

Proposition 2.10 (Markov subchains) Let $\mathcal{N}_n = \{1, 2, \dots, n\}$ and let $X_1 \rightarrow X_2 \rightarrow \dots \rightarrow X_n$ form a Markov chain. For any subset α of \mathcal{N}_n , denote $(X_i, i \in \alpha)$ by X_α . Then for any disjoint subsets $\alpha_1, \alpha_2, \dots, \alpha_m$ of \mathcal{N}_n such that

$$k_1 < k_2 < \dots < k_m$$

for all $k_j \in \alpha_j$, $j = 1, 2, \dots, m$,

$$X_{\alpha_1} \rightarrow X_{\alpha_2} \rightarrow \dots \rightarrow X_{\alpha_m}$$

forms a Markov chain. That is, a subchain of $X_1 \rightarrow X_2 \rightarrow \dots \rightarrow X_n$ is also a Markov chain. ([Exercise](#))

Example 2.11 Let $X_1 \rightarrow X_2 \rightarrow \dots \rightarrow X_{10}$ form a Markov chain and $\alpha_1 = \{1, 2\}$, $\alpha_2 = \{4\}$, $\alpha_3 = \{6, 8\}$, and $\alpha_4 = \{10\}$ be subsets of \mathcal{N}_{10} . Then Proposition 2.10 says that

$$(X_1, X_2) \rightarrow X_4 \rightarrow (X_6, X_8) \rightarrow X_{10}$$

also forms a Markov chain.