

5. Therefore,

$$R_I(D) = \min_{\hat{X}: Ed(X, \hat{X}) \leq D} I(X; \hat{X}) \geq h_b(\gamma) - h_b(D).$$

Now need to construct \hat{X} which is tight for (1) and (2), so that the above bound is achieved.

6. Observe that

- (1) tight $\Leftrightarrow Y$ independent of \hat{X}
- (2) tight $\Leftrightarrow \Pr\{X \neq \hat{X}\} = D$

The required \hat{X} can be specified by the following reverse BSC:

7. Therefore, we conclude that for $0 \leq \gamma \leq 1/2$,

$$R_I(D) = \begin{cases} h_b(\gamma) - h_b(D) & \text{if } 0 \leq D < \gamma \\ 0 & \text{if } D \geq \gamma. \end{cases}$$

8. For $1/2 \leq \gamma \leq 1$, by exchanging the roles of the symbols 0 and 1 and applying the same argument, we obtain $R_I(D)$ as above except that γ is replaced by $1 - \gamma$, i.e.,

$$R_I(D) = \begin{cases} h_b(1 - \gamma) - h_b(D) & \text{if } 0 \leq D < 1 - \gamma \\ 0 & \text{if } D \geq 1 - \gamma. \end{cases}$$

9. Combining the two cases, we have

$$R_I(D) = \begin{cases} h_b(\gamma) - h_b(D) & \text{if } 0 \leq D < \min(\gamma, 1 - \gamma) \\ 0 & \text{if } D \geq \min(\gamma, 1 - \gamma) \end{cases}$$

for $0 \leq \gamma \leq 1$.

Note that

$$\left(\frac{1 - \gamma - D}{1 - 2D} \right) D + \left(\frac{\gamma - D}{1 - 2D} \right) (1 - D) = \gamma$$

$$0 \leq \frac{\gamma - D}{1 - 2D} \leq 1, \text{ because } D < \gamma \leq 1/2$$