1. Consider X with distribution p such that

$$p(0) = 0.5, \;\; p(1) = 0.25, \;\; p(2) = 0.25.$$

2. Consider a sequence $\mathbf{x}$ of length n and let $q(x) = n^{-1} N(x; \mathbf{x})$ be the relative frequency of occurrence of symbol x in $\mathbf{x}$, $x = 0, 1, 2$.

3. In order for the sequence $\mathbf{x}$ to be weakly typical, we need the empirical entropy to be close to $H(X)$:

$$
\begin{aligned}
&-\frac{1}{n} \log p(\mathbf{x}) \quad \text{(empirical entropy)} \\
&= -\frac{1}{n} \log \prod_{k=1}^{n} p(x_k) \\
&= -\frac{1}{n} \sum_{k=1}^{n} \log p(x_k) \\
&= -\frac{1}{n} [N(0;\mathbf{x}) \log p(0) + N(1;\mathbf{x}) \log p(1) + N(2;\mathbf{x}) \log p(2)] \\
&= -\frac{N(0;\mathbf{x})}{n} \log p(0) - \frac{N(1;\mathbf{x})}{n} \log p(1) - \frac{N(2;\mathbf{x})}{n} \log p(2) \\
&= -q(0) \log 0.5 - q(1) \log 0.25 - q(2) \log 0.25 \qquad (1) \\
&\approx H(X) \\
&= -p(0) \log 0.5 - p(1) \log 0.25 - p(2) \log 0.25 \\
&= -(0.5) \log 0.5 - (0.25) \log 0.25 - (0.25) \log 0.25. \qquad (2)
\end{aligned}
$$

This can be satisfied by choosing $q(i) = p(i)$ for all i.

4. Alternatively, we can choose

$$q(0) = 0.5, \;\; q(1) = 0.5, \;\; q(2) = 0.$$

5. With such a choice of $\{q(i)\}$, the sequence $\mathbf{x}$ is weakly typical with respect to p because (1) and (2) are evaluated to the same value which implies

$$\text{empirical entropy} \approx H(X),$$

but obviously not strongly typical with respect to p, because

$$p \not\approx q.$$