Example 4.5 In this example, we illustrate a technique used in the proof of Theorem 4.4.

1. Consider the code $\mathcal{C}$ in Example 4.3.

2. Let $l_1 = l_2 = 1$ and $l_3 = l_4 = 2$. These are the lengths of the codewords in $\mathcal{C}$.

3. Consider the polynomial

$$\sum_{k=1}^{4} 2^{-l_k} = \left(2^{-1} + 2^{-1} + 2^{-2} + 2^{-2}\right)$$

raised to the power N.

4. For $N = 2$, we have

$$\begin{aligned}
&\left(\sum_{k=1}^{4} 2^{-l_k}\right)^2 \\
&\quad = \left(2^{-1} + 2^{-1} + 2^{-2} + 2^{-2}\right) \cdot \left(2^{-1} + 2^{-1} + 2^{-2} + 2^{-2}\right) \\
&\quad = 4 \cdot 2^{-2} + 8 \cdot 2^{-3} + 4 \cdot 2^{-4} \\
&\quad = A_2 \cdot 2^{-2} + A_3 \cdot 2^{-3} + A_4 \cdot 2^{-4},
\end{aligned}$$

where

$$A_2 = 4, \quad A_3 = 8, \quad A_4 = 4.$$

5. Then $A_2 = 4$ is the total number of sequences of $N = 2$ codewords with a total length of 2 code symbols. Specifically, the 4 sequences are

$$00(AA), \; 01(AB), \; 10(BA), \; 11(BB).$$

6. Similarly, $A_3 = 8$ and $A_4 = 4$ are the total number of sequences of 2 codewords with a total length of 3 and 4 code symbols, respectively.

Exercise Verify that $A_3 = 8$ and list the 8 sequences of 2 codewords with a total length of 3 code symbols.