

Proposition 2.12 Let X_1, X_2, X_3 , and X_4 be random variables such that $p(x_1, x_2, x_3, x_4)$ is strictly positive. Then

$$\left. \begin{array}{l} X_1 \perp X_4 | (X_2, X_3) \\ X_1 \perp X_3 | (X_2, X_4) \end{array} \right\} \Rightarrow X_1 \perp (X_3, X_4) | X_2.$$

- See textbook for a proof of the proposition.
- Not true if p is not strictly positive!
- Let $X_1 = Y$, $X_2 = Z$, and $X_3 = X_4 = (Y, Z)$, where $Y \perp Z$
- Then $X_1 \perp X_4 | (X_2, X_3)$, $X_1 \perp X_3 | (X_2, X_4)$, but $X_1 \not\perp (X_3, X_4) | X_2$.
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- p is not strictly positive because $p(x_1, x_2, x_3, x_4) = 0$ if $x_3 \neq (x_1, x_2)$ or $x_4 \neq (x_1, x_2)$.