

## An Interpretation of $I(X; Y|Z) \geq 0$

1. Fix  $\mathbf{z} \in S_{[Z]}^n$ . By the consistency of strong typicality,

$$(\mathbf{x}, \mathbf{y}, \mathbf{z}) \in T_{[XYZ]}^n \delta \implies \begin{cases} (\mathbf{x}, \mathbf{z}) \in T_{[XZ]}^n \delta \\ (\mathbf{y}, \mathbf{z}) \in T_{[YZ]}^n \delta \end{cases}$$

2. Equivalently,

$$(\mathbf{x}, \mathbf{y}) \in T_{[XY|Z]}^n \delta(\mathbf{z}) \implies \begin{cases} \mathbf{x} \in T_{[X|Z]}^n \delta(\mathbf{z}) \\ \mathbf{y} \in T_{[Y|Z]}^n \delta(\mathbf{z}) \end{cases}$$

3. Then

$$|T_{[XY|Z]}^n \delta(\mathbf{z})| \leq |T_{[X|Z]}^n \delta(\mathbf{z})| |T_{[Y|Z]}^n \delta(\mathbf{z})|.$$

4. By the conditional strong AEP,

$$2^{nH(X,Y|Z)} \leq 2^{nH(X|Z)} 2^{nH(Y|Z)}$$

or

$$H(X, Y|Z) \leq H(X|Z) + H(Y|Z).$$

5. This is equivalent to  $I(X; Y|Z) \geq 0$ .