Y})$.

Proof

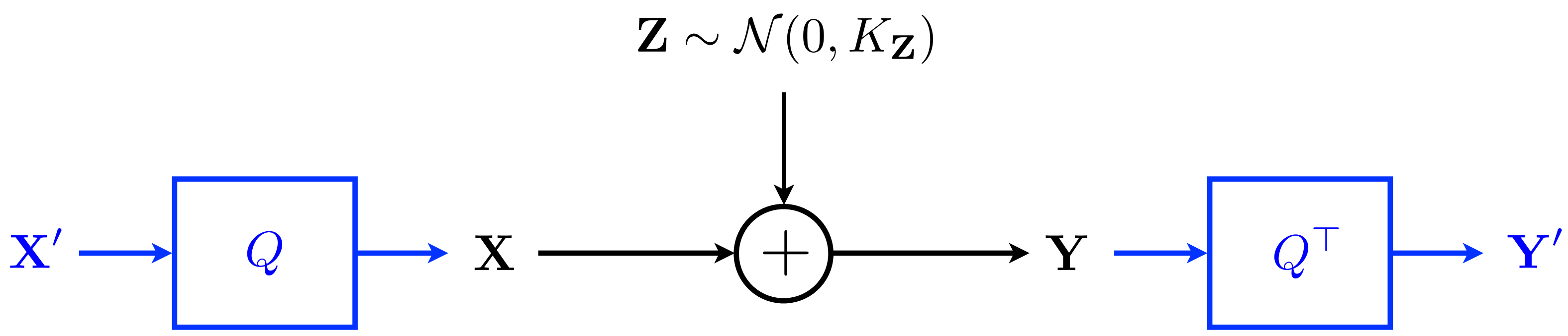
$$
\begin{aligned}
&I(\mathbf{X}'; \mathbf{Y}') \\
&\quad = \quad h(\mathbf{Y}') - \underline{h(\mathbf{Y}'|\mathbf{X}')} \\
&\quad = \quad h(\mathbf{Y}') - \underline{h(\mathbf{Z}'|\mathbf{X}')}
\end{aligned}
$$

Lemma 11.22 Let $Y = X + Z$. Then

$$h(Y|X) = h(Z|X)$$

provided that $f_{Z|X}(z|x)$ exists for all $x \in \mathcal{S}_X$.

Equivalence of Capacity

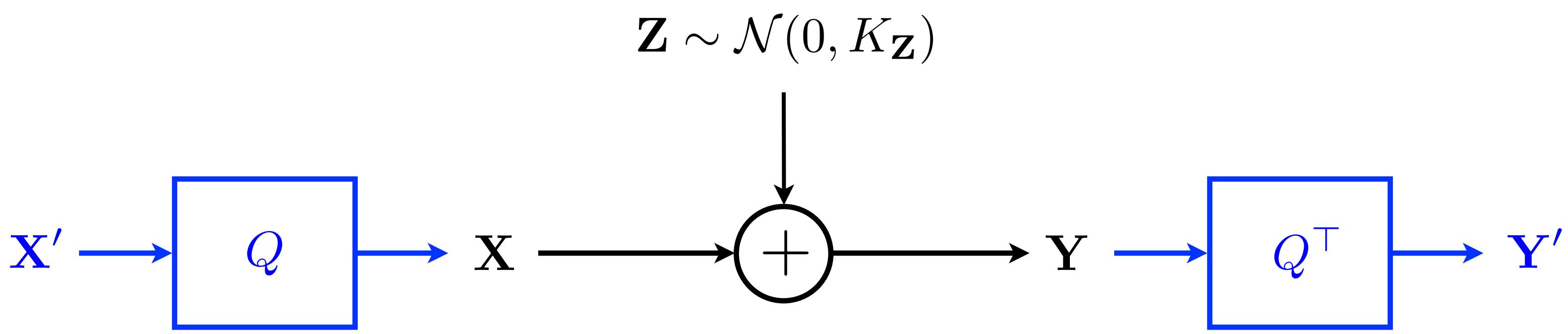


Proposition $I(\mathbf{X}'; \mathbf{Y}') = I(\mathbf{X}; \mathbf{Y})$.

Proof

$$
\begin{aligned}
&I(\mathbf{X}'; \mathbf{Y}') \\
&\quad = \quad h(\mathbf{Y}') - h(\mathbf{Y}'|\mathbf{X}') \\
&\quad = \quad h(\mathbf{Y}') - \underline{h(\mathbf{Z}'|\mathbf{X}')}
\end{aligned}
$$

Equivalence of Capacity

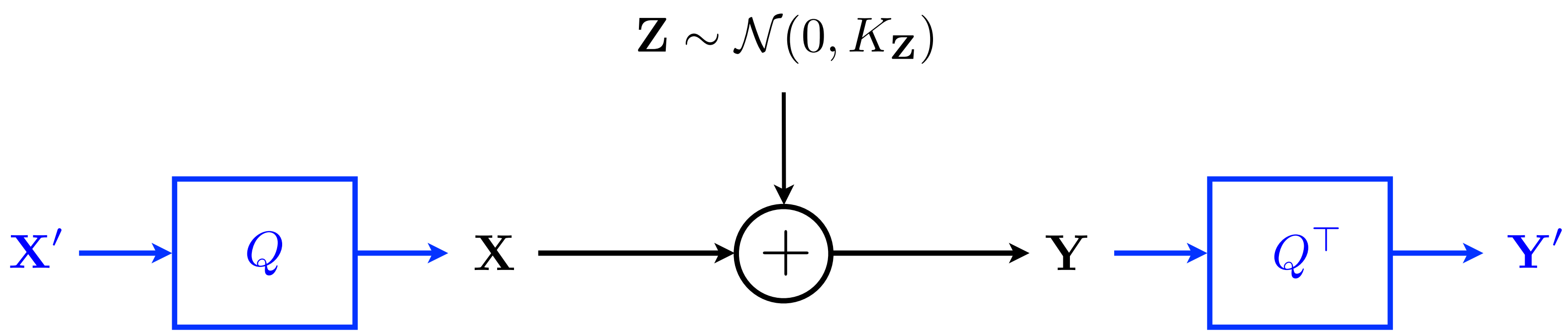


Proposition $I(\mathbf{X}'; \mathbf{Y}') = I(\mathbf{X}; \mathbf{Y})$.

Proof

$$
\begin{aligned}
& I(\mathbf{X}'; \mathbf{Y}') \\
&= h(\mathbf{Y}') - h(\mathbf{Y}'|\mathbf{X}') \\
&= h(\mathbf{Y}') - \underline{h(\mathbf{Z}'|\mathbf{X}')} \\
&= h(\mathbf{Y}') - \underline{h(\mathbf{Z}')}
\end{aligned}
$$

Equivalence of Capacity



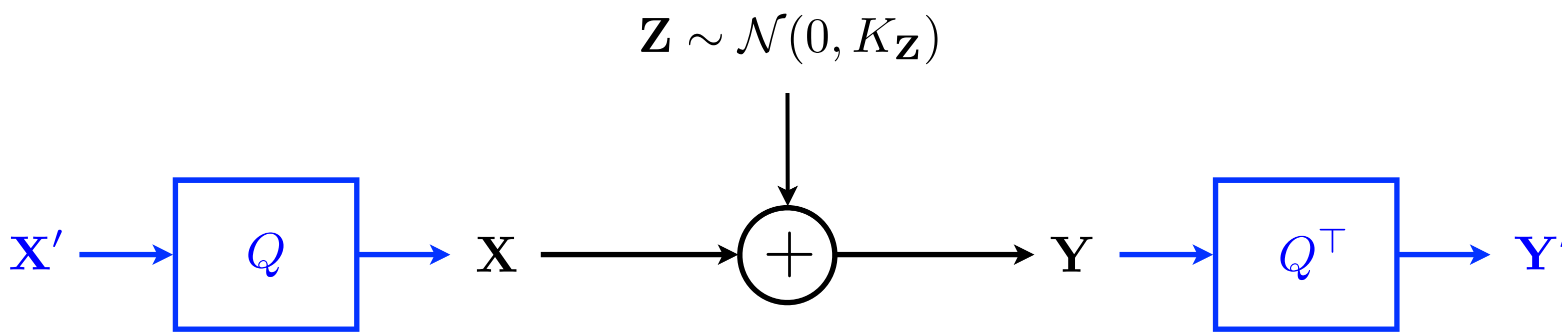
Proposition $I(\mathbf{X}'; \mathbf{Y}') = I(\mathbf{X}; \mathbf{Y})$.

