Parameter Settings

1. Fix $\epsilon > 0$ and input distribution $p(x)$. Let δ to be specified later.

2. Let M be an even integer satisfying

$$I(X;Y) - \frac{\epsilon}{2} < \frac{1}{n} \log M < I(X;Y) - \frac{\epsilon}{4},$$

where n is sufficiently large, i.e., $M \approx 2^{nI(X;Y)}$.

The Random Coding Scheme

1. Construct the codebook $\mathcal{C}$ of an (n, M) code by generating M codewords in $\mathcal{X}^n$ independently and identically according to $p(x)^n$. Denote these codewords by $\tilde{\mathbf{X}}(1), \tilde{\mathbf{X}}(2), \cdots, \tilde{\mathbf{X}}(M)$.

2. Reveal the codebook $\mathcal{C}$ to both the encoder and the decoder.

3. A message W is chosen from $\mathcal{W}$ according to the uniform distribution.

4. Transmit $\mathbf{X} = \tilde{\mathbf{X}}(W)$ through the channel.

5. The channel outputs a sequence $\mathbf{Y}$ according to

$$\Pr\{\mathbf{Y} = \mathbf{y} | \tilde{\mathbf{X}}(W) = \mathbf{x}\} = \prod_{i=1}^{n} p(y_i | x_i).$$

6. The sequence $\mathbf{Y}$ is decoded to the message w if

- $(\tilde{\mathbf{X}}(w), \mathbf{Y}) \in T^n_{[XY]\delta}$, and
- there does not exists $w' \neq w$ such that $(\tilde{\mathbf{X}}(w'), \mathbf{Y}) \in T^n_{[XY]\delta}$.

Otherwise, $\mathbf{Y}$ is decoded to a constant message in $\mathcal{W}$. Denote by $\hat{W}$ the message to which $\mathbf{Y}$ is decoded.