3. Then by Rayleigh's energy theorem, we have

$$\begin{aligned}\int_{-\infty}^{\infty} \operatorname{sinc}^2(2Wt)dt &= \left(\frac{1}{2W}\right)^2 \int_{-\infty}^{\infty} \operatorname{rect}^2\left(\frac{f}{2W}\right) df \\ &= \left(\frac{1}{2W}\right)^2 (2W) \\ &= \frac{1}{2W}.\end{aligned}$$

4. It then follows that

$$\int_{-\infty}^{\infty} \psi_i^2(t)dt = \int_{-\infty}^{\infty} \psi_0^2(t)dt = 1. \tag{1}$$

5. Since (1) implies that both $\operatorname{sinc}(2Wt - i)$ and $\operatorname{sinc}(2Wt - i')$ have finite energy, we can consider their cross-correlation function

$$R_{ii'}(\tau) = \int_{-\infty}^{\infty} \operatorname{sinc}(2Wt - i)\operatorname{sinc}(2W(t - \tau) - i')dt.$$

In particular,

$$R_{ii'}(0) = \int_{-\infty}^{\infty} \operatorname{sinc}(2Wt - i)\operatorname{sinc}(2Wt - i')dt.$$

6. Now

$$\begin{aligned}&\operatorname{sinc}(2Wt - i) \\ &\rightleftharpoons\quad e^{-j2\pi f\left(\frac{i}{2W}\right)} \left(\frac{1}{2W}\right) \operatorname{rect}\left(\frac{f}{2W}\right) := G_i(f)\end{aligned}$$

and

$$\begin{aligned}&\operatorname{sinc}(2Wt - i') \\ &\rightleftharpoons\quad e^{-j2\pi f\left(\frac{i'}{2W}\right)} \left(\frac{1}{2W}\right) \operatorname{rect}\left(\frac{f}{2W}\right) := G_{i'}(f).\end{aligned}$$

7. Then we have

$$R_{ii'}(\tau) \rightleftharpoons G_i(f)G_{i'}^*(f),$$

and so

$$R_{ii'}(\tau) = \int_{-\infty}^{\infty} G_i(f)G_{i'}^*(f)e^{j2\pi f\tau} df.$$

In particular,

$$\begin{aligned}R_{ii'}(0) &= \int_{-\infty}^{\infty} G_i(f)G_{i'}^*(f)df \\ &= 0 \quad \text{for } i \neq i' \quad \text{(exercise)}\end{aligned}$$

8. Therefore, for $i \neq i'$,

$$\begin{aligned}&\int_{-\infty}^{\infty} \psi_i(t)\psi_{i'}(t)dt \\ &= 2W \int_{-\infty}^{\infty} \operatorname{sinc}(2Wt - i)\operatorname{sinc}(2Wt - i')dt \\ &= (2W)R_{ii'}(0) \\ &= 0.\end{aligned}$$