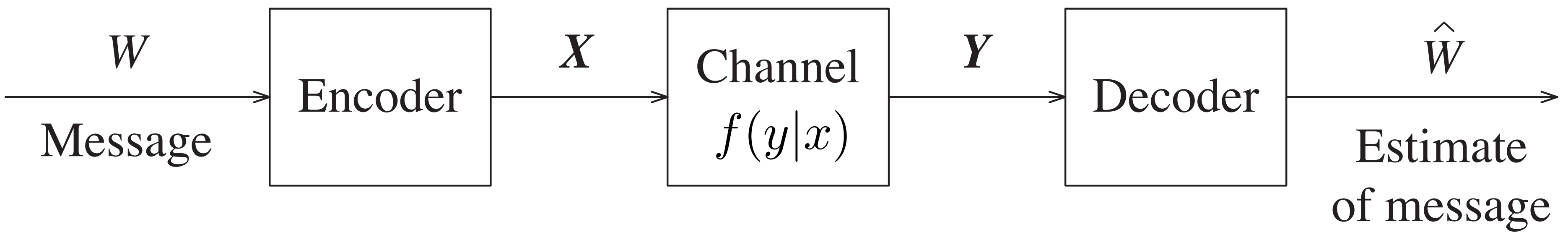
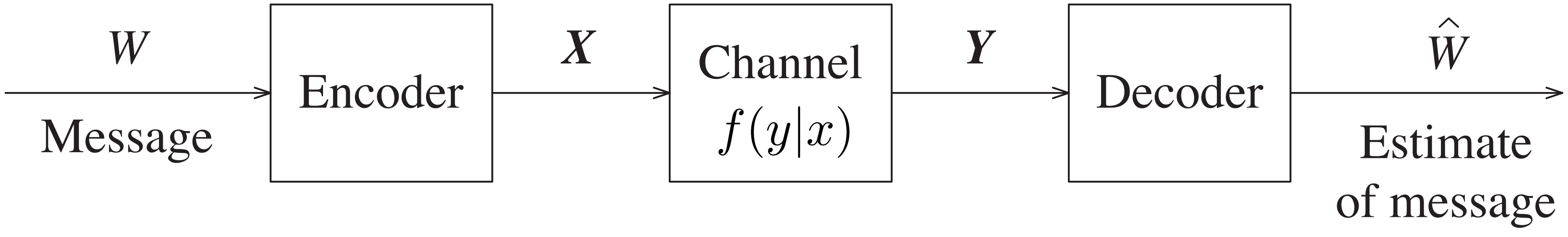
11.3.1 The Converse


W
Message
Encoder
$\boldsymbol{X}$
Channel
$f(y|x)$
$\boldsymbol{Y}$
Decoder
$\widehat{W}$
Estimate
of message

Lemma 11.15 (Data Processing Theorem) $I(W;\hat{W}) \leq I(\mathbf{X};\mathbf{Y})$.



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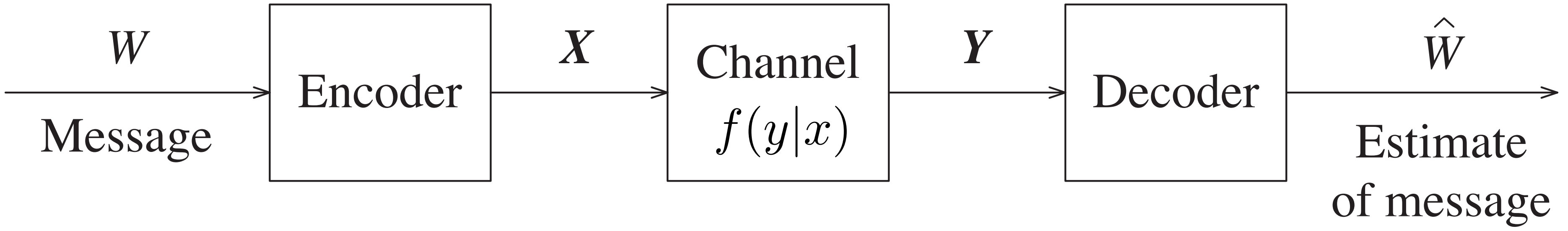
Sketch of Proof



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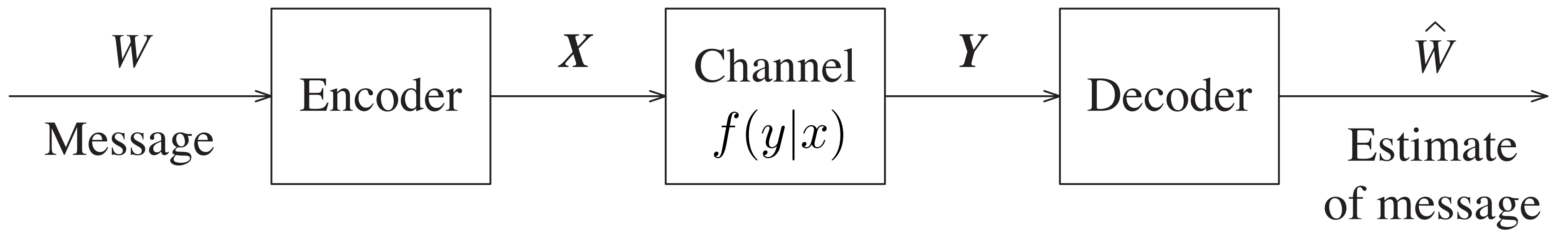
- $W \to \mathbf{X} \to \mathbf{Y} \to \hat{W}$ can be established like the discrete case.



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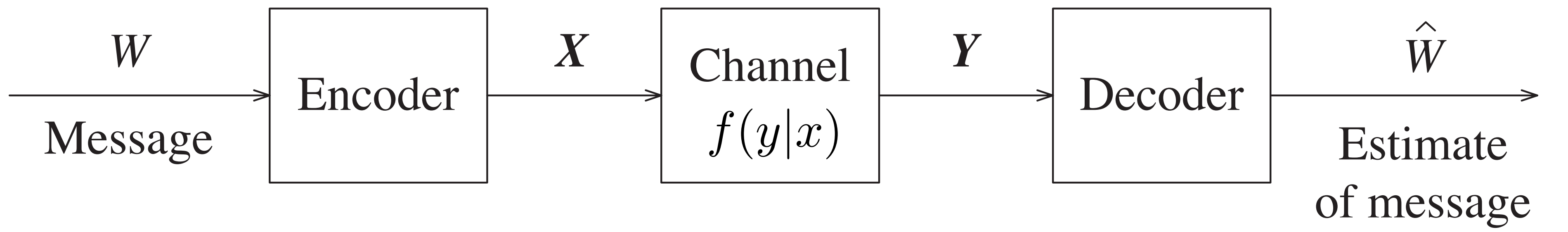

