

- According to the random coding scheme,

$$\Pr\{Err\} = \sum_{\mathcal{C}} \Pr\{\mathcal{C}\} \Pr\{Err|\mathcal{C}\}.$$

- Then there exists at least one codebook \mathcal{C}^* such that

$$P_e = \Pr\{Err|\mathcal{C}^*\} \leq \Pr\{Err\} < \frac{\epsilon}{2}.$$

- By construction, this codebook has rate

$$\frac{1}{n} \log M > I(X; Y) - \frac{\epsilon}{2}.$$