

Definition 2.4 (Conditional Independence) For random variables X, Y , and Z , X is independent of Z conditioning on Y , denoted by $X \perp Z|Y$, if

$$p(x, y, z) = \begin{cases} \frac{p(x, y)p(y, z)}{p(y)} = p(x, y)p(z|y) & \text{if } p(y) > 0 \\ 0 & \text{otherwise.} \end{cases}$$

Remark

- If $p(y) > 0$, then

$$p(x, y, z) = \frac{p(x, y)p(y, z)}{p(y)} = p(x, y)p(z|y).$$

- Conceptually, when $X \perp Z|Y$, X , Y , and Z are related as follows: