

**Theorem 4.6 (Entropy Bound)** Let  $\mathcal{C}$  be a  $D$ -ary uniquely decodable code for a source random variable  $X$  with entropy  $H_D(X)$ . Then the expected length of  $\mathcal{C}$  is lower bounded by  $H_D(X)$ , i.e.,

$$L \geq H_D(X).$$

This lower bound is tight if and only if  $l_i = -\log_D p_i$  for all  $i$ .