- II $\Rightarrow$ I $(\alpha, Z) \Rightarrow p(y|x)$: Obvious.

- I $\Rightarrow$ II:

  - For $x \in \mathcal{X}$, define Z_x with $\mathcal{Z}_x = \mathcal{Y}$ such that $\Pr\{Z_x = y\} = p(y|x)$.
  - Assume Z_x, $x \in \mathcal{X}$ are mutually independent and also independent of X.
  - Define the noise variable $Z = (Z_x : x \in \mathcal{X})$.
  - Let $Y = Z_x$ if $X = x$, so that $Y = \alpha(X, Z)$.
  - Then

$$\begin{aligned}
\Pr\{X = x, Y = y\} &= \Pr\{X = x\}\Pr\{Y = y|X = x\} \\
&= \Pr\{X = x\}\Pr\{Z_x = y|X = x\} \\
&= \Pr\{X = x\}\Pr\{Z_x = y\} \\
&= \Pr\{X = x\}p(y|x)
\end{aligned}$$