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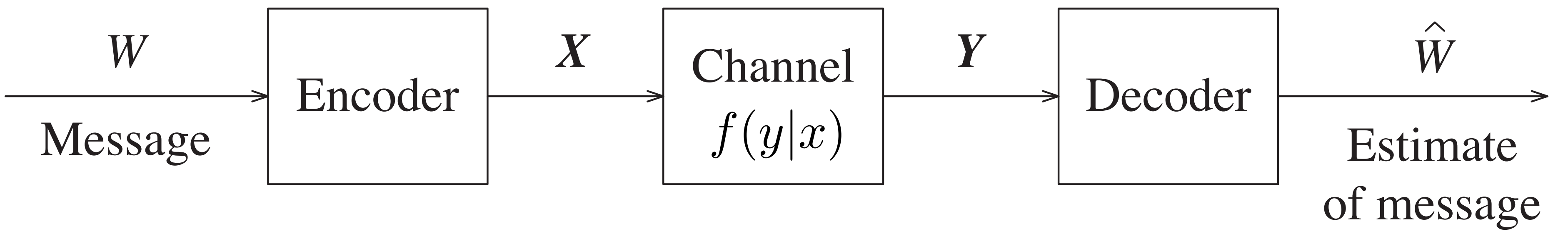
Sketch of Proof

- $W \to \mathbf{X} \to \mathbf{Y} \to \hat{W}$ can be established like the discrete case.
- W, $\hat{W}$ – discrete
- $\mathbf{X}$ – real but discrete



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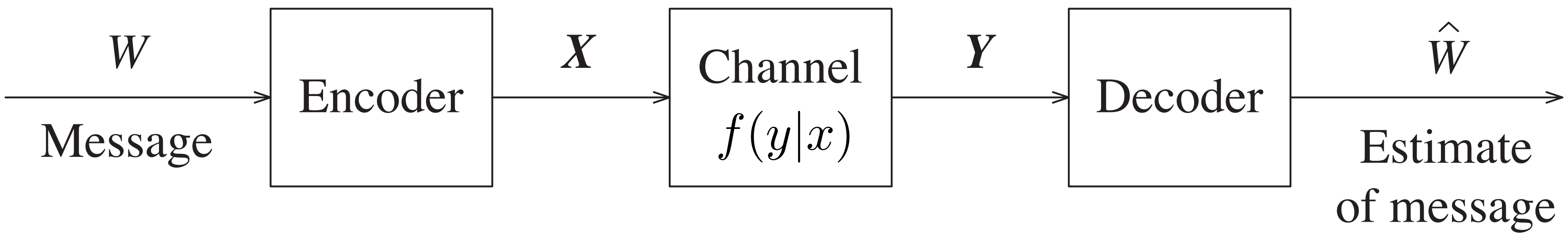
- $\mathbf{X}$ – real but discrete
- $\mathbf{Y}$ – real and continuous



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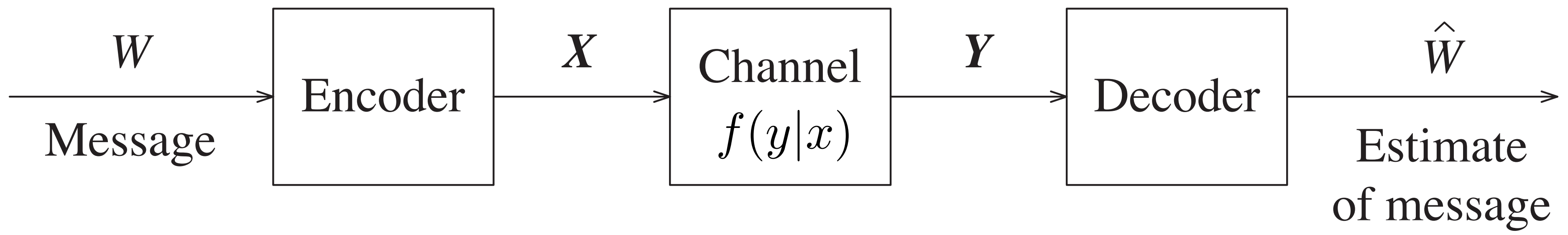
- $\mathbf{Y}$ needs to be handled with caution because it is continuous.



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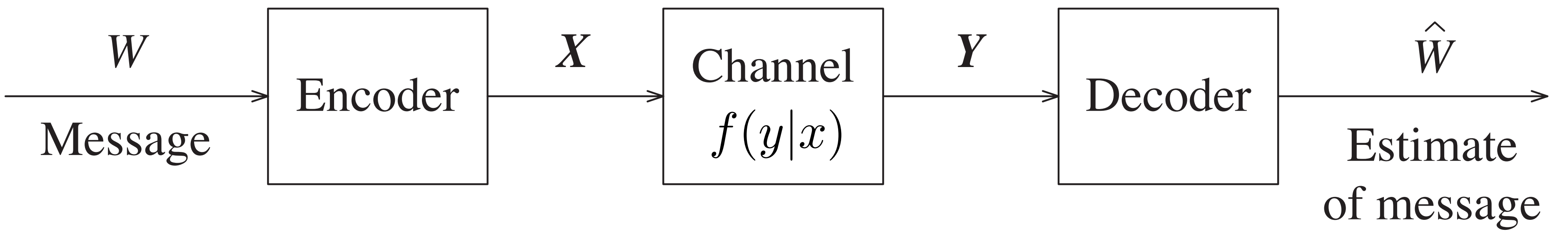
- In particular, the existence of the conditional pdf $f(\mathbf{y}|\hat{w})$ needs to be established so that $I(\mathbf{X};\mathbf{Y}|\hat{W})$ can be defined.



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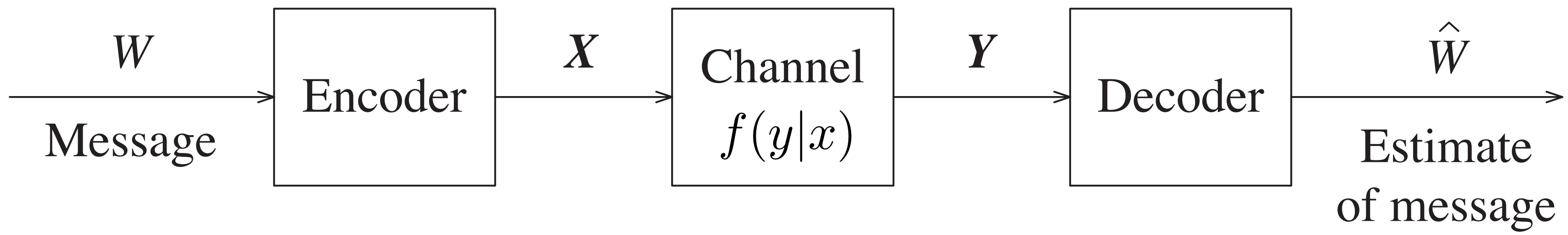
- In particular, the existence of the conditional pdf $f(\mathbf{y}|\hat{w})$ needs to be established so that $I(\mathbf{X};\mathbf{Y}|\hat{W})$ can be defined.
- Refer to the textbook for the technical details.



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1. Let R be an achievable rate, i.e., for any $\epsilon > 0$, there exists for sufficiently large n and (n, M) code such that

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$$\log M = H(W)$$

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3. Let V be a mixing random variable distributed uniformly on $\{1, 2, \cdots, n\}$ which is independent of X_i, $1 \leq i \leq n$.

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&= H(W|\hat{W}) + \sum_{i=1}^{n} I(X_i;Y_i).
\end{aligned}$$

3. Let V be a mixing random variable distributed uniformly on $\{1, 2, \cdots, n\}$ which is independent of X_i, $1 \leq i \leq n$.

