Corollary 4.7 (Theorem 2.43) $H(X) \leq \log |\mathcal{X}|$.

Proof

- Let $\mathcal{X} = \{0, 1, \cdots, |\mathcal{X}| - 1\}$.
- Let $\mathcal{C}$ be the identity code, i.e.,

x	0	1	$\cdots$	$\lvert\mathcal{X}\rvert - 1$
$\mathcal{C}(x)$	0	1	$\cdots$	$\lvert\mathcal{X}\rvert - 1$

- Evidently, $\mathcal{C}$ is an $|\mathcal{X}|$-ary uniquely decodable code, with expected length equals 1.
- By the entropy bound, we have

$$1 = L \geq H_{|\mathcal{X}|}(X).$$

- Leaving the base unspecified, we have

$$H(X) \leq \log |\mathcal{X}|,$$

recovering Theorem 2.43.