

**Proposition 2.8**  $X_1 \rightarrow X_2 \rightarrow \cdots \rightarrow X_n$  forms a Markov chain if and only if

$$X_1 \rightarrow X_2 \rightarrow X_3$$

$$(X_1, X_2) \rightarrow X_3 \rightarrow X_4$$

$$\vdots$$

$$(X_1, X_2, \cdots, X_{n-2}) \rightarrow X_{n-1} \rightarrow X_n$$

form Markov chains. ([Exercise](#))

**Proposition 2.9**  $X_1 \rightarrow X_2 \rightarrow \cdots \rightarrow X_n$  forms a Markov chain if and only if

$$p(x_1, x_2, \cdots, x_n) = f_1(x_1, x_2) f_2(x_2, x_3) \cdots f_{n-1}(x_{n-1}, x_n)$$

for all  $x_1, x_2, \cdots, x_n$  such that  $p(x_2), p(x_3), \cdots, p(x_{n-1}) > 0$ .