4. Let $X = X_V$ and Y be the output of the channel with X being the input.

Proof of Converse

1. Let R be an achievable rate, i.e., for any $\epsilon > 0$, there exists for sufficiently large n and (n, M) code such that

$$\frac{1}{n}\log M > R - \epsilon \quad \text{and} \quad \lambda_{max} < \epsilon.$$

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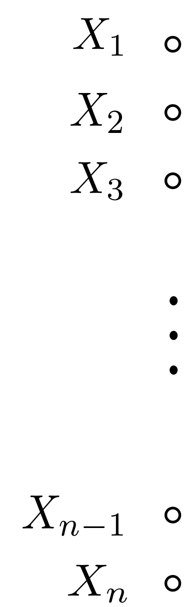

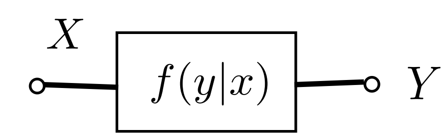

3. Let V be a mixing random variable distributed uniformly on $\{1, 2, \cdots, n\}$ which is independent of X_i, $1 \le i \le n$.

4. Let $X = X_V$ and Y be the output of the channel with X being the input.

Proof of Converse

1. Let R be an achievable rate, i.e., for any $\epsilon > 0$, there exists for sufficiently large n and (n, M) code such that

$$\frac{1}{n}\log M > R - \epsilon \quad \text{and} \quad \lambda_{max} < \epsilon.$$

2. Consider

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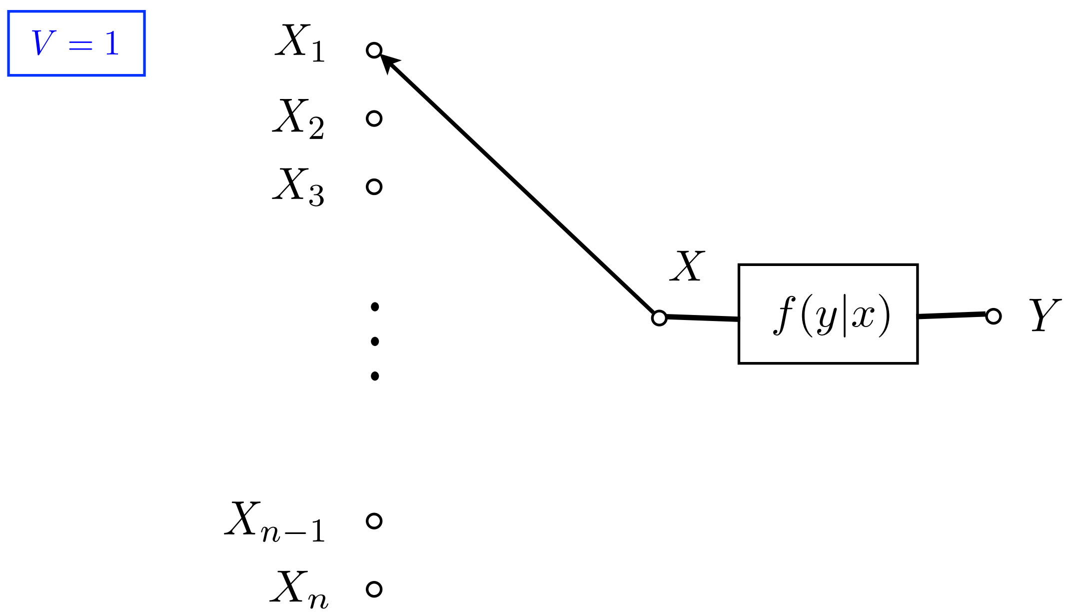
&= H(W|\hat{W}) + h(\mathbf{Y}) - h(\mathbf{Y}|\mathbf{X}) \\
&\leq H(W|\hat{W}) + \sum_{i=1}^{n} h(Y_i) - h(\mathbf{Y}|\mathbf{X}) \\
&= H(W|\hat{W}) + \sum_{i=1}^{n} h(Y_i) - \sum_{i=1}^{n} h(Y_i|X_i) \\
&= H(W|\hat{W}) + \sum_{i=1}^{n} I(X_i;Y_i).
\end{aligned}$$

3. Let V be a mixing random variable distributed uniformly on $\{1, 2, \cdots, n\}$ which is independent of X_i, $1 \leq i \leq n$.

4. Let $X = X_V$ and Y be the output of the channel with X being the input.



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&= H(W|\hat{W}) + \sum_{i=1}^{n} h(Y_i) - \sum_{i=1}^{n} h(Y_i|X_i) \\
&= H(W|\hat{W}) + \sum_{i=1}^{n} I(X_i;Y_i).
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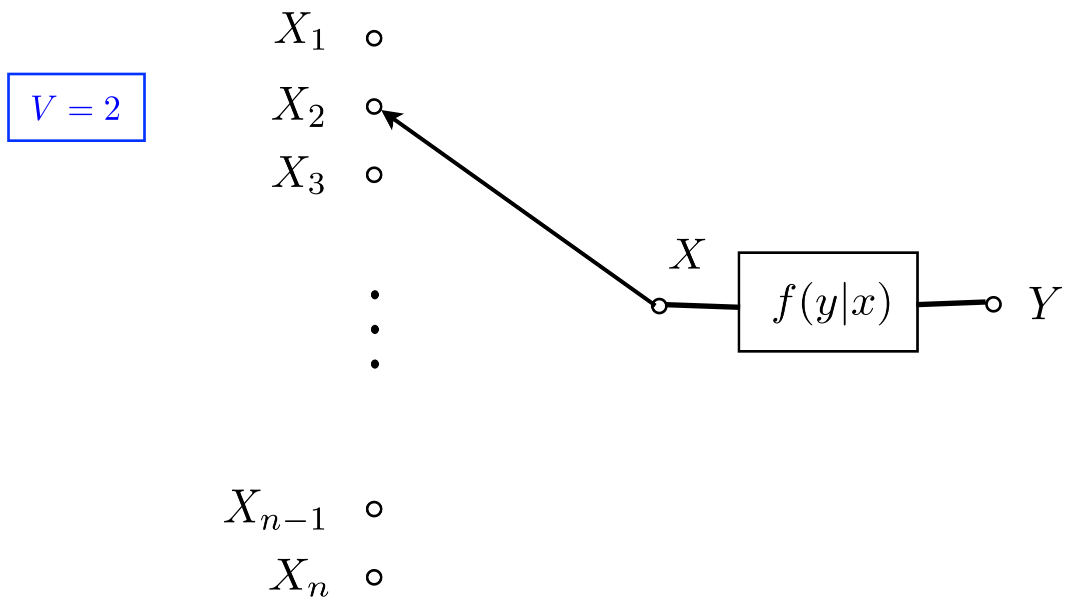

3. Let V be a mixing random variable distributed uniformly on $\{1, 2, \cdots, n\}$ which is independent of X_i, $1 \leq i \leq n$.

