

### Example 7.7 (BSC)

Alternative representation of a BSC with crossover probability  $\epsilon$ :

$$Y = X + Z \bmod 2$$

with

$$\Pr\{Z = 0\} = 1 - \epsilon \quad \text{and} \quad \Pr\{Z = 1\} = \epsilon$$

and  $Z$  is independent of  $X$ .

### Determination of $C$

1. Consider

$$\begin{aligned} I(X; Y) &= H(Y) - H(Y|X) \\ &= H(Y) - \sum_x p(x) H(Y|X = x) \\ &= H(Y) - \sum_x p(x) h_b(\epsilon) \\ &= H(Y) - h_b(\epsilon) \\ &\leq 1 - h_b(\epsilon). \end{aligned}$$

2. So,

$$C = \max_{p(x)} I(X; Y) \leq 1 - h_b(\epsilon).$$

3. The upper bound on  $I(X; Y)$  is tight if

$$H(Y) = 1.$$

This can be achieved by taking the uniform input distribution.

4. Therefore,  $C = 1 - h_b(\epsilon)$  bit per use.