Proof

$$
\begin{aligned}
I(\mathbf{X}'; \mathbf{Y}') & \\
&= h(\mathbf{Y}') - h(\mathbf{Y}' | \mathbf{X}') \\
&= h(\mathbf{Y}') - \underline{h(\mathbf{Z}' | \mathbf{X}')} \\
&= h(\mathbf{Y}') - \underline{h(\mathbf{Z}')}
\end{aligned}
$$

$\mathbf{Z} \perp \mathbf{X}$

Equivalence of Capacity



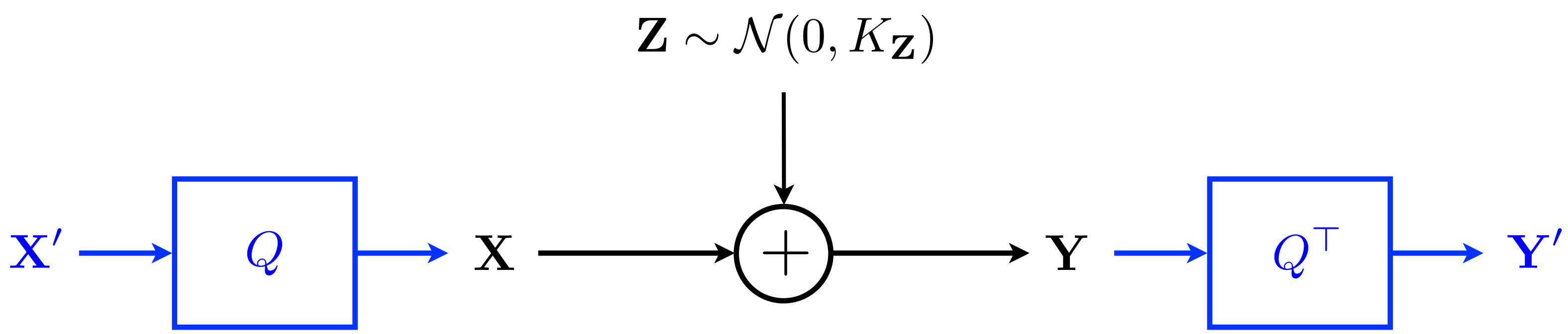
Proposition $I(\mathbf{X}';\mathbf{Y}') = I(\mathbf{X};\mathbf{Y})$.

Proof

$$\begin{aligned} I(\mathbf{X}';\mathbf{Y}') &\\ &= h(\mathbf{Y}') - h(\mathbf{Y}'|\mathbf{X}') \\ &= h(\mathbf{Y}') - \underline{h(\mathbf{Z}'|\mathbf{X}')} \\ &= h(\mathbf{Y}') - \underline{h(\mathbf{Z}')} \end{aligned}$$

$$\begin{array}{ccc} \mathbf{Z} & \perp & \mathbf{X} \\ Q^\top\mathbf{Z} & \perp & Q^\top\mathbf{X} \end{array}$$

Equivalence of Capacity



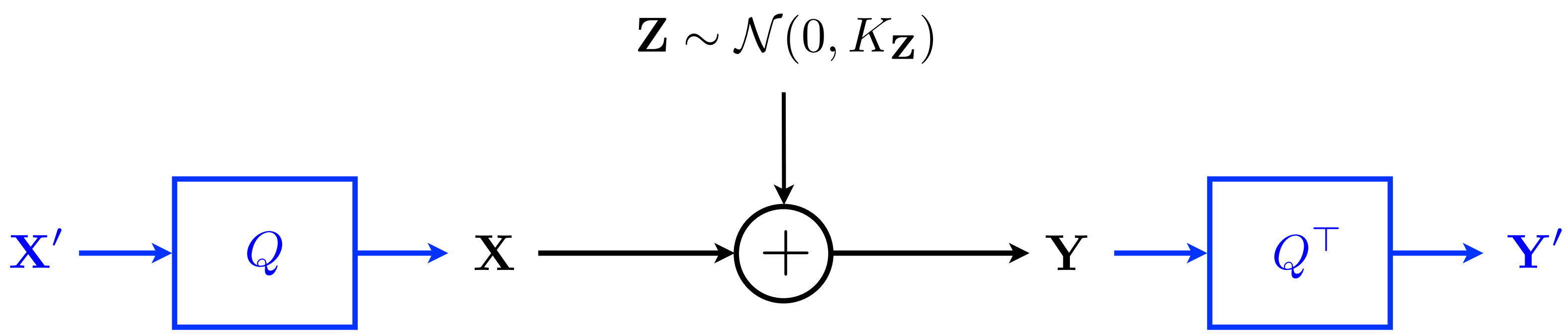
Proposition $I(\mathbf{X}'; \mathbf{Y}') = I(\mathbf{X}; \mathbf{Y})$.

Proof

$$
\begin{aligned}
I(\mathbf{X}'; \mathbf{Y}') \\
&= h(\mathbf{Y}') - h(\mathbf{Y}'|\mathbf{X}') \\
&= h(\mathbf{Y}') - \underline{h(\mathbf{Z}'|\mathbf{X}')} \\
&= h(\mathbf{Y}') - \underline{h(\mathbf{Z}')}
\end{aligned}
$$

$$
\begin{array}{ccc}
\mathbf{Z} & \perp & \mathbf{X} \\
Q^\top \mathbf{Z} & \perp & Q^\top \mathbf{X} \\
\mathbf{Z}' & \perp & \mathbf{X}'
\end{array}
$$

Equivalence of Capacity

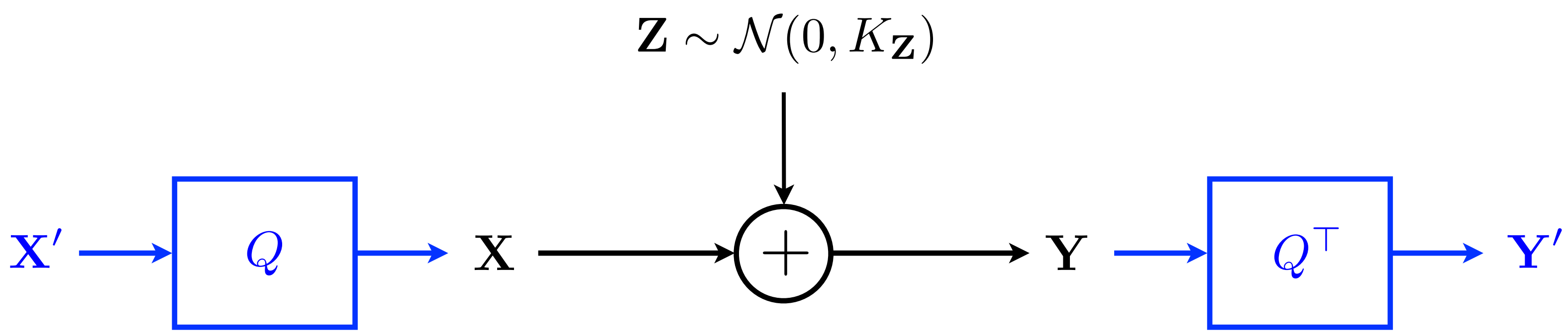


Proposition $I(\mathbf{X}'; \mathbf{Y}') = I(\mathbf{X}; \mathbf{Y})$.