4. Let $X = X_V$ and Y be the output of the channel with X being the input.

Proof of Converse

1. Let R be an achievable rate, i.e., for any $\epsilon > 0$, there exists for sufficiently large n and (n, M) code such that

$$\frac{1}{n}\log M > R - \epsilon \quad \text{and} \quad \lambda_{max} < \epsilon.$$

2. Consider

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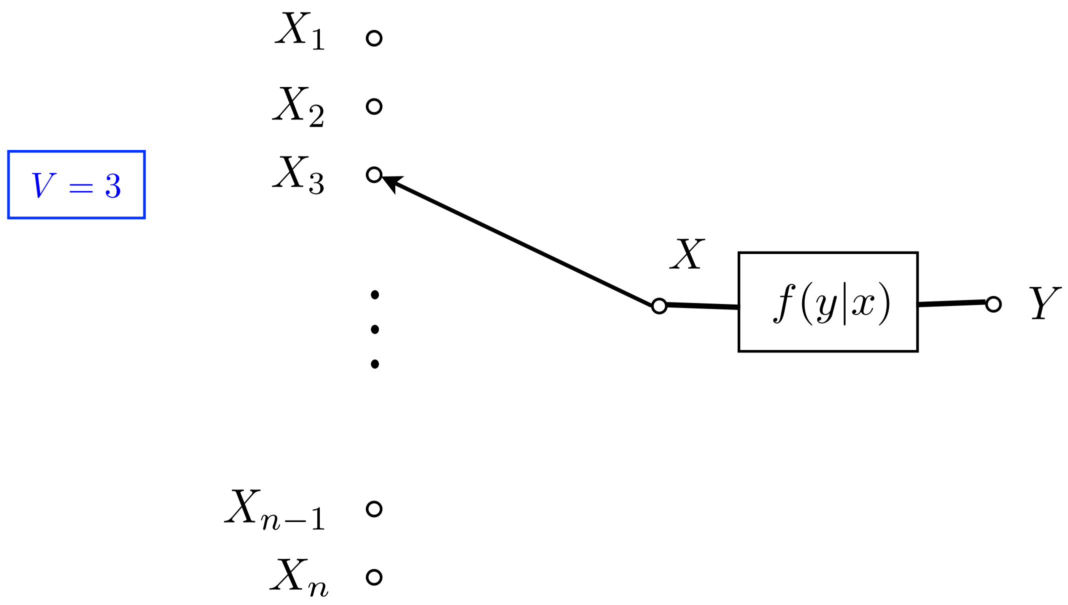
&\le H(W|\hat{W}) + \sum_{i=1}^{n} h(Y_i) - h(\mathbf{Y}|\mathbf{X}) \\
&= H(W|\hat{W}) + \sum_{i=1}^{n} h(Y_i) - \sum_{i=1}^{n} h(Y_i|X_i) \\
&= H(W|\hat{W}) + \sum_{i=1}^{n} I(X_i;Y_i).
\end{aligned}$$

3. Let V be a mixing random variable distributed uniformly on $\{1, 2, \cdots, n\}$ which is independent of X_i, $1 \le i \le n$.

4. Let $X = X_V$ and Y be the output of the channel with X being the input.



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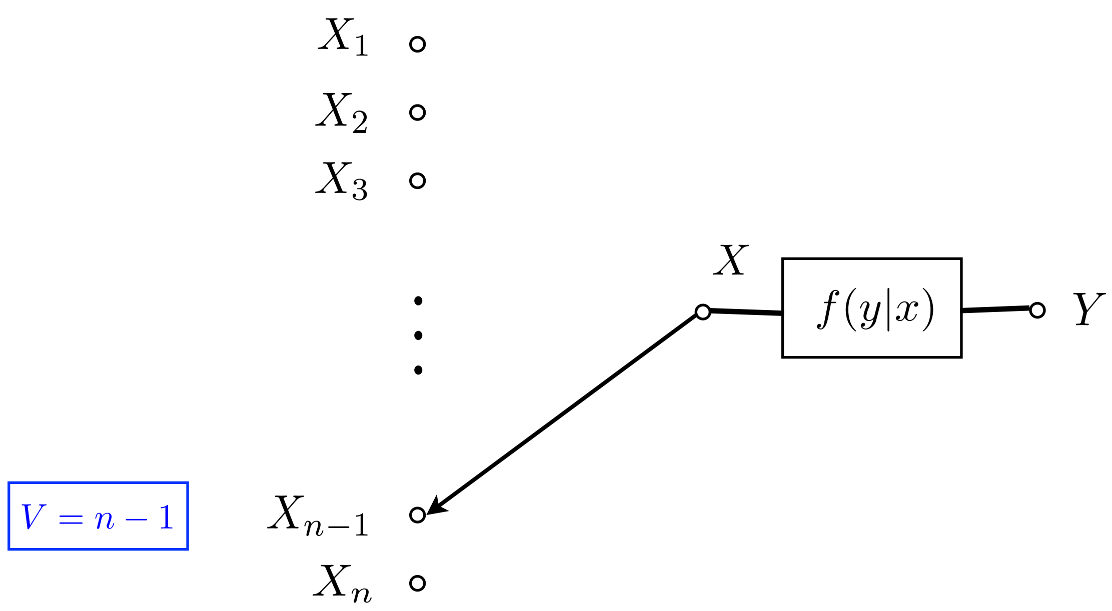

3. Let V be a mixing random variable distributed uniformly on $\{1, 2, \cdots, n\}$ which is independent of X_i, $1 \leq i \leq n$.

4. Let $X = X_V$ and Y be the output of the channel with X being the input.

Proof of Converse

1. Let R be an achievable rate, i.e., for any $\epsilon > 0$, there exists for sufficiently large n and (n, M) code such that

$$\frac{1}{n}\log M > R - \epsilon \quad \text{and} \quad \lambda_{max} < \epsilon.$$

2. Consider

$$\begin{aligned}
\log M &= H(W) \\
&= H(W|\hat{W}) + I(W;\hat{W}) \\
&\leq H(W|\hat{W}) + I(\mathbf{X};\mathbf{Y}) \\
&= H(W|\hat{W}) + h(\mathbf{Y}) - h(\mathbf{Y}|\mathbf{X}) \\
&\leq H(W|\hat{W}) + \sum_{i=1}^{n} h(Y_i) - h(\mathbf{Y}|\mathbf{X}) \\
&= H(W|\hat{W}) + \sum_{i=1}^{n} h(Y_i) - \sum_{i=1}^{n} h(Y_i|X_i) \\
&= H(W|\hat{W}) + \sum_{i=1}^{n} I(X_i;Y_i).
\end{aligned}$$

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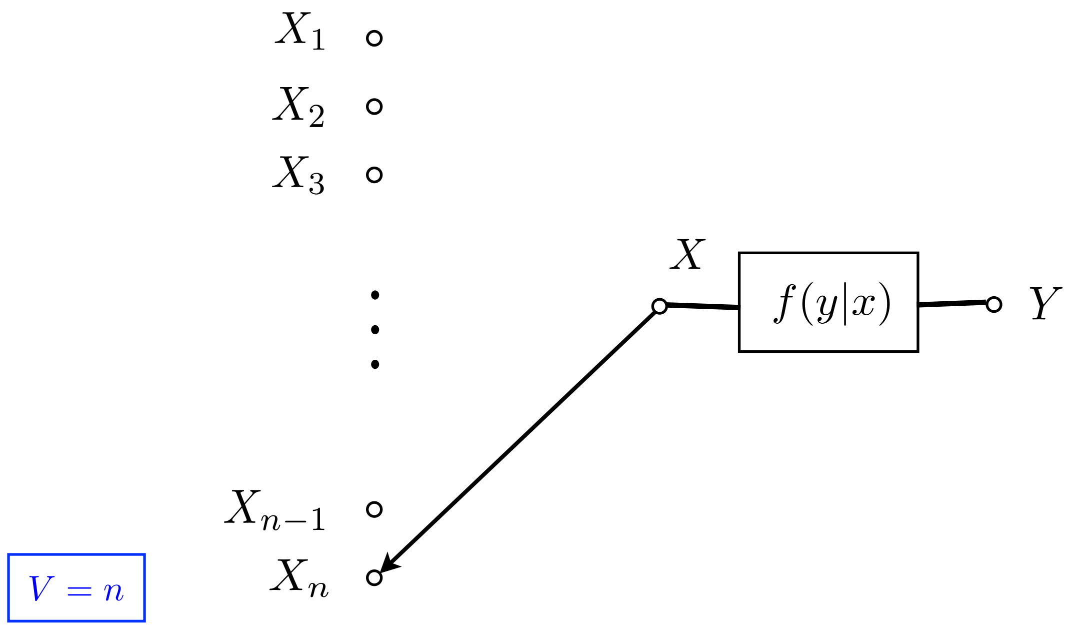

