香港中文大學
The Chinese University of Hong Kong

10.4 AEP for Continuous Random Variables

Theorem 10.35 (AEP I for Continuous Random Variables)

$$-\frac{1}{n}\log f(\mathbf{X}) \to h(X)$$

in probability as $n \to \infty$, i.e., for any $\epsilon > 0$, for n sufficiently large,

$$\Pr\left\{\left|-\frac{1}{n}\log f(\mathbf{X}) - h(X)\right| < \epsilon\right\} > 1 - \epsilon.$$

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Proof WWLN.

Definition 10.36 The typical set $W^n_{[X]\epsilon}$ with respect to $f(x)$ is the set of sequences $\mathbf{x} = (x_1, x_2, \cdots, x_n) \in \mathcal{X}^n$ such that

$$\left| -\frac{1}{n} \log f(\mathbf{x}) - h(X) \right| < \epsilon$$

or equivalently,

$$h(X) - \epsilon < -\frac{1}{n} \log f(\mathbf{x}) < h(X) + \epsilon$$

where ϵ is an arbitrarily small positive real number. The sequences in $W^n_{[X]\epsilon}$ are called ϵ-typical sequences.

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Empirical Differential Entropy:

$$-\frac{1}{n} \log f(\mathbf{x}) = -\frac{1}{n} \sum_{k=1}^{n} \log f(x_k)$$

The empirical differential entropy of a typical sequence is close to the true differential entropy $h(X)$.

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3) For n sufficiently large,

$$(1-\epsilon)2^{n(h(X)-\epsilon)} < \text{Vol}(W^n_{[X]\epsilon}) < 2^{n(h(X)+\epsilon)}$$

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2. The fact that $h(X)$ can be negative does not incur any difficulty because $2^{nh(X)}$ is always positive.

3. If the differential entropy is large, then the volume of the typical set is large.