

Assume both \mathcal{X} and \mathcal{Y} are finite.

Definition 7.6 The capacity of a discrete memoryless channel $p(y|x)$ is defined as

$$C = \max_{p(x)} I(X; Y),$$

where X and Y are respectively the input and the output of the generic discrete channel, and the maximum is taken over all input distributions $p(x)$.

Remarks:

- Since $I(X; Y)$ is a continuous functional of $p(x)$ and the set of all $p(x)$ is a compact set (i.e., closed and bounded) in $\mathbb{R}^{|\mathcal{X}|}$, the maximum value of $I(X; Y)$ can be attained.
- Will see that C is in fact the maximum rate at which information can be communicated reliably through a DMC.
- Can communicate through a channel at a positive rate while $P_e \rightarrow 0$!