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5. Then

$$E\kappa(X) = EE[\kappa(X)|V]$$

Proof of Converse

1. Let R be an achievable rate, i.e., for any $\epsilon > 0$, there exists for sufficiently large n and (n, M) code such that

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3. Let V be a mixing random variable distributed uniformly on $\{1, 2, \cdots, n\}$ which is independent of X_i, $1 \leq i \leq n$.

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5. Then

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3. Let V be a mixing random variable distributed uniformly on $\{1, 2, \cdots, n\}$ which is independent of X_i, $1 \leq i \leq n$.

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E\kappa(X) &= EE[\kappa(X)|V] \\
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&= \sum_{i=1}^{n} \Pr\{V = i\} E[\kappa(X_i)|\cancel{V = i}] \\
&= \sum_{i=1}^{n} \frac{1}{n} \underline{E}\kappa(X_i) \\
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&= E\left[\frac{1}{n}\sum_{i=1}^{n} \kappa(X_i)\right]
\end{aligned}$$

Each codeword satisfies the input constraint:

$$\frac{1}{n}\sum_{i=1}^{n} \kappa(x_i(w)) \le P \quad \text{for } 1 \le w \le M.$$

Therefore

$$\frac{1}{n}\sum_{i=1}^{n} \kappa(X_i) \le P$$

with probability 1.

Proof of Converse

1. Let R be an achievable rate, i.e., for any $\epsilon > 0$, there exists for sufficiently large n and (n, M) code such that

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3. Let V be a mixing random variable distributed uniformly on $\{1, 2, \cdots, n\}$ which is independent of X_i, $1 \leq i \leq n$.

4. Let $X = X_V$ and Y be the output of the channel with X being the input.

5. Then

$$\begin{aligned}
E\kappa(X) &= EE[\kappa(X)|V] \\
&= \sum_{i=1}^{n} \Pr\{V = i\}E[\kappa(X)|V = i] \\
&= \sum_{i=1}^{n} \Pr\{V = i\}E[\kappa(X_i)|V = i] \\
&= \sum_{i=1}^{n} \frac{1}{n}E\kappa(X_i) \\
&= E\left[\frac{1}{n}\sum_{i=1}^{n}\kappa(X_i)\right] \\
&\leq P.
\end{aligned}$$

Each codeword satisfies the power constraint:

$$\frac{1}{n}\sum_{i=1}^{n}\kappa(x_i(w)) \leq P \quad \text{for } 1 \leq w \leq M.$$

Therefore

$$\frac{1}{n}\sum_{i=1}^{n}\kappa(X_i) \leq P$$

with probability 1.

Proof of Converse

1. Let R be an achievable rate, i.e., for any $\epsilon > 0$, there exists for sufficiently large n and (n, M) code such that

$$\frac{1}{n}\log M > R - \epsilon \quad \text{and} \quad \lambda_{max} < \epsilon.$$

2. Consider

$$\begin{aligned}
\log M &= H(W) \\
&= H(W|\hat{W}) + I(W;\hat{W}) \\
&\leq H(W|\hat{W}) + I(\mathbf{X};\mathbf{Y}) \\
&= H(W|\hat{W}) + h(\mathbf{Y}) - h(\mathbf{Y}|\mathbf{X}) \\
&\leq H(W|\hat{W}) + \sum_{i=1}^{n} h(Y_i) - h(\mathbf{Y}|\mathbf{X}) \\
&= H(W|\hat{W}) + \sum_{i=1}^{n} h(Y_i) - \sum_{i=1}^{n} h(Y_i|X_i) \\
&= H(W|\hat{W}) + \sum_{i=1}^{n} I(X_i;Y_i).
\end{aligned}$$

3. Let V be a mixing random variable distributed uniformly on $\{1, 2, \cdots, n\}$ which is independent of X_i, $1 \leq i \leq n$.

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1. Let R be an achievable rate, i.e., for any $\epsilon > 0$, there exists for sufficiently large n and (n, M) code such that

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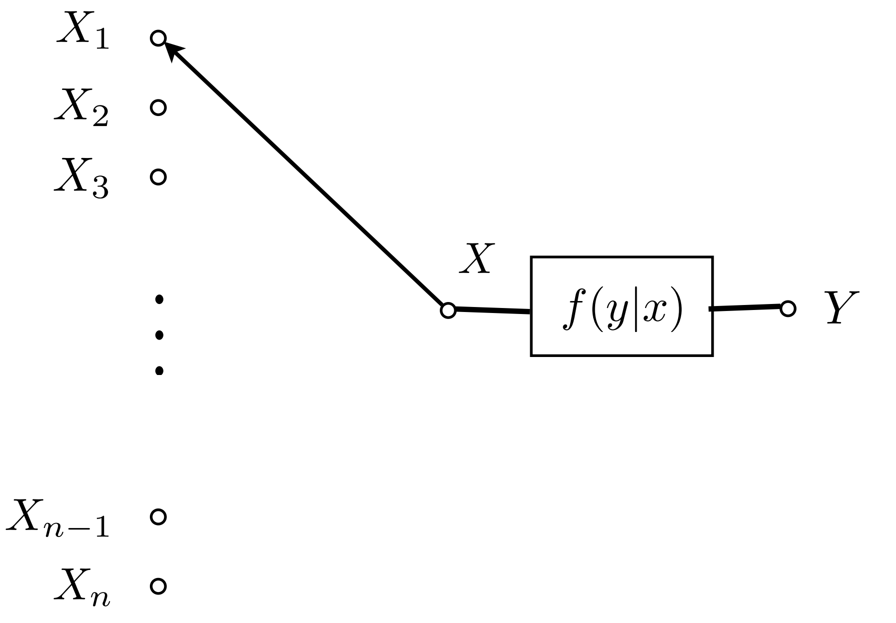
\end{aligned}$$

3. Let V be a mixing random variable distributed uniformly on $\{1, 2, \cdots, n\}$ which is independent of X_i, $1 \leq i \leq n$.

