Structure of μ^* for $X_1 \to X_2 \to X_3 \to X_4$

1. The Markov subchain $X_1 \to X_2 \to X_3$ implies

$$0 = I(X_1; X_3|X_2) = I(X_1; X_3; X_4|X_2)+I(X_1; X_3|X_2, X_4).$$

Let $I(X_1; X_3|X_2, X_4) = a \geq 0$. Then

$$I(X_1; X_3; X_4|X_2) = -a.$$

2. The Markov subchain $X_1 \to X_2 \to X_4$ implies

$$0 = I(X_1; X_4|X_2) = I(X_1; X_3; X_4|X_2)+I(X_1; X_4|X_2, X_3).$$

Since $I(X_1; X_3; X_4|X_2) = -a$,

$$I(X_1; X_4|X_2, X_3) = a.$$

3. The Markov subchain $X_1 \to X_3 \to X_4$ implies

$$0 = I(X_1; X_4|X_3) = I(X_1; X_2; X_4|X_3)+I(X_1; X_4|X_2, X_3).$$

Since $I(X_1; X_4|X_2, X_3) = a$,

$$I(X_1; X_2; X_4|X_3) = -a.$$

4. The Markov subchain $X_2 \to X_3 \to X_4$ implies

$$0 = I(X_2; X_4|X_3) = I(X_1; X_2; X_4|X_3)+I(X_2; X_4|X_1, X_3).$$

Since $I(X_1; X_2; X_4|X_3) = -a$,

$$I(X_2; X_4|X_1, X_3) = a.$$

5. The Markov subchain $(X_1, X_2) \to X_3 \to X_4$ implies

$$0 = I(X_1, X_2; X_4|X_3) = I(X_1; X_4|X_2, X_3) + I(X_1; X_2; X_4|X_3) + I(X_2; X_4|X_1, X_3).$$

Then

$$0 = a - a + a = a.$$

Therefore $a = 0$, and so μ^* vanishes on the corresponding 5 atoms as shown in the information diagram.