Proof

$$
\begin{aligned}
& I(\mathbf{X}'; \mathbf{Y}') \\
& \quad = \quad h(\mathbf{Y}') - h(\mathbf{Y}'|\mathbf{X}') \\
& \quad = \quad h(\mathbf{Y}') - h(\mathbf{Z}'|\mathbf{X}') \\
& \quad = \quad h(\mathbf{Y}') - h(\mathbf{Z}')
\end{aligned}
$$

Equivalence of Capacity

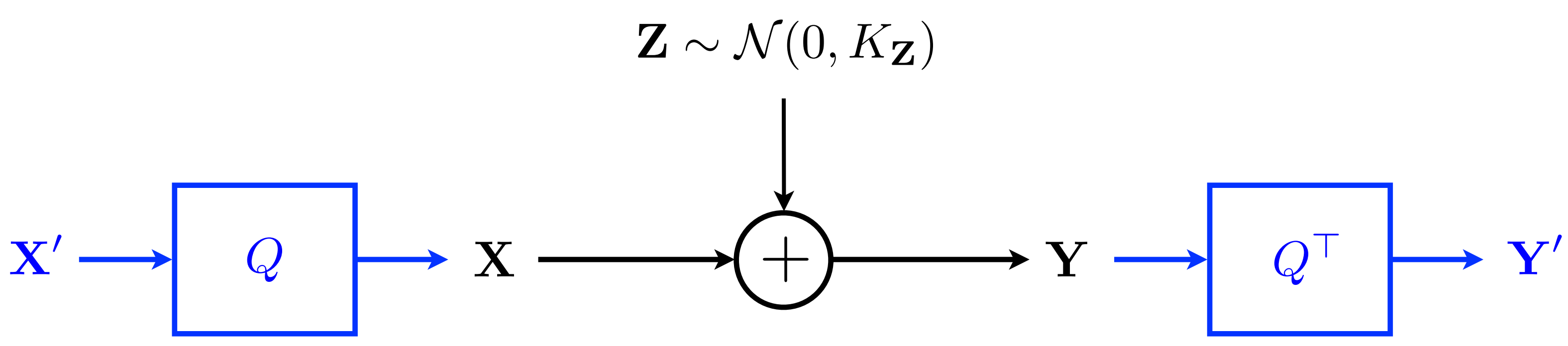


Proposition $I(\mathbf{X}'; \mathbf{Y}') = I(\mathbf{X}; \mathbf{Y})$.

Proof

$$
\begin{aligned}
& I(\mathbf{X}'; \mathbf{Y}') \\
&\quad = \quad h(\mathbf{Y}') - h(\mathbf{Y}'|\mathbf{X}') \\
&\quad = \quad h(\mathbf{Y}') - h(\mathbf{Z}'|\mathbf{X}') \\
&\quad = \quad h(\underline{\mathbf{Y}'}) - h(\mathbf{Z}')
\end{aligned}
$$

Equivalence of Capacity

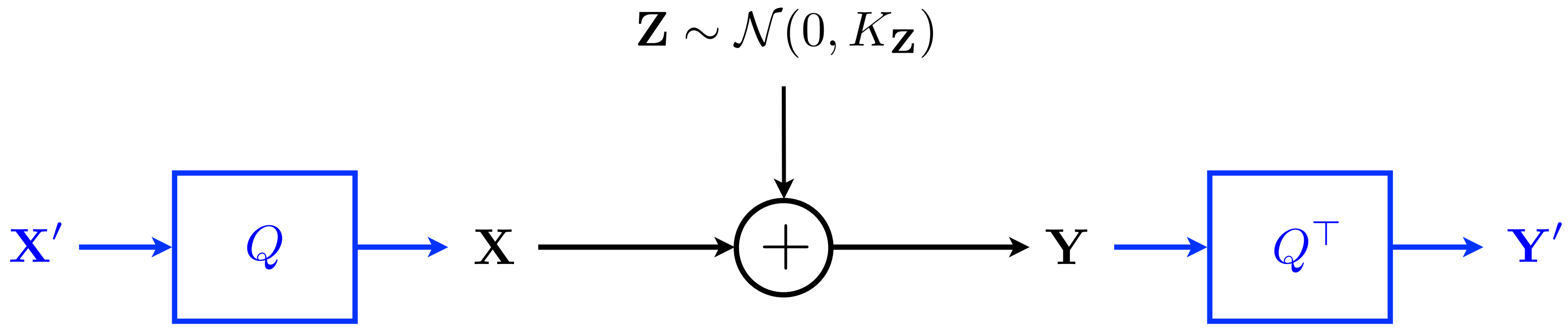


Proposition $I(\mathbf{X}';\mathbf{Y}') = I(\mathbf{X};\mathbf{Y})$.

Proof

$$\begin{aligned} I(\mathbf{X}';\mathbf{Y}') &\\ = \quad & h(\mathbf{Y}') - h(\mathbf{Y}'|\mathbf{X}') \\ = \quad & h(\mathbf{Y}') - h(\mathbf{Z}'|\mathbf{X}') \\ = \quad & h(\underline{\mathbf{Y}'}) - h(\mathbf{Z}') \\ = \quad & h(\underline{Q^\top \mathbf{Y}}) - h(Q^\top \mathbf{Z}) \end{aligned}$$

Equivalence of Capacity

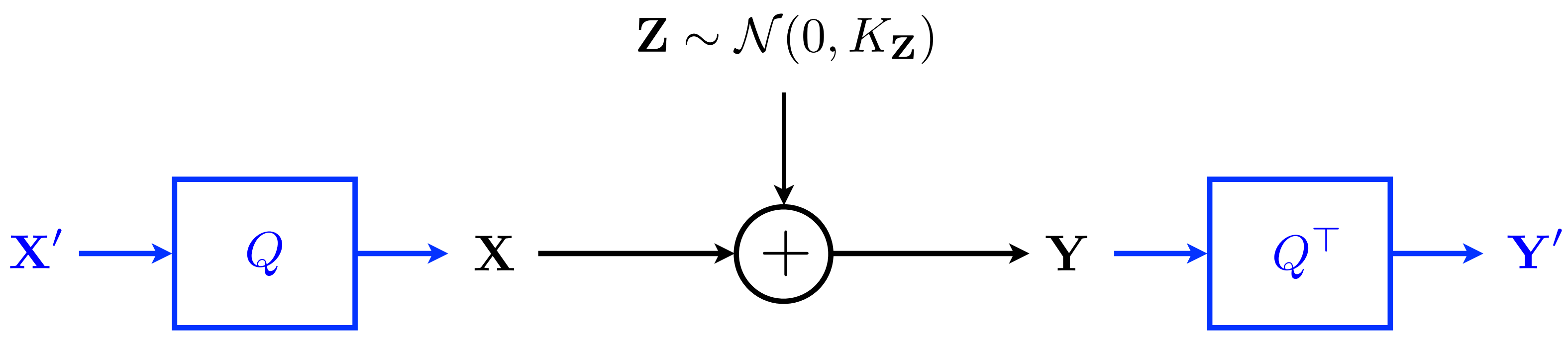


Proposition $I(\mathbf{X}'; \mathbf{Y}') = I(\mathbf{X}; \mathbf{Y})$.

Proof

$$
\begin{aligned}
& I(\mathbf{X}'; \mathbf{Y}') \\
&\quad = \quad h(\mathbf{Y}') - h(\mathbf{Y}'|\mathbf{X}') \\
&\quad = \quad h(\mathbf{Y}') - h(\mathbf{Z}'|\mathbf{X}') \\
&\quad = \quad h(\mathbf{Y}') - h(\underline{\mathbf{Z}'}) \\
&\quad = \quad h(Q^\top \mathbf{Y}) - h(Q^\top \mathbf{Z})
\end{aligned}
$$

Equivalence of Capacity

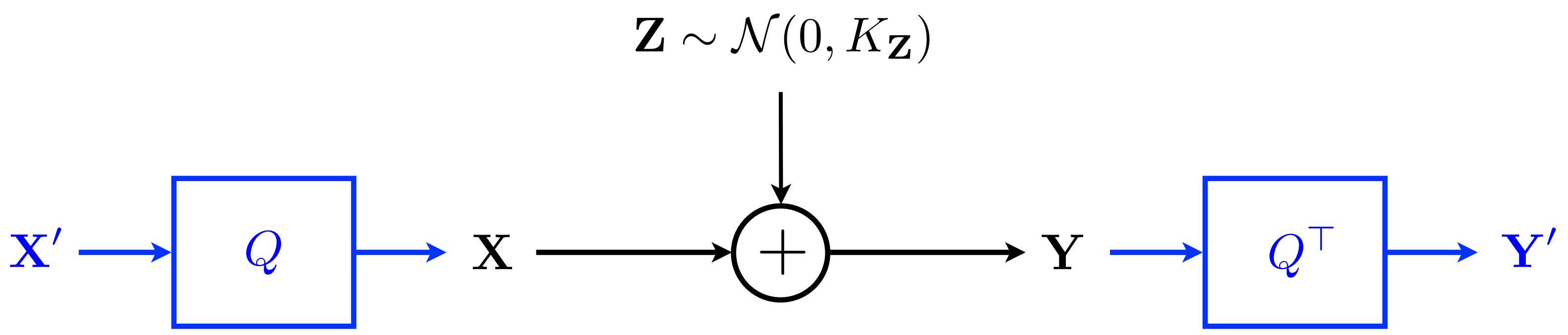


Proposition $I(\mathbf{X}'; \mathbf{Y}') = I(\mathbf{X}; \mathbf{Y})$.

Proof

$$
\begin{aligned}
& I(\mathbf{X}'; \mathbf{Y}') \\
&= h(\mathbf{Y}') - h(\mathbf{Y}'|\mathbf{X}') \\
&= h(\mathbf{Y}') - h(\mathbf{Z}'|\mathbf{X}') \\
&= h(\mathbf{Y}') - h(\underline{\mathbf{Z}'}) \\
&= h(Q^\top \mathbf{Y}) - h(\underline{Q^\top \mathbf{Z}})
\end{aligned}
$$

Equivalence of Capacity

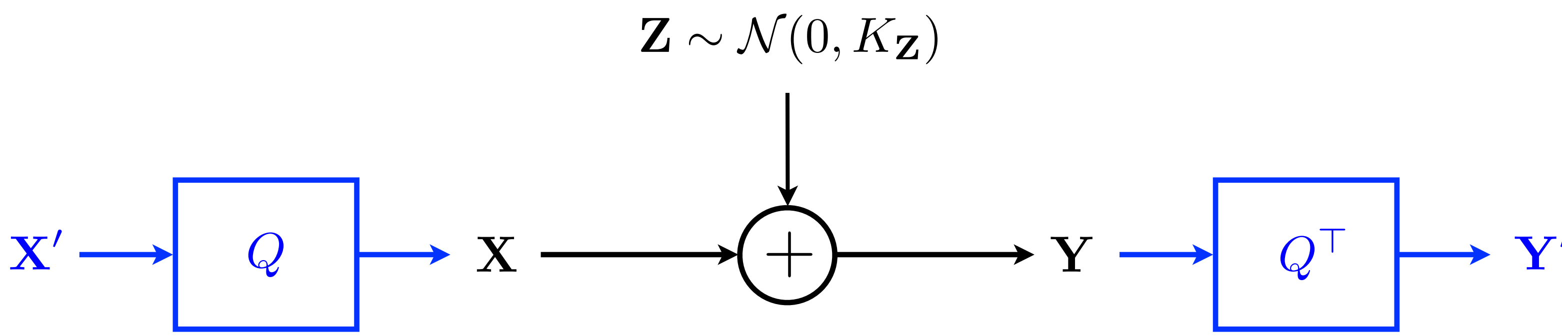


Proposition $I(\mathbf{X}';\mathbf{Y}') = I(\mathbf{X};\mathbf{Y})$.

Proof

$$
\begin{aligned}
& I(\mathbf{X}';\mathbf{Y}') \\
&= h(\mathbf{Y}') - h(\mathbf{Y}'|\mathbf{X}') \\
&= h(\mathbf{Y}') - h(\mathbf{Z}'|\mathbf{X}') \\
&= h(\mathbf{Y}') - h(\mathbf{Z}') \\
&= \underline{h(Q^\top \mathbf{Y})} - h(Q^\top \mathbf{Z})
\end{aligned}
$$

Equivalence of Capacity



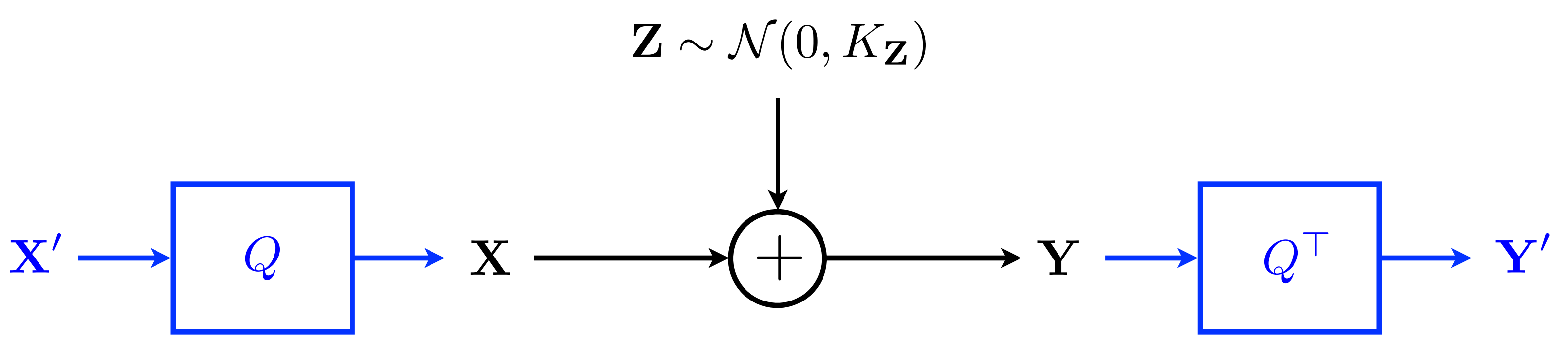
Proposition $I(\mathbf{X}'; \mathbf{Y}') = I(\mathbf{X}; \mathbf{Y})$.

Theorem 10.19 $h(AX) = h(X) + \log \lvert \det(Q^\top) \rvert$.

Proof

$$
\begin{aligned}
& I(\mathbf{X}'; \mathbf{Y}') \\
&= h(\mathbf{Y}') - h(\mathbf{Y}'|\mathbf{X}') \\
&= h(\mathbf{Y}') - h(\mathbf{Z}'|\mathbf{X}') \\
&= h(\mathbf{Y}') - h(\mathbf{Z}') \\
&= \underline{h(Q^\top \mathbf{Y})} - h(Q^\top \mathbf{Z}) \\
&= \left[\underline{h(\mathbf{Y}) + \log \lvert \det(Q^\top) \rvert} \right] \\
&\quad - \left[h(\mathbf{Z}) + \log \lvert \det(Q^\top) \rvert \right]
\end{aligned}
$$

Equivalence of Capacity



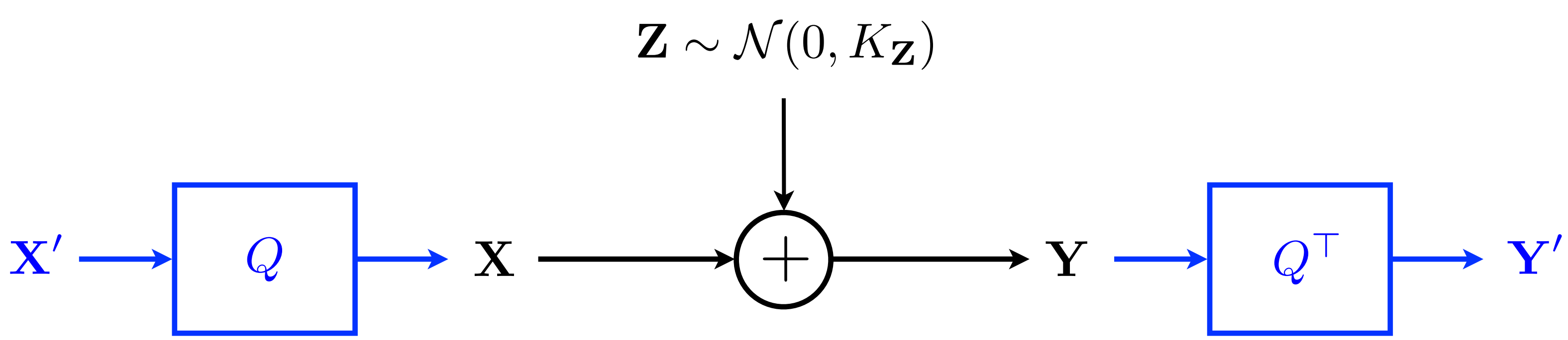
Proposition $I(\mathbf{X}'; \mathbf{Y}') = I(\mathbf{X}; \mathbf{Y})$.

Theorem 10.19 $h(AX) = h(X) + \log|\det(Q^{\top})|$.