4. Let $X = X_V$ and Y be the output of the channel with X being the input.



Proof of Converse

1. Let R be an achievable rate, i.e., for any $\epsilon > 0$, there exists for sufficiently large n and (n, M) code such that

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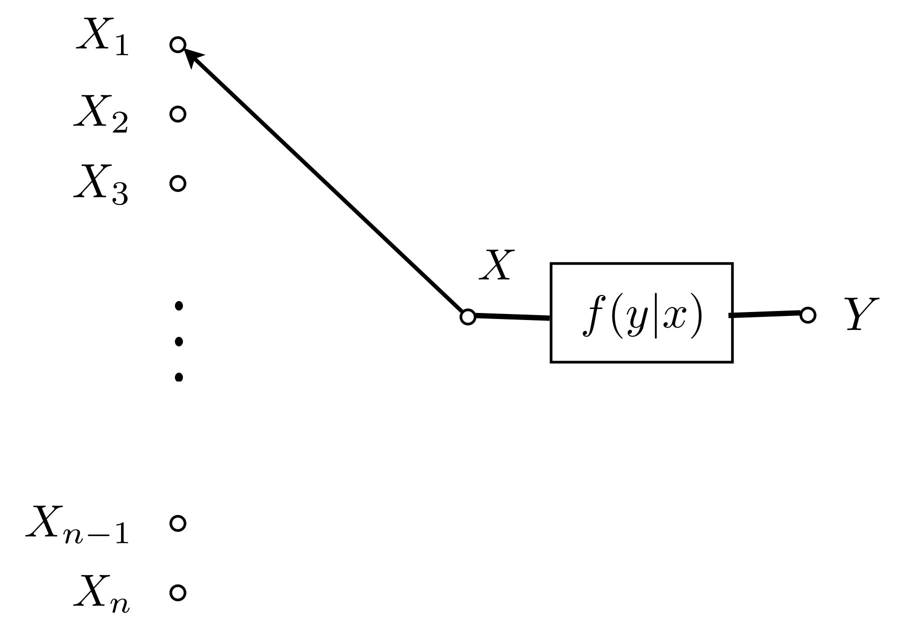
&= H(W|\hat{W}) + \sum_{i=1}^{n} I(X_i;Y_i).
\end{aligned}$$

3. Let V be a mixing random variable distributed uniformly on $\{1, 2, \cdots, n\}$ which is independent of X_i, $1 \leq i \leq n$.

4. Let $X = X_V$ and Y be the output of the channel with X being the input.



Proof of Converse

1. Let R be an achievable rate, i.e., for any $\epsilon > 0$, there exists for sufficiently large n and (n, M) code such that

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\end{aligned}$$

3. Let V be a mixing random variable distributed uniformly on $\{1, 2, \cdots, n\}$ which is independent of X_i, $1 \leq i \leq n$.

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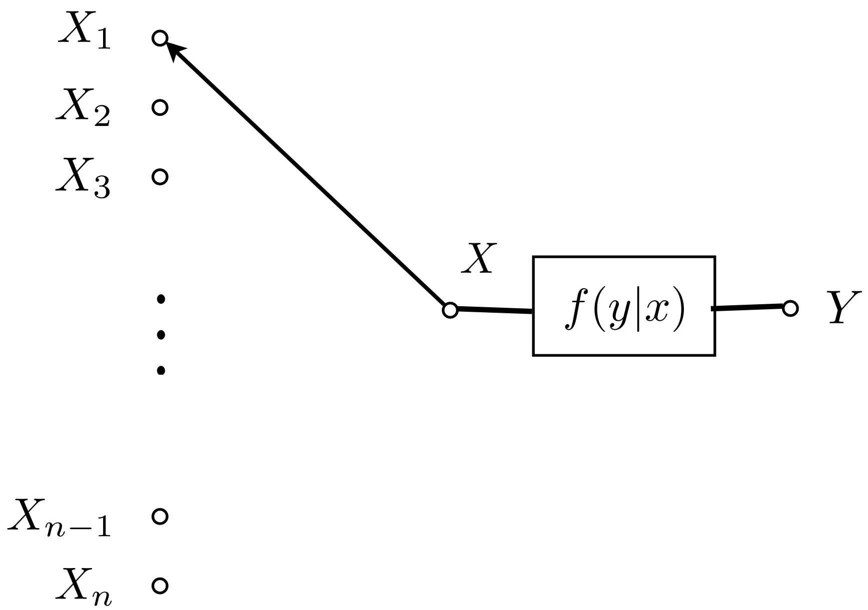

6. By the concavity of mutual information with respect to the input distribution,

Proof of Converse

1. Let R be an achievable rate, i.e., for any $\epsilon > 0$, there exists for sufficiently large n and (n, M) code such that

$$\frac{1}{n}\log M > R - \epsilon \quad \text{and} \quad \lambda_{max} < \epsilon.$$

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&\le H(W|\hat{W}) + \sum_{i=1}^{n} h(Y_i) - h(\mathbf{Y}|\mathbf{X}) \\
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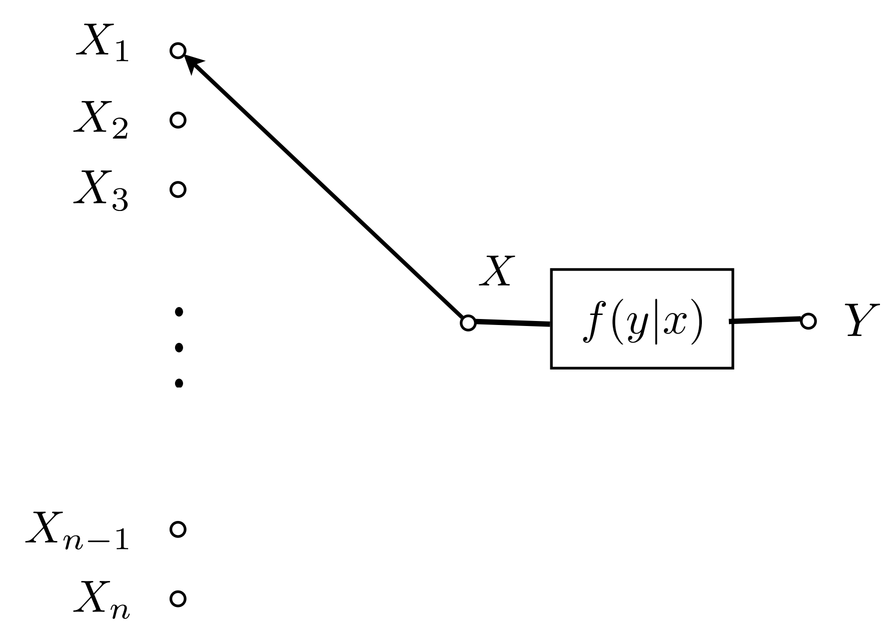
&= H(W|\hat{W}) + \sum_{i=1}^{n} I(X_i;Y_i).
\end{aligned}$$

3. Let V be a mixing random variable distributed uniformly on $\{1, 2, \cdots, n\}$ which is independent of X_i, $1 \le i \le n$.

