香港中文大學
The Chinese University of Hong Kong

11.2 The Channel Coding Theorem

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i.e., each codeword satisfies the input constraint.

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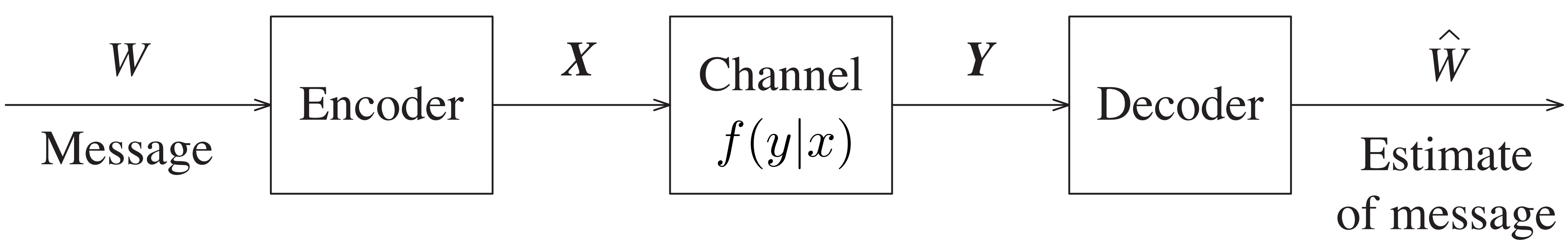

Error Probabilities

Definition 11.10 For all $1 \le w \le M$, let

$$\lambda_w = \Pr\{\hat{W} \neq w | W = w\} = \int_{\{\mathbf{y} \in \mathcal{Y}^n : g(\mathbf{y}) \neq w\}} f_{\mathbf{Y}|\mathbf{X}}(\mathbf{y}|e(w)) d\mathbf{y}$$

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Definition 11.12 The average probability of error of an (n, M) code is defined as

$$P_e = \Pr\{\hat{W} \neq W\}.$$

Definition 11.13 A rate R is (asymptotically) achievable for a continuous memoryless channel if for any $\epsilon > 0$, there exists for sufficiently large n an (n, M) code such that

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Theorem 11.14 A rate R is achievable for a continuous memoryless channel with input constraint (κ, P) if and only if $R \leq C(P)$, the capacity of the channel.