Proof

$$
\begin{aligned}
& I(\mathbf{X}'; \mathbf{Y}') \\
&= h(\mathbf{Y}') - h(\mathbf{Y}'|\mathbf{X}') \\
&= h(\mathbf{Y}') - h(\mathbf{Z}'|\mathbf{X}') \\
&= h(\mathbf{Y}') - h(\mathbf{Z}') \\
&= h(Q^{\top}\mathbf{Y}) - \underline{h(Q^{\top}\mathbf{Z})} \\
&= \left[h(\mathbf{Y}) + \log|\det(Q^{\top})| \right] \\
&\quad - \left[h(\mathbf{Z}) + \log|\det(Q^{\top})| \right]
\end{aligned}
$$

Equivalence of Capacity



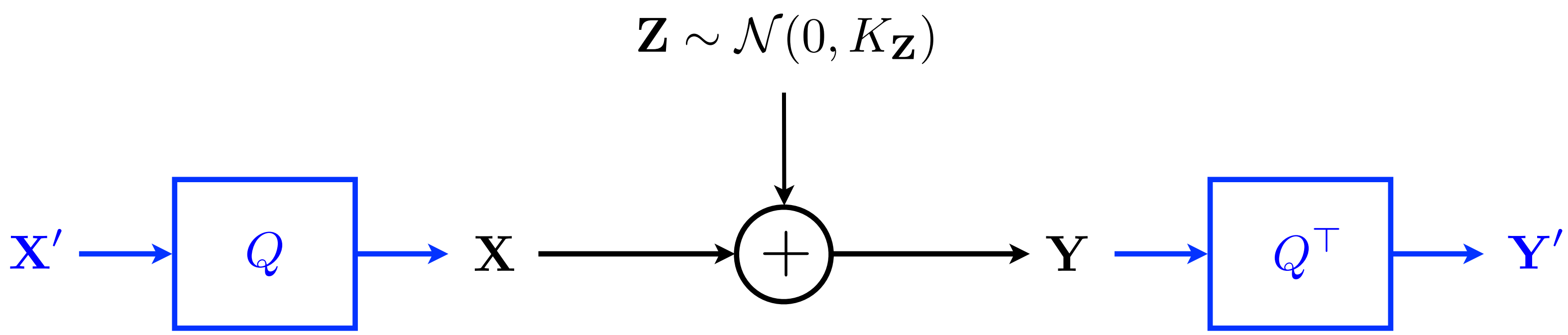
Proposition $I(\mathbf{X}';\mathbf{Y}') = I(\mathbf{X};\mathbf{Y})$.

Theorem 10.19 $h(AX) = h(X) + \log|\det(Q^\top)|$.

Proof

$$
\begin{aligned}
& I(\mathbf{X}';\mathbf{Y}') \\
&= h(\mathbf{Y}') - h(\mathbf{Y}'|\mathbf{X}') \\
&= h(\mathbf{Y}') - h(\mathbf{Z}'|\mathbf{X}') \\
&= h(\mathbf{Y}') - h(\mathbf{Z}') \\
&= h(Q^\top\mathbf{Y}) - \underline{h(Q^\top\mathbf{Z})} \\
&= \left[h(\mathbf{Y}) + \log|\det(Q^\top)| \right] \\
&\quad - \left[\underline{h(\mathbf{Z}) + \log|\det(Q^\top)|} \right]
\end{aligned}
$$

Equivalence of Capacity



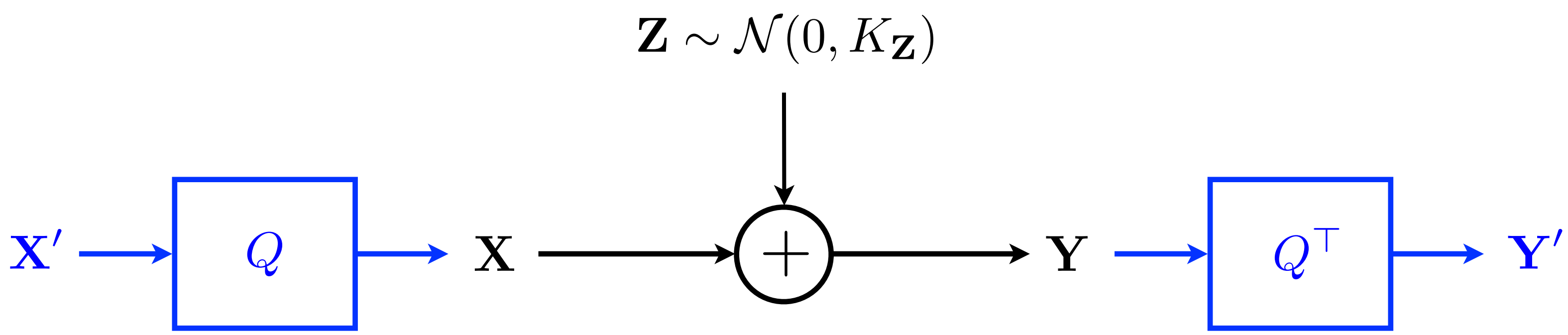
Proposition $I(\mathbf{X}'; \mathbf{Y}') = I(\mathbf{X}; \mathbf{Y})$.

Theorem 10.19 $h(AX) = h(X) + \log|\det(Q^\top)|$.

Proof

$$
\begin{aligned}
&I(\mathbf{X}'; \mathbf{Y}') \\
&\quad = \quad h(\mathbf{Y}') - h(\mathbf{Y}'|\mathbf{X}') \\
&\quad = \quad h(\mathbf{Y}') - h(\mathbf{Z}'|\mathbf{X}') \\
&\quad = \quad h(\mathbf{Y}') - h(\mathbf{Z}') \\
&\quad = \quad h(Q^\top \mathbf{Y}) - h(Q^\top \mathbf{Z}) \\
&\quad = \quad \left[h(\mathbf{Y}) + \log|\det(Q^\top)| \right] \\
&\qquad\qquad - \left[h(\mathbf{Z}) + \log|\det(Q^\top)| \right]
\end{aligned}
$$

Equivalence of Capacity



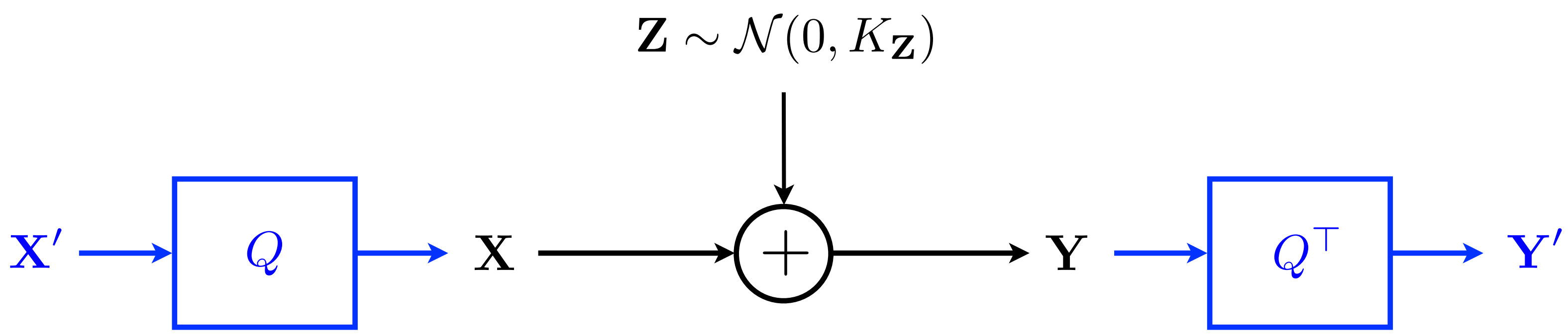
Proposition $I(\mathbf{X}'; \mathbf{Y}') = I(\mathbf{X}; \mathbf{Y})$.

Theorem 10.19 $h(AX) = h(X) + \log |\det(Q^\top)|$.

