Theorem 2.32 (Log-Sum Inequality) For positive numbers $a_1, a_2, \cdots$ and nonnegative numbers $b_1, b_2, \cdots$ such that $\sum_i a_i < \infty$ and $0 < \sum_i b_i < \infty$,

$$\sum_i a_i \log \frac{a_i}{b_i} \geq \left(\sum_i a_i \right) \log \frac{\sum_i a_i}{\sum_i b_i}$$

with the convention that $\log \frac{a_i}{0} = \infty$. Moreover, equality holds if and only if $\frac{a_i}{b_i} = \textit{constant}$ for all i.

Example:

$$a_1 \log \frac{a_1}{b_1} + a_2 \log \frac{a_2}{b_2} \geq (a_1 + a_2) \log \frac{a_1 + a_2}{b_1 + b_2}.$$