Proposition 6.5 For any $\mathbf{x} \in \mathcal{X}^n$, if $\mathbf{x} \in T^n_{[X]\delta}$, then $\mathbf{x} \in W^n_{[X]\eta}$, where $\eta \to 0$ as $\delta \to 0$.

Proof

1. If $\mathbf{x} \in T^n_{[X]\delta}$, by Property 1 of strong AEP, we have

$$2^{-n(H(X)+\eta)} \leq p(\mathbf{x}) \leq 2^{-n(H(X)-\eta)}.$$

2. This is equivalent to

$$H(X) - \eta \leq -\frac{1}{n} \log p(\mathbf{x}) \leq H(X) + \eta,$$

where $\eta \to 0$ as $\delta \to 0$ as asserted by the strong AEP.

3. Then $\mathbf{x} \in W^n_{[X]\eta}$ by Definition 5.2. The proposition is proved.