Proof

$$
\begin{aligned}
& I(\mathbf{X}'; \mathbf{Y}') \\
&= h(\mathbf{Y}') - h(\mathbf{Y}'|\mathbf{X}') \\
&= h(\mathbf{Y}') - h(\mathbf{Z}'|\mathbf{X}') \\
&= h(\mathbf{Y}') - h(\mathbf{Z}') \\
&= h(Q^\top \mathbf{Y}) - h(Q^\top \mathbf{Z}) \\
&= \left[h(\mathbf{Y}) + \log |\det(Q^\top)| \right] \\
&\quad - \left[h(\mathbf{Z}) + \log |\det(Q^\top)| \right]
\end{aligned}
$$

Equivalence of Capacity

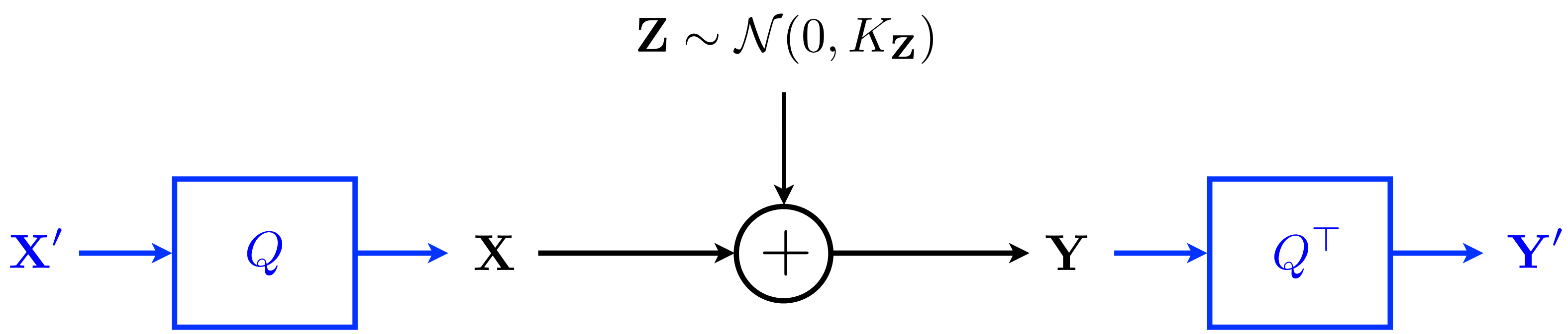


Proposition $I(\mathbf{X}'; \mathbf{Y}') = I(\mathbf{X}; \mathbf{Y})$.

Proof

$$
\begin{aligned}
I(\mathbf{X}'; \mathbf{Y}') & \\
&= h(\mathbf{Y}') - h(\mathbf{Y}'|\mathbf{X}') \\
&= h(\mathbf{Y}') - h(\mathbf{Z}'|\mathbf{X}') \\
&= h(\mathbf{Y}') - h(\mathbf{Z}') \\
&= h(Q^\top \mathbf{Y}) - h(Q^\top \mathbf{Z}) \\
&= \left[h(\mathbf{Y}) + \cancel{\log |\det(Q^\top)|} \right] \\
&\quad - \left[h(\mathbf{Z}) + \cancel{\log |\det(Q^\top)|} \right] \\
&= h(\mathbf{Y}) - h(\mathbf{Z})
\end{aligned}
$$

Equivalence of Capacity

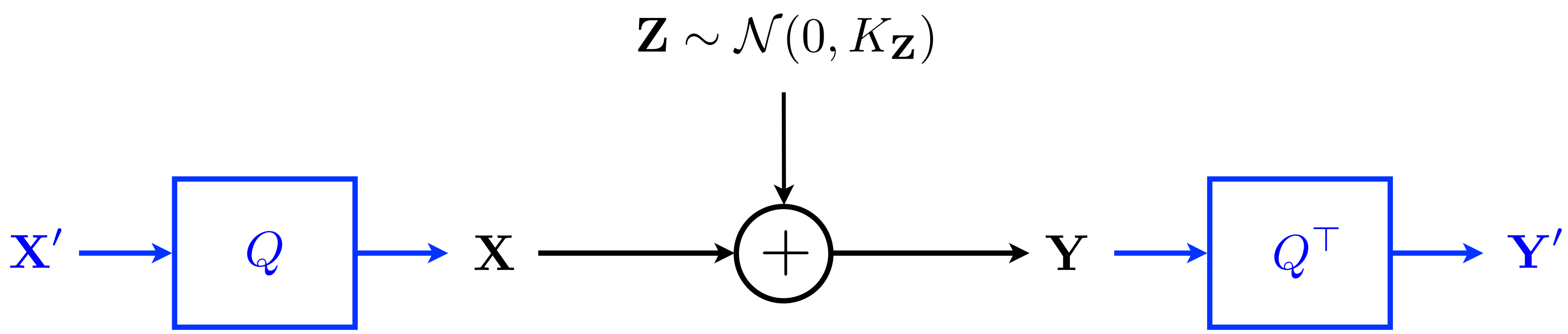


Proposition $I(\mathbf{X}'; \mathbf{Y}') = I(\mathbf{X}; \mathbf{Y})$.

Proof

$$
\begin{aligned}
& I(\mathbf{X}'; \mathbf{Y}') \\
&= h(\mathbf{Y}') - h(\mathbf{Y}'|\mathbf{X}') \\
&= h(\mathbf{Y}') - h(\mathbf{Z}'|\mathbf{X}') \\
&= h(\mathbf{Y}') - h(\mathbf{Z}') \\
&= h(Q^\top \mathbf{Y}) - h(Q^\top \mathbf{Z}) \\
&= \left[h(\mathbf{Y}) + \log |\det(Q^\top)| \right] \\
&\quad - \left[h(\mathbf{Z}) + \log |\det(Q^\top)| \right] \\
&= h(\mathbf{Y}) - h(\mathbf{Z}) \\
&= h(\mathbf{Y}) - h(\mathbf{Z}|\mathbf{X})
\end{aligned}
$$

Equivalence of Capacity

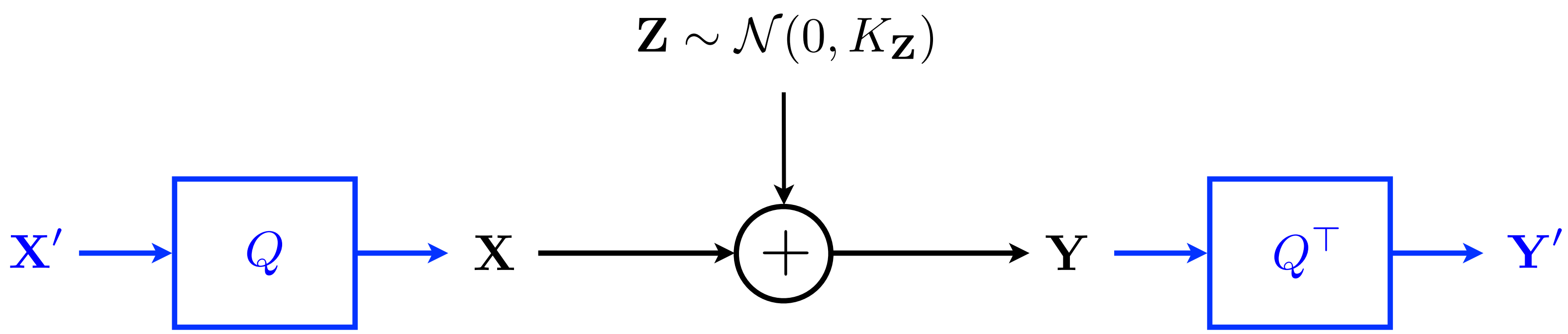


Proposition $I(\mathbf{X}'; \mathbf{Y}') = I(\mathbf{X}; \mathbf{Y})$.