4. Let $X = X_V$ and Y be the output of the channel with X being the input.



6. By the concavity of mutual information with respect to the input distribution,

$$\frac{1}{n}\sum_{i=1}^{n} I(X_i;Y_i) \le I(X;Y) \le C(P).$$

Proof of Converse

1. Let R be an achievable rate, i.e., for any $\epsilon > 0$, there exists for sufficiently large n and (n, M) code such that

$$\frac{1}{n}\log M > R - \epsilon \quad \text{and} \quad \lambda_{max} < \epsilon.$$

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\log M &= H(W) \\
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&\leq H(W|\hat{W}) + \sum_{i=1}^{n} h(Y_i) - h(\mathbf{Y}|\mathbf{X}) \\
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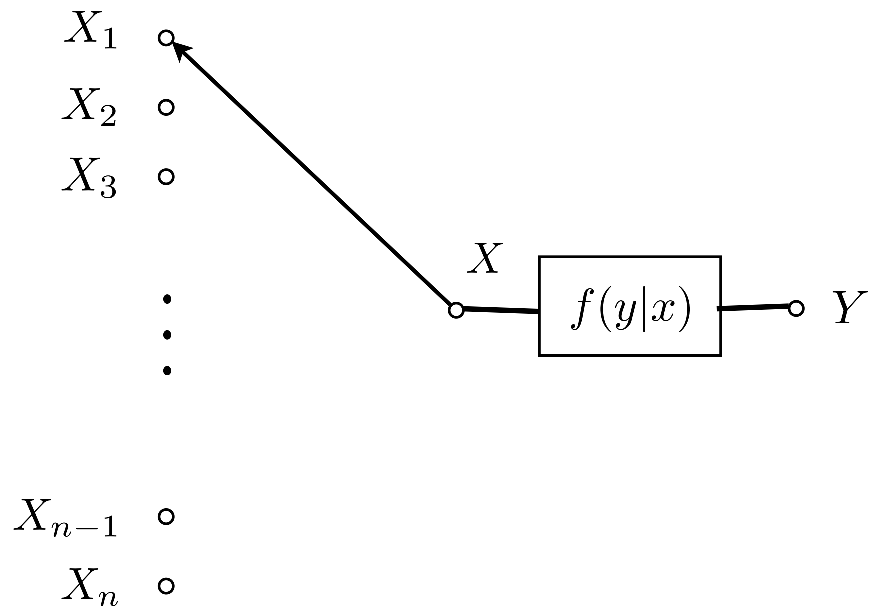
&= H(W|\hat{W}) + \sum_{i=1}^{n} I(X_i;Y_i).
\end{aligned}$$

3. Let V be a mixing random variable distributed uniformly on $\{1, 2, \cdots, n\}$ which is independent of X_i, $1 \leq i \leq n$.

4. Let $X = X_V$ and Y be the output of the channel with X being the input.



6. By the concavity of mutual information with respect to the input distribution,

$$\frac{1}{n}\sum_{i=1}^{n} I(X_i;Y_i) \leq I(X;Y) \leq C(P).$$

Proof of Converse

1. Let R be an achievable rate, i.e., for any $\epsilon > 0$, there exists for sufficiently large n and (n, M) code such that

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3. Let V be a mixing random variable distributed uniformly on $\{1, 2, \cdots, n\}$ which is independent of X_i, $1 \leq i \leq n$.

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5. Then

$$\begin{aligned}
E\kappa(X) &= EE[\kappa(X)|V] \\
&= \sum_{i=1}^{n} \Pr\{V = i\} E[\kappa(X)|V = i] \\
&= \sum_{i=1}^{n} \Pr\{V = i\} E[\kappa(X_i)|V = i] \\
&= \sum_{i=1}^{n} \frac{1}{n} E\kappa(X_i) \\
&= E\left[\frac{1}{n}\sum_{i=1}^{n} \kappa(X_i)\right] \\
&\leq P.
\end{aligned}$$

6. By the concavity of mutual information with respect to the input distribution,

$$\frac{1}{n}\sum_{i=1}^{n} I(X_i;Y_i) \leq I(X;Y) \leq C(P).$$

Proof of Converse

1. Let R be an achievable rate, i.e., for any $\epsilon > 0$, there exists for sufficiently large n and (n, M) code such that

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\end{aligned}$$

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6. By the concavity of mutual information with respect to the input distribution,

$$\frac{1}{n} \sum_{i=1}^{n} I(X_i;Y_i) \leq I(X;Y) \leq C(P).$$

Proof of Converse

1. Let R be an achievable rate, i.e., for any $\epsilon > 0$, there exists for sufficiently large n and (n, M) code such that

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3. Let V be a mixing random variable distributed uniformly on $\{1, 2, \cdots, n\}$ which is independent of X_i, $1 \leq i \leq n$.

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Proof of Converse

1. Let R be an achievable rate, i.e., for any $\epsilon > 0$, there exists for sufficiently large n and (n, M) code such that

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&= H(W|\hat{W}) + \sum_{i=1}^{n} I(X_i;Y_i). \quad \le nC(P)
\end{aligned}$$

3. Let V be a mixing random variable distributed uniformly on $\{1, 2, \cdots, n\}$ which is independent of X_i, $1 \le i \le n$.

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&= E\left[\frac{1}{n}\sum_{i=1}^{n} \kappa(X_i)\right] \\
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\end{aligned}$$

6. By the concavity of mutual information with respect to the input distribution,

$$\frac{1}{n}\sum_{i=1}^{n} I(X_i;Y_i) \leq I(X;Y) \leq C(P).$$

7. It follows that

$$n(R - \epsilon) < \log M \leq H(W|\hat{W}) + nC(P).$$

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\end{aligned}$$

3. Let V be a mixing random variable distributed uniformly on $\{1, 2, \cdots, n\}$ which is independent of X_i, $1 \le i \le n$.

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7. It follows that

$$n(R - \epsilon) < \underline{\log M \le H(W|\hat{W}) + nC(P)}.$$

8. Invoke Fano's inequality to conclude that $R \le C(P)$.

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1. Let R be an achievable rate, i.e., for any $\epsilon > 0$, there exists for sufficiently large n and (n, M) code such that

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\end{aligned}$$

3. Let V be a mixing random variable distributed uniformly on $\{1, 2, \cdots, n\}$ which is independent of X_i, $1 \le i \le n$.

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&\le P.
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\end{aligned}$$

3. Let V be a mixing random variable distributed uniformly on $\{1, 2, \cdots, n\}$ which is independent of X_i, $1 \le i \le n$.

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6. By the concavity of mutual information with respect to the input distribution,

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7. It follows that

$$n(R - \epsilon) < \log M \le H(W|\hat{W}) + nC(P).$$

8. Invoke Fano's inequality to conclude that $R \le C(P)$.

Proof of Converse

1. Let R be an achievable rate, i.e., for any $\epsilon > 0$, there exists for sufficiently large n and (n, M) code such that

$$\frac{1}{n}\log M > R - \epsilon \quad \text{and} \quad \lambda_{max} < \epsilon.$$

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3. Let V be a mixing random variable distributed uniformly on $\{1, 2, \cdots, n\}$ which is independent of X_i, $1 \leq i \leq n$.

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11.3.2 Achievability

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- Recall that for any input distribution $F(x)$, $f(y)$ exists as long as $f(y|x)$ exists. Hence
$$I(X;Y) = E \log \frac{f(Y|X)}{f(Y)}.$$

Definition 11.16 The mutually typical set $\Psi^n_{[XY]\delta}$ with respect to $F(x, y)$ is the set of $(\mathbf{x}, \mathbf{y}) \in \mathcal{X}^n \times \mathcal{Y}^n$ such that

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and δ is an arbitrarily small positive number. A pair of sequences $(\mathbf{x}, \mathbf{y})$ is called mutually δ-typical if it is in $\Psi^n_{[XY]\delta}$.

