

Proposition 2.5 For random variables X , Y , and Z , $X \perp Z|Y$ if and only if

$$p(x, y, z) = a(x, y)b(y, z) \quad (1)$$

for all x , y , and z such that $p(y) > 0$.

Proof

A. ‘Only if’

1. Assume $p(x, y, z)$ takes the form in Definition 2.4.
2. For all x and for all y such that $p(y) > 0$, let

$$a(x, y) = \frac{p(x, y)}{p(y)}.$$

3. For all y such that $p(y) > 0$ and for and all z , let

$$b(y, z) = p(y, z).$$

4. Then (1) and hence the ‘only if’ part is proved.
5. Note that the choice of $a(x, y)$ and $b(y, z)$ is not unique. For example, one can choose

$$a(x, y) = p(x, y) \quad \text{and} \quad b(y, z) = \frac{p(y, z)}{p(y)}.$$