Nonnegatvity of μ^* for $X_1 \rightarrow X_2 \rightarrow X_3 \rightarrow X_4$

1. We have proved that μ^* vanishes on the 5 atoms shown in the information diagram.

2. Suppress these atoms by setting them to $\emptyset$ to obtain the information diagram below.

3. From this information diagram, it can readily be checked that the values of μ^* on the remaining

$$(2^4 - 1) - 5 = 10$$

nonempty atoms are equal to

$$H(X_1|X_2, X_3, X_4),\ I(X_1; X_2|X_3, X_4)$$
$$I(X_1; X_3|X_4),\ I(X_1; X_4)$$
$$H(X_2|X_1, X_3, X_4),\ I(X_2; X_3|X_1; X_4)$$
$$I(X_2; X_4|X_1),\ H(X_3|X_1, X_2, X_4)$$
$$I(X_3; X_4|X_1, X_2),\ H(X_4|X_1, X_2, X_3).$$

3. There are all Shannon's information measures which are always nonnegative. Therefore, μ^* is a measure.

Exercise: Identify these 10 atoms in the information diagram at the bottom.