Let $\hat{X}$ be any estimate of X which takes values in $\hat{\mathcal{X}}$. Then

$$\begin{aligned}
Ed(X,\hat{X}) &= \sum_x \sum_{\hat{x}} p(x,\hat{x}) d(x,\hat{x}) \\
&= \sum_x \sum_{\hat{x}} p(x,\hat{x}) \left[\tilde{d}(x,\hat{x}) + c_x\right] \\
&= E\tilde{d}(X,\hat{X}) + \sum_x p(x) \sum_{\hat{x}} p(\hat{x}|x) c_x \\
&= E\tilde{d}(X,\hat{X}) + \sum_x p(x) c_x \left(\sum_{\hat{x}} p(\hat{x}|x)\right) \\
&= E\tilde{d}(X,\hat{X}) + \sum_x p(x) c_x \\
&= E\tilde{d}(X,\hat{X}) + \Delta,
\end{aligned}$$

where

$$\Delta = \sum_x p(x) c_x$$

is a constant which depends only on $p(x)$ and d but not on the conditional distribution $p(\hat{x}|x)$.