

**Definition 2.6 (Markov Chain)** For random variables  $X_1, X_2, \dots, X_n$ , where  $n \geq 3$ ,  $X_1 \rightarrow X_2 \rightarrow \dots \rightarrow X_n$  forms a Markov chain if

$$p(x_1, x_2, \dots, x_n) = \begin{cases} p(x_1, x_2)p(x_3|x_2) \cdots p(x_n|x_{n-1}) & \text{if } p(x_2), p(x_3), \dots, p(x_{n-1}) > 0 \\ 0 & \text{otherwise.} \end{cases}$$

**Remark**  $X_1 \rightarrow X_2 \rightarrow X_3$  is equivalent to  $X_1 \perp X_3 | X_2$ .

**Proposition 2.7**  $X_1 \rightarrow X_2 \rightarrow \dots \rightarrow X_n$  forms a Markov chain if and only if  $X_n \rightarrow X_{n-1} \rightarrow \dots \rightarrow X_1$  forms a Markov chain. (Exercise)