Lemma 2.29 (Fundamental Inequality) For any $a > 0$,

$$\ln a \leq a - 1$$

with equality if and only if $a = 1$.

Corollary 2.30 For any $a > 0$,

$$\ln a \geq 1 - \frac{1}{a}$$

with equality if and only if $a = 1$.

Proof Let $a = \frac{1}{b}$ in the fundamental inequality, where $b > 0$. Then

$$\begin{aligned} \ln \frac{1}{b} &\leq \frac{1}{b} - 1 \\ -\ln b &\leq \frac{1}{b} - 1 \\ \ln b &\geq 1 - \frac{1}{b} \end{aligned}$$

Equality holds if and only if $\frac{1}{b} = a = 1$, or $b = 1$.