Definition 2.20 For random variables X, Y and Z, the mutual information between X and Y conditioning on Z is defined as

$$I(X;Y|Z) = \sum_{x,y,z} p(x,y,z) \log \frac{p(x,y|z)}{p(x|z)p(y|z)} = E \log \frac{p(X,Y|Z)}{p(X|Z)p(Y|Z)}.$$

Remark $I(X;Y|Z)$ is symmetrical in X and Y.

Similar to entropy, we have

$$I(X;Y|Z) = \sum_{z} p(z) I(X;Y|Z=z),$$

where

$$I(X;Y|Z=z) = \sum_{x,y} p(x,y|z) \log \frac{p(x,y|z)}{p(x|z)p(y|z)}.$$