Proof

$$
\begin{aligned}
& I(\mathbf{X}'; \mathbf{Y}') \\
&= h(\mathbf{Y}') - h(\mathbf{Y}'|\mathbf{X}') \\
&= h(\mathbf{Y}') - h(\mathbf{Z}'|\mathbf{X}') \\
&= h(\mathbf{Y}') - h(\mathbf{Z}') \\
&= h(Q^\top \mathbf{Y}) - h(Q^\top \mathbf{Z}) \\
&= \left[h(\mathbf{Y}) + \log|\det(Q^\top)| \right] \\
&\quad - \left[h(\mathbf{Z}) + \log|\det(Q^\top)| \right] \\
&= h(\mathbf{Y}) - h(\mathbf{Z}) \\
&= h(\mathbf{Y}) - h(\mathbf{Z}|\mathbf{X}) \\
&= h(\mathbf{Y}) - h(\mathbf{Y}|\mathbf{X})
\end{aligned}
$$

Equivalence of Capacity

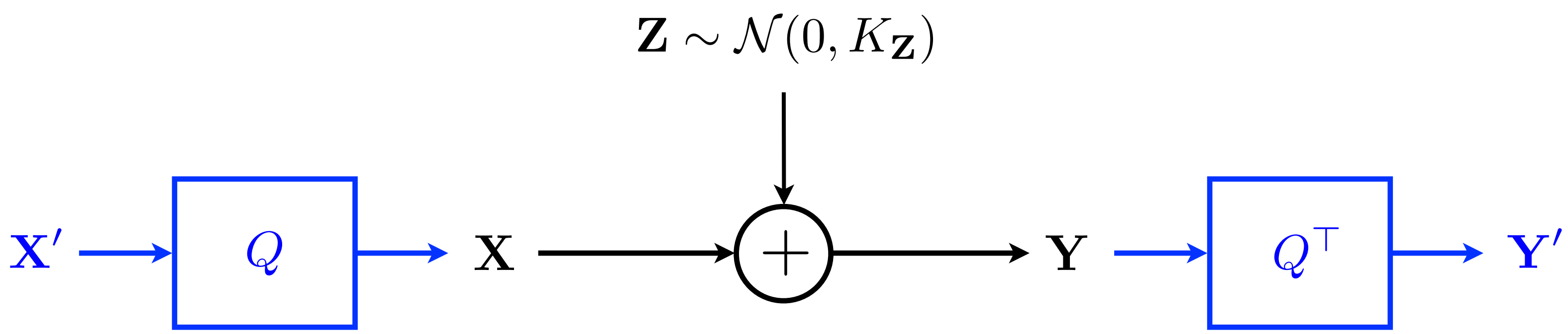


Proposition $I(\mathbf{X}'; \mathbf{Y}') = I(\mathbf{X}; \mathbf{Y})$.

Proof

$$
\begin{aligned}
& I(\mathbf{X}'; \mathbf{Y}') \\
&= h(\mathbf{Y}') - h(\mathbf{Y}'|\mathbf{X}') \\
&= h(\mathbf{Y}') - h(\mathbf{Z}'|\mathbf{X}') \\
&= h(\mathbf{Y}') - h(\mathbf{Z}') \\
&= h(Q^\top \mathbf{Y}) - h(Q^\top \mathbf{Z}) \\
&= \left[h(\mathbf{Y}) + \log|\det(Q^\top)| \right] \\
&\quad - \left[h(\mathbf{Z}) + \log|\det(Q^\top)| \right] \\
&= h(\mathbf{Y}) - h(\mathbf{Z}) \\
&= h(\mathbf{Y}) - h(\mathbf{Z}|\mathbf{X}) \\
&= h(\mathbf{Y}) - h(\mathbf{Y}|\mathbf{X}) \\
&= I(\mathbf{X}; \mathbf{Y}).
\end{aligned}
$$

Equivalence of Capacity

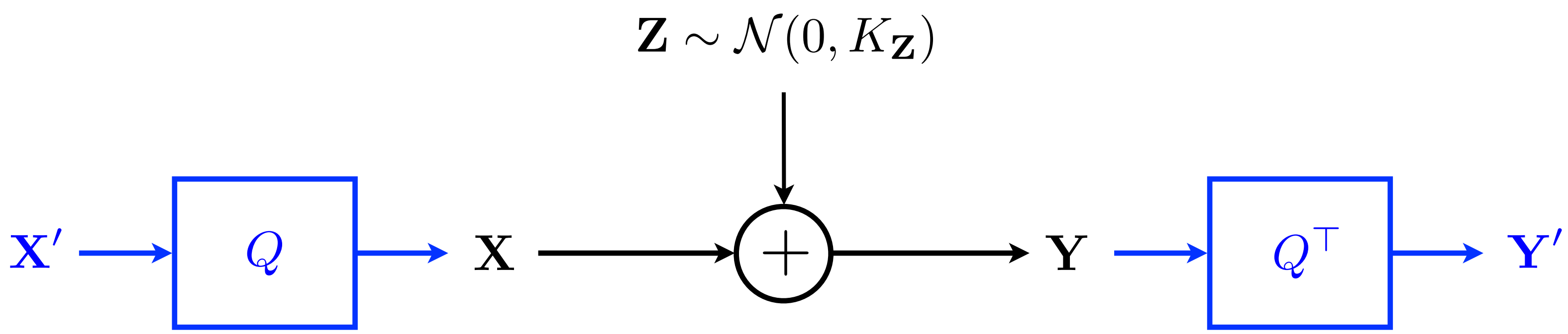


Proposition $I(\mathbf{X}'; \mathbf{Y}') = I(\mathbf{X}; \mathbf{Y})$.

Proof

$$
\begin{aligned}
& I(\mathbf{X}'; \mathbf{Y}') \\
&= h(\mathbf{Y}') - h(\mathbf{Y}'|\mathbf{X}') \\
&= h(\mathbf{Y}') - h(\mathbf{Z}'|\mathbf{X}') \\
&= h(\mathbf{Y}') - h(\mathbf{Z}') \\
&= h(Q^\top \mathbf{Y}) - h(Q^\top \mathbf{Z}) \\
&= \left[h(\mathbf{Y}) + \log |\det(Q^\top)| \right] \\
&\quad - \left[h(\mathbf{Z}) + \log |\det(Q^\top)| \right] \\
&= h(\mathbf{Y}) - h(\mathbf{Z}) \\
&= h(\mathbf{Y}) - h(\mathbf{Z}|\mathbf{X}) \\
&= h(\mathbf{Y}) - h(\mathbf{Y}|\mathbf{X}) \\
&= I(\mathbf{X}; \mathbf{Y}).
\end{aligned}
$$

Equivalence of Capacity



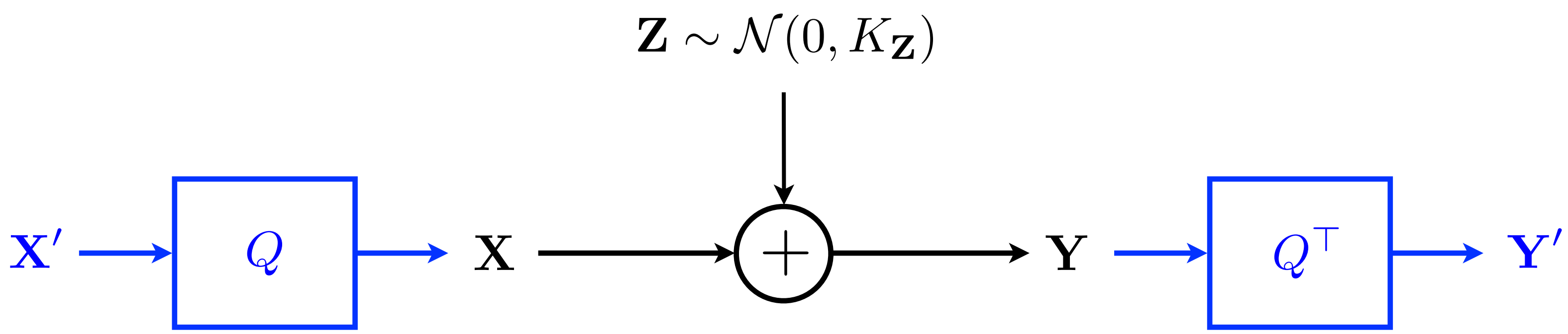
Proposition $I(\mathbf{X}'; \mathbf{Y}') = I(\mathbf{X}; \mathbf{Y})$.

Proof

$$
\begin{aligned}
& I(\mathbf{X}'; \mathbf{Y}') \\
= \quad & h(\mathbf{Y}') - h(\mathbf{Y}'|\mathbf{X}') \\
= \quad & h(\mathbf{Y}') - h(\mathbf{Z}'|\mathbf{X}') \\
= \quad & h(\mathbf{Y}') - h(\mathbf{Z}') \\
= \quad & h(Q^\top \mathbf{Y}) - h(Q^\top \mathbf{Z}) \\
= \quad & \left[h(\mathbf{Y}) + \log |\det(Q^\top)| \right] \\
& - \left[h(\mathbf{Z}) + \log |\det(Q^\top)| \right] \\
= \quad & h(\mathbf{Y}) - h(\mathbf{Z}) \\
= \quad & h(\mathbf{Y}) - h(\mathbf{Z}|\mathbf{X}) \\
= \quad & h(\mathbf{Y}) - h(\mathbf{Y}|\mathbf{X}) \\
= \quad & I(\mathbf{X}; \mathbf{Y}).
\end{aligned}
$$

- Therefore, the equivalent system and the original system have the same capacity.

