

- Write

$$\begin{aligned} H(Y|X) &= - \sum_{x,y} p(x,y) \log p(y|x) \\ &= - \sum_x \sum_y p(x)p(y|x) \log p(y|x) \\ &= \sum_x p(x) \left[- \sum_y p(y|x) \log p(y|x) \right] \end{aligned}$$

- The inner sum is the entropy of Y conditioning on a fixed $x \in \mathcal{S}_X$.
- Denoting the inner sum by $H(Y|X = x)$, we have

$$H(Y|X) = \sum_x p(x) H(Y|X = x),$$