Lemma 11.17 For any $\delta > 0$, for sufficiently large n,

$$\Pr\{(\mathbf{X}, \mathbf{Y}) \in \Psi^n_{[XY]\delta})\} \geq 1 - \delta.$$

Proof

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Proof

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2. By WLLN,

$$\frac{1}{n} \sum_{i=1}^{n} \log \frac{f(Y_i|X_i)}{f(Y_i)} \;\rightarrow\; E \log \frac{f(Y|X)}{f(Y)} = I(X;Y)$$

   in probability.

Lemma 11.18 Let $(\mathbf{X}', \mathbf{Y}')$ be n i.i.d. copies of a pair of generic random variables (X', Y'), where X' and Y' are independent and have the same marginal distributions as X and Y, respectively. Then

$$\Pr\{(\mathbf{X}', \mathbf{Y}') \in \Psi^n_{[XY]\delta}\} \leq 2^{-n(I(X;Y)-\delta)}.$$

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$$\Pr\{(\mathbf{X}', \mathbf{Y}') \in \Psi^n_{[XY]\delta}\} \le 2^{-n(I(X;Y)-\delta)}.$$

Proof

1. For any $(\mathbf{x}, \mathbf{y}) \in \Psi^n_{[XY]\delta}$, by definition,

$$\left| \frac{1}{n} \log \frac{f(\mathbf{y}|\mathbf{x})}{f(\mathbf{y})} - I(X;Y) \right| \le \delta.$$

2. Then

$$\frac{1}{n} \log \frac{f(\mathbf{y}|\mathbf{x})}{f(\mathbf{y})} \ge I(X;Y) - \delta$$

which implies

$$\frac{f(\mathbf{y}|\mathbf{x})}{f(\mathbf{y})} \ge 2^{n(I(X;Y)-\delta)}$$

or

$$f(\mathbf{y}|\mathbf{x}) \ge f(\mathbf{y}) 2^{n(I(X;Y)-\delta)}.$$

3. Consider

$$\begin{aligned} 1 &\ge \Pr\{(\mathbf{X}, \mathbf{Y}) \in \Psi^n_{[XY]\delta})\} \\ &= \int\int_{\Psi^n_{[XY]\delta}} f(\mathbf{y}|\mathbf{x}) dF(\mathbf{x})\, d\mathbf{y} \\ &\ge 2^{n(I(X;Y)-\delta)} \int\int_{\Psi^n_{[XY]\delta}} f(\mathbf{y}) dF(\mathbf{x}) d\mathbf{y} \end{aligned}$$

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4. Hence

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Random Coding Scheme

Random Coding Scheme

Parameter Settings

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1. Fix $\epsilon > 0$ and input distribution $F(x)$. Let δ to be specified later.

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$$C(P) = \sup_{F(x):E\kappa(X)\le P} I(X;Y)$$

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The Random Coding Scheme

1. Construct the codebook $\mathcal{C}$ of an (n, M) code randomly by generating M codewords in $\Re^n$ independently and identically according to $F(x)^n$.

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6. The channel outputs a sequence $\mathbf{Y}$ according to

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Otherwise, $\mathbf{Y}$ is decoded to a constant message in $\mathcal{W}$. Denote by $\hat{W}$ the message to which $\mathbf{Y}$ is decoded.

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$$E_e = \left\{ \frac{1}{n} \sum_{i=1}^{n} \kappa(\tilde{X}_i(W)) > P \right\}$$

and

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3. Consider

$$\begin{aligned} \Pr\{Err\} &= \Pr\{Err|W=1\} \\ &\leq \Pr\{E_e|W=1\} + \Pr\{E_d|W=1\}. \end{aligned}$$

4. The analysis of $\Pr\{E_d|W=1\}$ is exactly the same as the analysis of the decoding error in the discrete case. So we can choose δ to be small to make

$$\Pr\{E_d|W=1\} \leq \frac{\epsilon}{4}$$

for sufficiently large n.

5. By WLLN, for sufficiently large n,

$$\begin{aligned} &\Pr\{E_e|W=1\} \\ &= \Pr\left\{ \frac{1}{n} \sum_{i=1}^{n} \kappa(\tilde{X}_i(1)) > P \,\middle|\, W=1 \right\} \\ &= \Pr\left\{ \frac{1}{n} \sum_{i=1}^{n} \kappa(\tilde{X}_i(1)) > P \right\} \\ &= \Pr\left\{ \frac{1}{n} \sum_{i=1}^{n} \kappa(\tilde{X}_i(1)) > (P-\gamma)+\gamma \right\} \\ &\leq \Pr\left\{ \frac{1}{n} \sum_{i=1}^{n} \kappa(\tilde{X}_i(1)) > E\kappa(X)+\gamma \right\} \\ &\leq \frac{\epsilon}{4}. \end{aligned}$$

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Performance Analysis

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Performance Analysis

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10. Since $Err = E_e \cup E_d$, we have

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11. Observe that conditioning on $\{\mathcal{C}^*, W=w\}$, the codeword $\tilde{\mathbf{X}}(w)$ is deterministic, so either

Performance Analysis

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8. After discarding the worst half of the codewords in $\mathcal{C}^*$, if a codeword $\tilde{\mathbf{X}}(w)$ remains in $\mathcal{C}^*$, then

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9. However, it is not clear whether $\tilde{\mathbf{X}}(w)$ satisfies both $\lambda_w \le \epsilon$ and the input constraint.

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Performance Analysis

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11. Observe that conditioning on $\{\mathcal{C}^*, W=w\}$, the codeword $\tilde{\mathbf{X}}(w)$ is deterministic, so either

$$\Pr\{E_e|\mathcal{C}^*, W=w\} = 0$$

or

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Performance Analysis

5. By WLLN, for sufficiently large n,

$$\begin{aligned}
&\Pr\{E_e|W=1\} \\
&= \Pr\left\{ \frac{1}{n}\sum_{i=1}^{n} \kappa(\tilde{X}_i(1)) > P \,\middle|\, W=1 \right\} \\
&= \Pr\left\{ \frac{1}{n}\sum_{i=1}^{n} \kappa(\tilde{X}_i(1)) > P \right\} \\
&= \Pr\left\{ \frac{1}{n}\sum_{i=1}^{n} \kappa(\tilde{X}_i(1)) > (P-\gamma)+\gamma \right\} \\
&\leq \Pr\left\{ \frac{1}{n}\sum_{i=1}^{n} \kappa(\tilde{X}_i(1)) > E\kappa(X)+\gamma \right\} \\
&\leq \frac{\epsilon}{4}.
\end{aligned}$$

6. Therefore,

$$\Pr\{Err\} \leq \frac{\epsilon}{2}$$

which implies for some codebook $\mathcal{C}^*$,

$$\Pr\{Err|\mathcal{C}^*\} \leq \frac{\epsilon}{2}.$$

7. Rank the codewords in $\mathcal{C}^*$ in ascending order according to $\Pr\{Err|\mathcal{C}^*, W=w\}$.

8. After discarding the worst half of the codewords in $\mathcal{C}^*$, if a codeword $\tilde{\mathbf{X}}(w)$ remains in $\mathcal{C}^*$, then

$$\Pr\{Err|\mathcal{C}^*, W=w\} \leq \epsilon.$$

9. However, it is not clear whether $\tilde{\mathbf{X}}(w)$ satisfies both $\lambda_w \leq \epsilon$ and the input constraint.

10. Since $Err = E_e \cup E_d$, we have

$$\lambda_w = \Pr\{E_d|\mathcal{C}^*, W=w\} \leq \epsilon$$

and

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