- Let $\mathcal{X} = \{1, 2, \cdots, \}$, a countably infinite alphabet.
- Let $P_X = \{1, 0, 0, \cdots\}$, and let

$$P_{X_n} = \left\{1 - \frac{1}{\sqrt{\log n}}, \frac{1}{n\sqrt{\log n}}, \cdots, \frac{1}{n\sqrt{\log n}}, 0, 0, \cdots\right\}.$$

- As $n \to \infty$,

$$V(P_X, P_{X_n}) = \sum_i |P_X(i) - P_{X_n}(i)| = \frac{2}{\sqrt{\log n}} \to 0.$$

- However,

$$H\left(\lim_{n\to\infty} P_{X_n}\right) = H(P_X) = 0$$

but

$$\lim_{n\to\infty} H(P_{X_n}) = \infty.$$