Equivalence of Capacity



Proposition $I(\mathbf{X}'; \mathbf{Y}') = I(\mathbf{X}; \mathbf{Y})$.

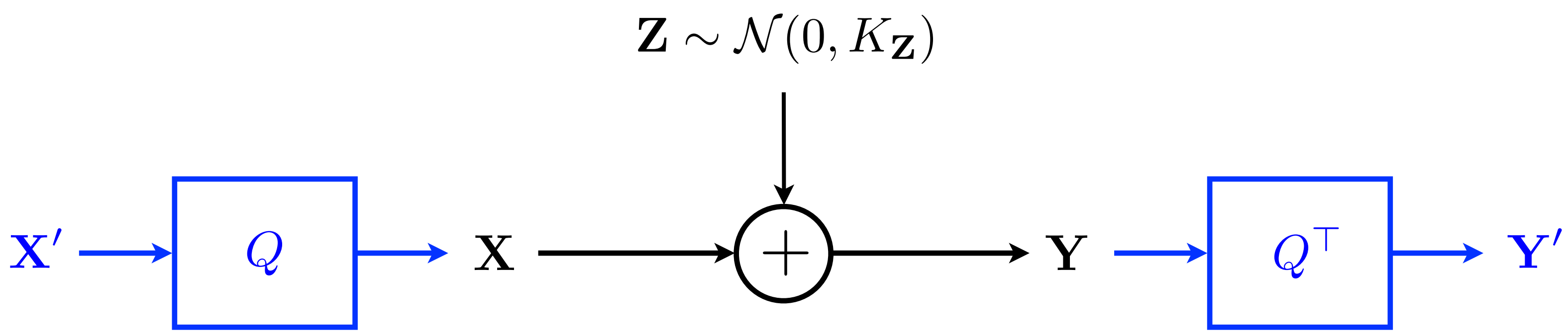
Proof

$$
\begin{aligned}
& I(\mathbf{X}'; \mathbf{Y}') \\
&\quad = \quad h(\mathbf{Y}') - h(\mathbf{Y}'|\mathbf{X}') \\
&\quad = \quad h(\mathbf{Y}') - h(\mathbf{Z}'|\mathbf{X}') \\
&\quad = \quad h(\mathbf{Y}') - h(\mathbf{Z}') \\
&\quad = \quad h(Q^\top \mathbf{Y}) - h(Q^\top \mathbf{Z}) \\
&\quad = \quad \Big[h(\mathbf{Y}) + \log|\det(Q^\top)| \Big] \\
&\qquad\qquad - \Big[h(\mathbf{Z}) + \log|\det(Q^\top)| \Big] \\
&\quad = \quad h(\mathbf{Y}) - h(\mathbf{Z}) \\
&\quad = \quad h(\mathbf{Y}) - h(\mathbf{Z}|\mathbf{X}) \\
&\quad = \quad h(\mathbf{Y}) - h(\mathbf{Y}|\mathbf{X}) \\
&\quad = \quad I(\mathbf{X}; \mathbf{Y}).
\end{aligned}
$$

- Therefore, the equivalent system and the original system have the same capacity.

- Hence, the capacity of a system of correlated Gaussian channels is given by

Equivalence of Capacity



Proposition $I(\mathbf{X}'; \mathbf{Y}') = I(\mathbf{X}; \mathbf{Y})$.

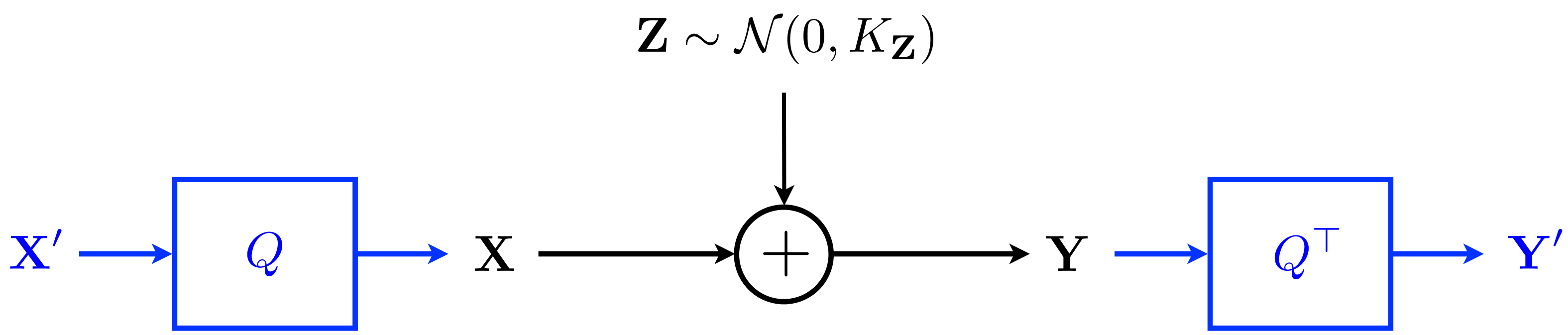
Proof

$$
\begin{aligned}
& I(\mathbf{X}'; \mathbf{Y}') \\
&= h(\mathbf{Y}') - h(\mathbf{Y}'|\mathbf{X}') \\
&= h(\mathbf{Y}') - h(\mathbf{Z}'|\mathbf{X}') \\
&= h(\mathbf{Y}') - h(\mathbf{Z}') \\
&= h(Q^\top \mathbf{Y}) - h(Q^\top \mathbf{Z}) \\
&= \left[h(\mathbf{Y}) + \log |\det(Q^\top)| \right] \\
&\quad - \left[h(\mathbf{Z}) + \log |\det(Q^\top)| \right] \\
&= h(\mathbf{Y}) - h(\mathbf{Z}) \\
&= h(\mathbf{Y}) - h(\mathbf{Z}|\mathbf{X}) \\
&= h(\mathbf{Y}) - h(\mathbf{Y}|\mathbf{X}) \\
&= I(\mathbf{X}; \mathbf{Y}).
\end{aligned}
$$

- Therefore, the equivalent system and the original system have the same capacity.
- Hence, the capacity of a system of correlated Gaussian channels is given by

$$
\frac{1}{2} \sum_{i=1}^{k} \log \left(1 + \frac{a_i^*}{\lambda_i} \right),
$$

Equivalence of Capacity



Proposition $I(\mathbf{X}'; \mathbf{Y}') = I(\mathbf{X}; \mathbf{Y})$.

Proof

$$
\begin{aligned}
& I(\mathbf{X}'; \mathbf{Y}') \\
&= h(\mathbf{Y}') - h(\mathbf{Y}'|\mathbf{X}') \\
&= h(\mathbf{Y}') - h(\mathbf{Z}'|\mathbf{X}') \\
&= h(\mathbf{Y}') - h(\mathbf{Z}') \\
&= h(Q^\top \mathbf{Y}) - h(Q^\top \mathbf{Z}) \\
&= \left[h(\mathbf{Y}) + \log |\det(Q^\top)| \right] \\
&\quad - \left[h(\mathbf{Z}) + \log |\det(Q^\top)| \right] \\
&= h(\mathbf{Y}) - h(\mathbf{Z}) \\
&= h(\mathbf{Y}) - h(\mathbf{Z}|\mathbf{X}) \\
&= h(\mathbf{Y}) - h(\mathbf{Y}|\mathbf{X}) \\
&= I(\mathbf{X}; \mathbf{Y}).
\end{aligned}
$$

- Therefore, the equivalent system and the original system have the same capacity.
- Hence, the capacity of a system of correlated Gaussian channels is given by

$$
\frac{1}{2} \sum_{i=1}^{k} \log \left(1 + \frac{a_i^*}{\lambda_i} \right),
$$

where a_i^* is the optimal power allocated to the ith channel in the equivalent system, and its value can be obtained by water-filling.