

Lemma 7.22 For all $1 \leq i \leq n$,

$$(W, \mathbf{Y}^{i-1}) \rightarrow X_i \rightarrow Y_i$$

forms a Markov chain.

Proof

1. The Markov chain

$$(W, \mathbf{X}^{i-1}, \mathbf{Y}^{i-1}) \rightarrow X_i \rightarrow Y_i$$

holds because the channel is memoryless.

2. Then

$$\begin{aligned} 0 &= I(W, \mathbf{X}^{i-1}, \mathbf{Y}^{i-1}; Y_i | X_i) \\ &= I(W, \mathbf{Y}^{i-1}; Y_i | X_i) + I(\mathbf{X}^{i-1}; Y_i | W, X_i, \mathbf{Y}^{i-1}) \end{aligned}$$

which implies $I(W, \mathbf{Y}^{i-1}; Y_i | X_i) = 0$, or $(W, \mathbf{Y}^{i-1}) \rightarrow X_i \rightarrow Y_i$.