

11. The channel capacity is equal to

$$\sum_{i=1}^k \Delta_k \log \left(1 + \frac{P_i^*}{2S_{Z,i} \Delta_k} \right) = \sum_{i=1}^k \Delta_k \log \left(1 + \frac{\frac{P_i^*}{2\Delta_k}}{S_{Z,i}} \right)$$

where by Proposition 11.23,

$$\frac{P_i^*}{2\Delta_k} = (\nu - S_{Z,i})^+ \quad \text{with} \quad \sum_{i=1}^k P_i^* = P.$$

12. As $k \rightarrow \infty$,

$$\begin{aligned} & \sum_{i=1}^k \Delta_k \log \left(1 + \frac{\frac{P_i^*}{2\Delta_k}}{S_{Z,i}} \right) \\ & \rightarrow \int_0^W \log \left(1 + \frac{(\nu - S_Z(f))^+}{S_Z(f)} \right) df \\ & = \frac{1}{2} \int_{-W}^W \log \left(1 + \frac{(\nu - S_Z(f))^+}{S_Z(f)} \right) df \end{aligned}$$

bits per unit time

since $S_Z(-f) = S_Z(f)$ (see Problem 8).

13. As $k \rightarrow \infty$,

$$\begin{aligned} \sum_{i=1}^k P_i^* &= \sum_{i=1}^k 2\Delta_k (\nu - S_{Z,i})^+ \\ &\rightarrow 2 \int_0^W (\nu - S_Z(f))^+ df \\ &\rightarrow \int_{-W}^W (\nu - S_Z(f))^+ df \end{aligned}$$

since $S_Z(f) = S_Z(-f)$.

14. Therefore,

$$\sum_{i=1}^k P_i^* = P \rightarrow \int_{-W}^W (\nu - S_Z(f))^+ df = P. \quad (1)$$

15. The optimal power allocation given in (1) has a water-filling interpretation.