

$$\begin{aligned}\int (\psi_i(t) + \psi_j(t))^2 dt &= \int [\psi_i^2(t) + \psi_j^2(t) + 2\psi_i(t)\psi_j(t)] dt \\ &= \int \psi_i^2(t) dt + \int \psi_j^2(t) dt + 2 \int \psi_i(t)\psi_j(t) dt \\ &= \int \psi_i^2(t) dt + \int \psi_j^2(t) dt\end{aligned}$$