

$\mu^*$  for  $X_1 \rightarrow X_2 \rightarrow X_3$

1. In this information diagram,

$$\begin{aligned} I(X_1; X_3 | X_2) &= \mu^*(\tilde{X}_1 \cap \tilde{X}_3 - \tilde{X}_2) \\ &= \mu^*(\emptyset) \\ &= 0. \end{aligned}$$

2. Also,

$$\begin{aligned} \mu^*(\tilde{X}_1 \cap \tilde{X}_2 \cap \tilde{X}_3) &= \mu^*(\tilde{X}_1 \cap \tilde{X}_3) \\ &= I(X_1; X_3) \\ &\geq 0. \end{aligned}$$

3. Since the values of  $\mu^*$  on all the remaining atoms correspond to Shannon's information measures and hence are nonnegative, we conclude that  $\mu^*$  is a measure.