- We want a code with $\lambda_{max} < \epsilon$, not just $P_e < \epsilon/2$.
- Without loss of generality, assume $\lambda_1 \le \lambda_2 \le \cdots \le \lambda_M$. Consider

$$\frac{1}{M}\sum_{w=1}^{M}\lambda_w < \frac{\epsilon}{2} \iff \sum_{w=1}^{M}\lambda_w < \left(\frac{M}{2}\right)\epsilon.$$

- Since M is even, $M/2$ is an integer. Then

$$\sum_{w=M/2+1}^{M}\lambda_w \le \sum_{w=1}^{M}\lambda_w < \left(\frac{M}{2}\right)\epsilon \implies \frac{1}{M/2}\sum_{w=M/2+1}^{M}\lambda_w < \epsilon.$$

- Hence,

$$\lambda_{M/2+1} < \epsilon \implies \lambda_{M/2} < \epsilon.$$

- Conclusion: If $P_e < \epsilon/2$, then $\lambda_1 \le \lambda_2 \le \cdots \le \lambda_{M/2} < \epsilon$.
- Discard the worst half of the codewords in $\mathcal{C}^*$ to achieve $\lambda_{max} < \epsilon$.